CHAPTER 5
INTRODUCTION TO VALUATION: THE TIME VALUE OF MONEY

Answers to Concepts Review and Critical Thinking Questions

1. The four parts are the present value (PV), the future value (FV), the discount rate (*r*), and the life of the investment (*t*).

2. Compounding refers to the growth of a dollar amount through time via reinvestment of interest earned. It is also the process of determining the future value of an investment. Discounting is the process of determining the value today of an amount to be received in the future.

3. Future values grow (assuming a positive rate of return); present values shrink.

4. The future value rises (assuming it's positive); the present value falls.

5. It would appear to be both deceptive and unethical to run such an ad without a disclaimer or explanation.

6. It's a reflection of the time value of money. TMCC gets to use the $24,099. If TMCC uses it wisely, it will be worth more than $100,000 in thirty years.

7. This will probably make the security less desirable. TMCC will only repurchase the security prior to maturity if it is to its advantage, i.e. interest rates decline. Given the drop in interest rates needed to make this viable for TMCC, it is unlikely the company will repurchase the security. This is an example of a "call" feature. Such features are discussed at length in a later chapter.

8. The key considerations would be: (1) Is the rate of return implicit in the offer attractive relative to other, similar risk investments? and (2) How risky is the investment; i.e., how certain are we that we will actually get the $100,000? Thus, our answer does depend on who is making the promise to repay.

9. The Treasury security would have a somewhat higher price because the Treasury is the strongest of all borrowers.

10. The price would be higher because, as time passes, the price of the security will tend to rise toward $100,000. This rise is just a reflection of the time value of money. As time passes, the time until receipt of the $100,000 grows shorter, and the present value rises. In 2019, the price will probably be higher for the same reason. We cannot be sure, however, because interest rates could be much higher, or TMCC's financial position could deteriorate. Either event would tend to depress the security's price.

Solutions to Questions and Problems

NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The simple interest per year is:

$5,000 × .08 = $400

So after 10 years you will have:

$400 × 10 = $4,000 in interest.

The total balance will be $5,000 + 4,000 = $9,000

With compound interest we use the future value formula:

$FV = PV(1 + r)^t$
$FV = \$5,000(1.08)^{10} = \$10,794.62$

The difference is:

$10,794.62 – 9,000 = $1,794.62

2. To find the FV of a lump sum, we use:

$FV = PV(1 + r)^t$

$FV = \$2,250(1.10)^{11}$ = $ 6,419.51
$FV = \$8,752(1.08)^{7}$ = $ 14,999.39
$FV = \$76,355(1.17)^{14}$ = $687,764.17
$FV = \$183,796(1.07)^{8}$ = $315,795.75

3. To find the PV of a lump sum, we use:

$PV = FV / (1 + r)^t$

$PV = \$15,451 / (1.07)^{6}$ = $ 10,295.65
$PV = \$51,557 / (1.13)^{7}$ = $ 21,914.85
$PV = \$886,073 / (1.14)^{23}$ = $ 43,516.90
$PV = \$550,164 / (1.09)^{18}$ = $116,631.32

4. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for *r*, we get:

$r = (FV / PV)^{1/t} - 1$

$FV = \$297 = \$240(1 + r)^2$;	$r = (\$297 / \$240)^{1/2} - 1$	= 11.24%
$FV = \$1,080 = \$360(1 + r)^{10}$;	$r = (\$1,080 / \$360)^{1/10} - 1$	= 11.61%
$FV = \$185,382 = \$39,000(1 + r)^{15}$;	$r = (\$185,382 / \$39,000)^{1/15} - 1$	= 10.95%
$FV = \$531,618 = \$38,261(1 + r)^{30}$;	$r = (\$531,618 / \$38,261)^{1/30} - 1$	= 9.17%

5. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for *t*, we get:

$t = \ln(FV / PV) / \ln(1 + r)$

$FV = \$1,284 = \$560(1.09)^t$;	$t = \ln(\$1,284/ \$560) / \ln 1.09$	= 9.63 years
$FV = \$4,341 = \$810(1.10)^t$;	$t = \ln(\$4,341/ \$810) / \ln 1.10$	= 17.61 years
$FV = \$364,518 = \$18,400(1.17)^t$;	$t = \ln(\$364,518 / \$18,400) / \ln 1.17$	= 19.02 years
$FV = \$173,439 = \$21,500(1.15)^t$;	$t = \ln(\$173,439 / \$21,500) / \ln 1.15$	= 14.94 years

6. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for *r*, we get:

$r = (FV / PV)^{1/t} - 1$
$r = (\$290,000 / \$55,000)^{1/18} - 1 = .0968$ or 9.68%

7. To find the length of time for money to double, triple, etc., the present value and future value are irrelevant as long as the future value is twice the present value for doubling, three times as large for tripling, etc. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for *t*, we get:

$t = \ln(FV / PV) / \ln(1 + r)$

The length of time to double your money is:

$FV = \$2 = \$1(1.07)^t$
$t = \ln 2 / \ln 1.07 = 10.24$ years

The length of time to quadruple your money is:

$FV = \$4 = \$1(1.07)^t$
$t = \ln 4 / \ln 1.07 = 20.49$ years

Notice that the length of time to quadruple your money is twice as long as the time needed to double your money (the difference in these answers is due to rounding). This is an important concept of time value of money.

8. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for *r*, we get:

$r = (FV / PV)^{1 / t} - 1$
$r = (\$314{,}600 / \$200{,}300)^{1/7} - 1 = .0666$ or 6.66%

9. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for *t*, we get:

$t = \ln(FV / PV) / \ln(1 + r)$
$t = \ln (\$170{,}000 / \$40{,}000) / \ln 1.053 = 28.02$ years

10. To find the PV of a lump sum, we use:

$PV = FV / (1 + r)^t$
$PV = \$650{,}000{,}000 / (1.074)^{20} = \$155{,}893{,}400.13$

11. To find the PV of a lump sum, we use:

$PV = FV / (1 + r)^t$
$PV = \$1{,}000{,}000 / (1.10)^{80} = \$488.19$

12. To find the FV of a lump sum, we use:

$FV = PV(1 + r)^t$
$FV = \$50(1.045)^{105} = \$5{,}083.71$

13. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for *r*, we get:

$r = (FV / PV)^{1/t} - 1$
$r = (\$1{,}260{,}000 / \$150)^{1/112} - 1 = .0840$ or 8.40%

To find the FV of the first prize, we use:

$FV = PV(1 + r)^t$
$FV = \$1{,}260{,}000(1.0840)^{33} = \$18{,}056{,}409.94$

14. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for *r*, we get:

$r = (FV / PV)^{1/t} - 1$
$r = (\$43{,}125 / \$1)^{1/113} - 1 = .0990$ or 9.90%

15. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for *r*, we get:

$r = (FV / PV)^{1/t} - 1$
$r = (\$10{,}311{,}500 / \$12{,}377{,}500)^{1/4} - 1 = -4.46\%$

Notice that the interest rate is negative. This occurs when the FV is less than the PV.

Intermediate

16. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for *r*, we get:

$r = (FV / PV)^{1/t} - 1$

a. $PV = \$100,000 / (1 + r)^{30} = \$24,099$
$r = (\$100,000 / \$24,099)^{1/30} - 1 = .0486$ or 4.86%

b. $PV = \$38,260 / (1 + r)^{12} = \$24,099$
$r = (\$38,260 / \$24,099)^{1/12} - 1 = .0393$ or 3.93%

c. $PV = \$100,000 / (1 + r)^{18} = \$38,260$
$r = (\$100,000 / \$38,260)^{1/18} - 1 = .0548$ or 5.48%

17. To find the PV of a lump sum, we use:

$PV = FV / (1 + r)^t$
$PV = \$170,000 / (1.12)^9 = \$61,303.70$

18. To find the FV of a lump sum, we use:

$FV = PV(1 + r)^t$

$FV = \$4,000(1.11)^{45} = \$438,120.97$

$FV = \$4,000(1.11)^{35} = \$154,299.40$

Better start early!

19. We need to find the FV of a lump sum. However, the money will only be invested for six years, so the number of periods is six.

$FV = PV(1 + r)^t$
$FV = \$20,000(1.084)^6 = \$32,449.33$

20. To answer this question, we can use either the FV or the PV formula. Both will give the same answer since they are the inverse of each other. We will use the FV formula, that is:

$FV = PV(1 + r)^t$

Solving for *t*, we get:

$t = \ln(FV / PV) / \ln(1 + r)$
$t = \ln(\$75{,}000 / \$10{,}000) / \ln(1.11) = 19.31$

So, the money must be invested for 19.31 years. However, you will not receive the money for another two years. From now, you'll wait:

2 years + 19.31 years = 21.31 years

Calculator Solutions

1.

	N	I/Y	PV	PMT	FV
Enter	10	8%	\$5,000		
Solve for					\$10,794.62

\$10,794.62 – 9,000 = \$1,794.62

2.

	N	I/Y	PV	PMT	FV
Enter	11	10%	\$2,250		
Solve for					\$6,419.51

	N	I/Y	PV	PMT	FV
Enter	7	8%	\$8,752		
Solve for					\$14,999.39

	N	I/Y	PV	PMT	FV
Enter	14	17%	\$76,355		
Solve for					\$687,764.17

	N	I/Y	PV	PMT	FV
Enter	8	7%	\$183,796		
Solve for					\$315,795.75

3.

	N	I/Y	PV	PMT	FV
Enter	6	7%			\$15,451
Solve for			\$10,295.65		

	N	I/Y	PV	PMT	FV
Enter	7	13%			$51,557
Solve for			$21,914.85		

	N	I/Y	PV	PMT	FV
Enter	23	14%			$886,073
Solve for			$43,516.90		

	N	I/Y	PV	PMT	FV
Enter	18	9%			$550,164
Solve for			$116,631.32		

4.

	N	I/Y	PV	PMT	FV
Enter	2		$240		±$297
Solve for		11.24%			

	N	I/Y	PV	PMT	FV
Enter	10		$360		±$1,080
Solve for		11.61%			

	N	I/Y	PV	PMT	FV
Enter	15		$39,000		±$185,382
Solve for		10.95%			

	N	I/Y	PV	PMT	FV
Enter	30		$38,261		±$531,618
Solve for		9.17%			

5.

	N	I/Y	PV	PMT	FV
Enter		9%	$560		±$1,284
Solve for	9.63				

	N	I/Y	PV	PMT	FV
Enter		10%	$810		±$4,341
Solve for	17.61				

	N	I/Y	PV	PMT	FV
Enter		17%	$18,400		±$364,518
Solve for	19.02				

	N	I/Y	PV	PMT	FV
Enter		15%	$21,500		±$173,439
Solve for	14.94				

6.

	N	I/Y	PV	PMT	FV
Enter	18		$55,000		±$290,000
Solve for		9.68%			

7.

	N	I/Y	PV	PMT	FV
Enter		7%	$1		±$2
Solve for	10.24				

	N	I/Y	PV	PMT	FV
Enter		7%	$1		±$4
Solve for	20.49				

8.

	N	I/Y	PV	PMT	FV
Enter	7		$200,300		±$314,600
Solve for		6.66%			

9.

	N	I/Y	PV	PMT	FV
Enter		5.30%	$40,000		±$170,000
Solve for	28.02				

10.

	N	I/Y	PV	PMT	FV
Enter	20	7.4%			$650,000,000
Solve for			$155,893,400.13		

11.

	N	I/Y	PV	PMT	FV
Enter	80	10%			$1,000,000
Solve for			$488.19		

12.

	N	I/Y	PV	PMT	FV
Enter	105	4.50%	$50		
Solve for					$5,083.71

13.

	N	I/Y	PV	PMT	FV
Enter	112		±$150		$1,260,000
Solve for		8.40%			

	N	I/Y	PV	PMT	FV
Enter	33	8.40%	$1,260,000		
Solve for					$18,056,404.94

14.

	N	I/Y	PV	PMT	FV
Enter	113		$1		±$43,125
Solve for		9.90%			

15.

	N	I/Y	PV	PMT	FV
Enter	4		±$12,377,500		$10,311,500
Solve for		–4.46%			

16. *a.*

	N	I/Y	PV	PMT	FV
Enter	30		±$24,099		$100,000
Solve for		4.86%			

16. *b.*

	N	I/Y	PV	PMT	FV
Enter	12		±$24,099		$38,260
Solve for		3.93%			

16. *c.*

	N	I/Y	PV	PMT	FV
Enter	18		±$38,260		$100,000
Solve for		5.48%			

17.

	N	I/Y	PV	PMT	FV
Enter	9	12%			$170,000
Solve for			$61,303.70		

18.

	N	I/Y	PV	PMT	FV
Enter	45	11%	$4,000		
Solve for					$438,120.97

	N	I/Y	PV	PMT	FV
Enter	35	11%	$4,000		
Solve for					$154,299.40

19.

Enter	6	8.40%	$20,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$32,449.33

20.

Enter		11%	±$10,000		$75,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for	19.31				

From now, you'll wait 2 + 19.31 = 21.31 years

CHAPTER 6
DISCOUNTED CASH FLOW VALUATION

Answers to Concepts Review and Critical Thinking Questions

1. The four pieces are the present value (PV), the periodic cash flow (*C*), the discount rate (*r*), and the number of payments, or the life of the annuity, *t*.

2. Assuming positive cash flows, both the present and the future values will rise.

3. Assuming positive cash flows, the present value will fall and the future value will rise.

4. It's deceptive, but very common. The basic concept of time value of money is that a dollar today is not worth the same as a dollar tomorrow. The deception is particularly irritating given that such lotteries are usually government sponsored!

5. If the total money is fixed, you want as much as possible as soon as possible. The team (or, more accurately, the team owner) wants just the opposite.

6. The better deal is the one with equal installments.

7. Yes, they should. APRs generally don't provide the relevant rate. The only advantage is that they are easier to compute, but, with modern computing equipment, that advantage is not very important.

8. A freshman does. The reason is that the freshman gets to use the money for much longer before interest starts to accrue. The subsidy is the present value (on the day the loan is made) of the interest that would have accrued up until the time it actually begins to accrue.

9. The problem is that the subsidy makes it easier to repay the loan, not obtain it. However, ability to repay the loan depends on future employment, not current need. For example, consider a student who is currently needy, but is preparing for a career in a high-paying area (such as corporate finance!). Should this student receive the subsidy? How about a student who is currently not needy, but is preparing for a relatively low-paying job (such as becoming a college professor)?

10. In general, viatical settlements are ethical. In the case of a viatical settlement, it is simply an exchange of cash today for payment in the future, although the payment depends on the death of the seller. The purchaser of the life insurance policy is bearing the risk that the insured individual will live longer than expected. Although viatical settlements are ethical, they may not be the best choice for an individual. In a *Business Week* article (October 31, 2005), options were examined for a 72 year old male with a life expectancy of 8 years and a $1 million dollar life insurance policy with an annual premium of $37,000. The four options were: 1) Cash the policy today for $100,000. 2) Sell the policy in a viatical settlement for $275,000. 3) Reduce the death benefit to $375,000, which would keep the policy in force for 12 years without premium payments. 4) Stop paying premiums and don't reduce the death benefit. This will run the cash value of the policy to zero in 5 years, but the viatical settlement would be worth $475,000 at that time. If he died within 5 years, the beneficiaries would receive $1 million. Ultimately, the decision rests on the individual on what they perceive as best for themselves. The values that will affect the value of the viatical settlement are the discount rate, the face value of the policy, and the health of the individual selling the policy.

Solutions to Questions and Problems

NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. To solve this problem, we must find the PV of each cash flow and add them. To find the PV of a lump sum, we use:

$PV = FV / (1 + r)^t$

PV@10% = $950 / 1.10 + $1,040 / 1.10^2 + $1,130 / 1.10^3 + $1,075 / 1.10^4 = $3,306.37

PV@18% = $950 / 1.18 + $1,040 / 1.18^2 + $1,130 / 1.18^3 + $1,075 / 1.18^4 = $2,794.22

PV@24% = $950 / 1.24 + $1,040 / 1.24^2 + $1,130 / 1.24^3 + $1,075 / 1.24^4 = $2,489.88

2. To find the PVA, we use the equation:

$PVA = C(\{1 - [1/(1 + r)]^t\} / r)$

At a 5 percent interest rate:

X@5%: PVA = $6,000{[1 – $(1/1.05)^9$] / .05 } = $42,646.93

Y@5%: PVA = $8,000{[1 – $(1/1.05)^6$] / .05 } = $40,605.54

And at a 15 percent interest rate:

X@15%: $PVA = \$6{,}000\{[1 - (1/1.15)^{9}] / .15\} = \$28{,}629.50$

Y@15%: $PVA = \$8{,}000\{[1 - (1/1.15)^{6}] / .15\} = \$30{,}275.86$

Notice that the PV of cash flow X has a greater PV at a 5 percent interest rate, but a lower PV at a 15 percent interest rate. The reason is that X has greater total cash flows. At a lower interest rate, the total cash flow is more important since the cost of waiting (the interest rate) is not as great. At a higher interest rate, Y is more valuable since it has larger cash flows. At the higher interest rate, these bigger cash flows early are more important since the cost of waiting (the interest rate) is so much greater.

3. To solve this problem, we must find the FV of each cash flow and add them. To find the FV of a lump sum, we use:

$FV = PV(1 + r)^{t}$

FV@8% $= \$940(1.08)^{3} + \$1{,}090(1.08)^{2} + \$1{,}340(1.08) + \$1{,}405 = \$5{,}307.71$

FV@11% $= \$940(1.11)^{3} + \$1{,}090(1.11)^{2} + \$1{,}340(1.11) + \$1{,}405 = \$5{,}520.96$

FV@24% $= \$940(1.24)^{3} + \$1{,}090(1.24)^{2} + \$1{,}340(1.24) + \$1{,}405 = \$6{,}534.81$

Notice we are finding the value at Year 4, the cash flow at Year 4 is simply added to the FV of the other cash flows. In other words, we do not need to compound this cash flow.

4. To find the PVA, we use the equation:

$PVA = C(\{1 - [1/(1 + r)]^{t}\} / r)$

PVA@15 yrs: $PVA = \$5{,}300\{[1 - (1/1.07)^{15}] / .07\} = \$48{,}271.94$

PVA@40 yrs: $PVA = \$5{,}300\{[1 - (1/1.07)^{40}] / .07\} = \$70{,}658.06$

PVA@75 yrs: $PVA = \$5{,}300\{[1 - (1/1.07)^{75}] / .07\} = \$75{,}240.70$

To find the PV of a perpetuity, we use the equation:

$PV = C / r$

$PV = \$5{,}300 / .07 = \$75{,}714.29$

Notice that as the length of the annuity payments increases, the present value of the annuity approaches the present value of the perpetuity. The present value of the 75 year annuity and the present value of the perpetuity imply that the value today of all perpetuity payments beyond 75 years is only $473.59.

5. Here we have the PVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the PVA equation:

$\text{PVA} = C(\{1 - [1/(1 + r)]^t\} / r)$
$\text{PVA} = \$34{,}000 = \$C\{[1 - (1/1.0765)^{15}] / .0765\}$

We can now solve this equation for the annuity payment. Doing so, we get:

$C = \$34{,}000 / 8.74548 = \$3{,}887.72$

6. To find the PVA, we use the equation:

$\text{PVA} = C(\{1 - [1/(1 + r)]^t\} / r)$
$\text{PVA} = \$73{,}000\{[1 - (1/1.085)^8] / .085\} = \$411{,}660.36$

7. Here we need to find the FVA. The equation to find the FVA is:

$\text{FVA} = C\{[(1 + r)^t - 1] / r\}$

FVA for 20 years $= \$4{,}000[(1.112^{20} - 1) / .112] = \$262{,}781.16$

FVA for 40 years $= \$4{,}000[(1.112^{40} - 1) / .112] = \$2{,}459{,}072.63$

Notice that because of exponential growth, doubling the number of periods does not merely double the FVA.

8. Here we have the FVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the FVA equation:

$\text{FVA} = C\{[(1 + r)^t - 1] / r\}$
$\$90{,}000 = \$C[(1.068^{10} - 1) / .068]$

We can now solve this equation for the annuity payment. Doing so, we get:

$C = \$90{,}000 / 13.68662 = \$6{,}575.77$

9. Here we have the PVA, the length of the annuity, and the interest rate. We want to calculate the annuity payment. Using the PVA equation:

$\text{PVA} = C(\{1 - [1/(1 + r)]^t\} / r)$
$\$50{,}000 = C\{[1 - (1/1.075)^7] / .075\}$

We can now solve this equation for the annuity payment. Doing so, we get:

$C = \$50{,}000 / 5.29660 = \$9{,}440.02$

10. This cash flow is a perpetuity. To find the PV of a perpetuity, we use the equation:

$\text{PV} = C / r$
$\text{PV} = \$25{,}000 / .072 = \$347{,}222.22$

11. Here we need to find the interest rate that equates the perpetuity cash flows with the PV of the cash flows. Using the PV of a perpetuity equation:

PV = C / r
\$375,000 = \$25,000 / r

We can now solve for the interest rate as follows:

r = \$25,000 / \$375,000 = .0667 or 6.67%

12. For discrete compounding, to find the EAR, we use the equation:

$$EAR = [1 + (APR / m)]^m - 1$$

$$EAR = [1 + (.08 / 4)]^4 - 1 = .0824 \text{ or } 8.24\%$$

$$EAR = [1 + (.16 / 12)]^{12} - 1 = .1723 \text{ or } 17.23\%$$

$$EAR = [1 + (.12 / 365)]^{365} - 1 = .1275 \text{ or } 12.75\%$$

To find the EAR with continuous compounding, we use the equation:

$$EAR = e^q - 1$$
$$EAR = e^{.15} - 1 = .1618 \text{ or } 16.18\%$$

13. Here we are given the EAR and need to find the APR. Using the equation for discrete compounding:

$$EAR = [1 + (APR / m)]^m - 1$$

We can now solve for the APR. Doing so, we get:

$$APR = m[(1 + EAR)^{1/m} - 1]$$

$$EAR = .0860 = [1 + (APR / 2)]^2 - 1 \qquad APR = 2[(1.0860)^{1/2} - 1] = .0842 \text{ or } 8.42\%$$

$$EAR = .1980 = [1 + (APR / 12)]^{12} - 1 \qquad APR = 12[(1.1980)^{1/12} - 1] = .1820 \text{ or } 18.20\%$$

$$EAR = .0940 = [1 + (APR / 52)]^{52} - 1 \qquad APR = 52[(1.0940)^{1/52} - 1] = .0899 \text{ or } 8.99\%$$

Solving the continuous compounding EAR equation:

$$EAR = e^q - 1$$

We get:

$$APR = \ln(1 + EAR)$$
$$APR = \ln(1 + .1650)$$
$$APR = .1527 \text{ or } 15.27\%$$

14. For discrete compounding, to find the EAR, we use the equation:

$EAR = [1 + (APR / m)]^m - 1$

So, for each bank, the EAR is:

First National: $EAR = [1 + (.1420 / 12)]^{12} - 1 = .1516$ or 15.16%

First United: $EAR = [1 + (.1450 / 2)]^2 - 1 = .1503$ or 15.03%

Notice that the higher APR does not necessarily mean the higher EAR. The number of compounding periods within a year will also affect the EAR.

15. The reported rate is the APR, so we need to convert the EAR to an APR as follows:

$EAR = [1 + (APR / m)]^m - 1$

$APR = m[(1 + EAR)^{1/m} - 1]$
$APR = 365[(1.16)^{1/365} - 1] = .1485$ or 14.85%

This is deceptive because the borrower is actually paying annualized interest of 16% per year, not the 14.85% reported on the loan contract.

16. For this problem, we simply need to find the FV of a lump sum using the equation:

$FV = PV(1 + r)^t$

It is important to note that compounding occurs semiannually. To account for this, we will divide the interest rate by two (the number of compounding periods in a year), and multiply the number of periods by two. Doing so, we get:

$FV = \$2,100[1 + (.084/2)]^{34} = \$8,505.93$

17. For this problem, we simply need to find the FV of a lump sum using the equation:

$FV = PV(1 + r)^t$

It is important to note that compounding occurs daily. To account for this, we will divide the interest rate by 365 (the number of days in a year, ignoring leap year), and multiply the number of periods by 365. Doing so, we get:

FV in 5 years $= \$4,500[1 + (.093/365)]^{5(365)} = \$7,163.64$

FV in 10 years $= \$4,500[1 + (.093/365)]^{10(365)} = \$11,403.94$

FV in 20 years $= \$4,500[1 + (.093/365)]^{20(365)} = \$28,899.97$

18. For this problem, we simply need to find the PV of a lump sum using the equation:

$PV = FV / (1 + r)^t$

It is important to note that compounding occurs daily. To account for this, we will divide the interest rate by 365 (the number of days in a year, ignoring leap year), and multiply the number of periods by 365. Doing so, we get:

$PV = \$58,000 / [(1 + .10/365)^{7(365)}] = \$28,804.71$

19. The APR is simply the interest rate per period times the number of periods in a year. In this case, the interest rate is 30 percent per month, and there are 12 months in a year, so we get:

$APR = 12(30\%) = 360\%$

To find the EAR, we use the EAR formula:

$EAR = [1 + (APR / m)]^m - 1$

$EAR = (1 + .30)^{12} - 1 = 2,229.81\%$

Notice that we didn't need to divide the APR by the number of compounding periods per year. We do this division to get the interest rate per period, but in this problem we are already given the interest rate per period.

20. We first need to find the annuity payment. We have the PVA, the length of the annuity, and the interest rate. Using the PVA equation:

$PVA = C(\{1 - [1/(1 + r)]^t\} / r)$
$\$68,500 = \$C[1 - \{1 / [1 + (.069/12)]^{60}\} / (.069/12)]$

Solving for the payment, we get:

$C = \$68,500 / 50.622252 = \$1,353.15$

To find the EAR, we use the EAR equation:

$EAR = [1 + (APR / m)]^m - 1$
$EAR = [1 + (.069 / 12)]^{12} - 1 = .0712$ or 7.12%

21. Here we need to find the length of an annuity. We know the interest rate, the PV, and the payments. Using the PVA equation:

$PVA = C(\{1 - [1/(1 + r)]^t\} / r)$
$\$18,000 = \$500\{[1 - (1/1.013)^t] / .013\}$

Now we solve for *t*:

$1/1.013^t = 1 - \{[(\$18,000)/(\$500)](.013)\}$
$1/1.013^t = 0.532$
$1.013^t = 1/(0.532) = 1.8797$
$t = \ln 1.8797 / \ln 1.013 = 48.86$ months

22. Here we are trying to find the interest rate when we know the PV and FV. Using the FV equation:

$FV = PV(1 + r)$
$\$4 = \$3(1 + r)$
$r = 4/3 - 1 = 33.33\%$ per week

The interest rate is 33.33% per week. To find the APR, we multiply this rate by the number of weeks in a year, so:

APR = (52)33.33% = 1,733.33%

And using the equation to find the EAR:

$EAR = [1 + (APR / m)]^m - 1$
$EAR = [1 + .3333]^{52} - 1 = 313,916,515.69\%$

23. Here we need to find the interest rate that equates the perpetuity cash flows with the PV of the cash flows. Using the PV of a perpetuity equation:

$PV = C / r$
$\$95,000 = \$1,800 / r$

We can now solve for the interest rate as follows:

$r = \$1,800 / \$95,000 = .0189$ or 1.89% per month

The interest rate is 1.89% per month. To find the APR, we multiply this rate by the number of months in a year, so:

APR = (12)1.89% = 22.74%

And using the equation to find an EAR:

$EAR = [1 + (APR / m)]^m - 1$
$EAR = [1 + .0189]^{12} - 1 = 25.26\%$

24. This problem requires us to find the FVA. The equation to find the FVA is:

$FVA = C\{[(1 + r)^t - 1] / r\}$
$FVA = \$300[\{[1 + (.10/12)]^{360} - 1\} / (.10/12)] = \$678,146.38$

25. In the previous problem, the cash flows are monthly and the compounding period is monthly. This assumption still holds. Since the cash flows are annual, we need to use the EAR to calculate the future value of annual cash flows. It is important to remember that you have to make sure the compounding periods of the interest rate is the same as the timing of the cash flows. In this case, we have annual cash flows, so we need the EAR since it is the true annual interest rate you will earn. So, finding the EAR:

$EAR = [1 + (APR / m)]^m - 1$
$EAR = [1 + (.10/12)]^{12} - 1 = .1047$ or 10.47%

Using the FVA equation, we get:

$FVA = C\{[(1 + r)^t - 1] / r\}$
$FVA = \$3{,}600[(1.1047^{30} - 1) / .1047] = \$647{,}623.45$

26. The cash flows are simply an annuity with four payments per year for four years, or 16 payments. We can use the PVA equation:

$PVA = C(\{1 - [1/(1 + r)]^t\} / r)$
$PVA = \$2{,}300\{[1 - (1/1.0065)^{16}] / .0065\} = \$34{,}843.71$

27. The cash flows are annual and the compounding period is quarterly, so we need to calculate the EAR to make the interest rate comparable with the timing of the cash flows. Using the equation for the EAR, we get:

$EAR = [1 + (APR / m)]^m - 1$
$EAR = [1 + (.11/4)]^4 - 1 = .1146$ or 11.46%

And now we use the EAR to find the PV of each cash flow as a lump sum and add them together:

$PV = \$725 / 1.1146 + \$980 / 1.1146^2 + \$1{,}360 / 1.1146^4 = \$2{,}320.36$

28. Here the cash flows are annual and the given interest rate is annual, so we can use the interest rate given. We simply find the PV of each cash flow and add them together.

$PV = \$1{,}650 / 1.0845 + \$4{,}200 / 1.0845^3 + \$2{,}430 / 1.0845^4 = \$6{,}570.86$

Intermediate

29. The total interest paid by First Simple Bank is the interest rate per period times the number of periods. In other words, the interest by First Simple Bank paid over 10 years will be:

$.07(10) = .7$

First Complex Bank pays compound interest, so the interest paid by this bank will be the FV factor of $1, or:

$(1 + r)^{10}$

Setting the two equal, we get:

$(.07)(10) = (1 + r)^{10} - 1$

$r = 1.7^{1/10} - 1 = .0545$ or 5.45%

30. Here we need to convert an EAR into interest rates for different compounding periods. Using the equation for the EAR, we get:

$EAR = [1 + (APR / m)]^m - 1$

EAR = .17 = $(1 + r)^2 - 1$; $r = (1.17)^{1/2} - 1 = .0817$ or 8.17% per six months

EAR = .17 = $(1 + r)^4 - 1$; $r = (1.17)^{1/4} - 1 = .0400$ or 4.00% per quarter

EAR = .17 = $(1 + r)^{12} - 1$; $r = (1.17)^{1/12} - 1 = .0132$ or 1.32% per month

Notice that the effective six month rate is not twice the effective quarterly rate because of the effect of compounding.

31. Here we need to find the FV of a lump sum, with a changing interest rate. We must do this problem in two parts. After the first six months, the balance will be:

$FV = \$5{,}000 [1 + (.015/12)]^6 = \$5{,}037.62$

This is the balance in six months. The FV in another six months will be:

$FV = \$5{,}037.62[1 + (.18/12)]^6 = \$5{,}508.35$

The problem asks for the interest accrued, so, to find the interest, we subtract the beginning balance from the FV. The interest accrued is:

Interest = \$5,508.35 – 5,000.00 = \$508.35

32. We need to find the annuity payment in retirement. Our retirement savings ends and the retirement withdrawals begin, so the PV of the retirement withdrawals will be the FV of the retirement savings. So, we find the FV of the stock account and the FV of the bond account and add the two FVs.

Stock account: $FVA = \$700[\{[1 + (.11/12)]^{360} - 1\} / (.11/12)] = \$1{,}963{,}163.82$

Bond account: $FVA = \$300[\{[1 + (.06/12)]^{360} - 1\} / (.06/12)] = \$301{,}354.51$

So, the total amount saved at retirement is:

\$1,963,163.82 + 301,354.51 = \$2,264,518.33

Solving for the withdrawal amount in retirement using the PVA equation gives us:

$PVA = \$2{,}264{,}518.33 = \$C[1 - \{1 / [1 + (.09/12)]^{300}\} / (.09/12)]$
$C = \$2{,}264{,}518.33 / 119.1616 = \$19{,}003.763$ withdrawal per month

33. We need to find the FV of a lump sum in one year and two years. It is important that we use the number of months in compounding since interest is compounded monthly in this case. So:

FV in one year = $\$1(1.0117)^{12} = \$1.15$

FV in two years = $\$1(1.0117)^{24} = \$1.32$

There is also another common alternative solution. We could find the EAR, and use the number of years as our compounding periods. So we will find the EAR first:

EAR = $(1 + .0117)^{12} - 1 = .1498$ or 14.98%

Using the EAR and the number of years to find the FV, we get:

FV in one year = $\$1(1.1498)^{1} = \$1.15$

FV in two years = $\$1(1.1498)^{2} = \$1.32$

Either method is correct and acceptable. We have simply made sure that the interest compounding period is the same as the number of periods we use to calculate the FV.

34. Here we are finding the annuity payment necessary to achieve the same FV. The interest rate given is a 12 percent APR, with monthly deposits. We must make sure to use the number of months in the equation. So, using the FVA equation:

Starting today:
FVA = $C[\{[1 + (.12/12)]^{480} - 1\} / (.12/12)]$
C = \$1,000,000 / 11,764.77 = \$85.00

Starting in 10 years:
FVA = $C[\{[1 + (.12/12)]^{360} - 1\} / (.12/12)]$
C = \$1,000,000 / 3,494.96 = \$286.13

Starting in 20 years:
FVA = $C[\{[1 + (.12/12)]^{240} - 1\} / (.12/12)]$
C = \$1,000,000 / 989.255 = \$1,010.86

Notice that a deposit for half the length of time, i.e. 20 years versus 40 years, does not mean that the annuity payment is doubled. In this example, by reducing the savings period by one-half, the deposit necessary to achieve the same ending value is about twelve times as large.

35. Since we are looking to quadruple our money, the PV and FV are irrelevant as long as the FV is three times as large as the PV. The number of periods is four, the number of quarters per year. So:

FV = $\$3 = \$1(1 + r)^{(12/3)}$
r = .3161 or 31.61%

36. Since we have an APR compounded monthly and an annual payment, we must first convert the interest rate to an EAR so that the compounding period is the same as the cash flows.

EAR = $[1 + (.10 / 12)]^{12} - 1 = .104713$ or 10.4713%

$PVA_1 = \$95{,}000\ \{[1 - (1 / 1.104713)^2] / .104713\} = \$163{,}839.09$

$PVA_2 = \$45{,}000 + \$70{,}000\{[1 - (1/1.104713)^2] / .104713\} = \$165{,}723.54$

You would choose the second option since it has a higher PV.

37. We can use the present value of a growing perpetuity equation to find the value of your deposits today. Doing so, we find:

$PV = C\ \{[1/(r - g)] - [1/(r - g)] \times [(1 + g)/(1 + r)]^t\}$
$PV = \$1{,}000{,}000\{[1/(.08 - .05)] - [1/(.08 - .05)] \times [(1 + .05)/(1 + .08)]^{30}\}$
$PV = \$19{,}016{,}563.18$

38. Since your salary grows at 4 percent per year, your salary next year will be:

Next year's salary = $\$50{,}000\ (1 + .04)$
Next year's salary = $\$52{,}000$

This means your deposit next year will be:

Next year's deposit = $\$52{,}000(.05)$
Next year's deposit = $\$2{,}600$

Since your salary grows at 4 percent, you deposit will also grow at 4 percent. We can use the present value of a growing perpetuity equation to find the value of your deposits today. Doing so, we find:

$PV = C\ \{[1/(r - g)] - [1/(r - g)] \times [(1 + g)/(1 + r)]^t\}$
$PV = \$2{,}600\{[1/(.11 - .04)] - [1/(.11 - .04)] \times [(1 + .04)/(1 + .11)]^{40}\}$
$PV = \$34{,}399.45$

Now, we can find the future value of this lump sum in 40 years. We find:

$FV = PV(1 + r)^t$
$FV = \$34{,}366.45(1 + .11)^{40}$
$FV = \$2{,}235{,}994.31$

This is the value of your savings in 40 years.

39. The relationship between the PVA and the interest rate is:

PVA falls as *r* increases, and PVA rises as *r* decreases
FVA rises as *r* increases, and FVA falls as *r* decreases

The present values of $9,000 per year for 10 years at the various interest rates given are:

PVA@10% = $9,000{[1 – (1/1.10)^{15}] / .10} = $68,454.72

PVA@5% = $9,000{[1 – (1/1.05)^{15}] / .05} = $93,416.92

PVA@15% = $9,000{[1 – (1/1.15)^{15}] / .15} = $52,626.33

40. Here we are given the FVA, the interest rate, and the amount of the annuity. We need to solve for the number of payments. Using the FVA equation:

FVA = $20,000 = $340[{[1 + (.06/12)]^{t} – 1 } / (.06/12)]

Solving for *t*, we get:

1.005^{t} = 1 + [($20,000)/($340)](.06/12)
t = ln 1.294118 / ln 1.005 = 51.69 payments

41. Here we are given the PVA, number of periods, and the amount of the annuity. We need to solve for the interest rate. Using the PVA equation:

PVA = $73,000 = $1,450[{1 – [1 / (1 + *r*)]^{60}}/ *r*]

To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate lowers the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

r = 0.594%

The APR is the periodic interest rate times the number of periods in the year, so:

APR = 12(0.594%) = 7.13%

42. The amount of principal paid on the loan is the PV of the monthly payments you make. So, the present value of the $1,150 monthly payments is:

PVA = $1,150[(1 – {1 / [1 + (.0635/12)]}^{360}) / (.0635/12)] = $184,817.42

The monthly payments of $1,150 will amount to a principal payment of $184,817.42. The amount of principal you will still owe is:

$240,000 – 184,817.42 = $55,182.58

This remaining principal amount will increase at the interest rate on the loan until the end of the loan period. So the balloon payment in 30 years, which is the FV of the remaining principal will be:

Balloon payment = $55,182.58[1 + (.0635/12)]^{360} = $368,936.54

43. We are given the total PV of all four cash flows. If we find the PV of the three cash flows we know, and subtract them from the total PV, the amount left over must be the PV of the missing cash flow. So, the PV of the cash flows we know are:

PV of Year 1 CF: $1,700 / 1.10 = $1,545.45

PV of Year 3 CF: $2,100 / 1.10^3 = $1,577.76

PV of Year 4 CF: $2,800 / 1.10^4 = $1,912.44

So, the PV of the missing CF is:

$6,550 – 1,545.45 – 1,577.76 – 1,912.44 = $1,514.35

The question asks for the value of the cash flow in Year 2, so we must find the future value of this amount. The value of the missing CF is:

$1,514.35(1.10)^2 = $1,832.36

44. To solve this problem, we simply need to find the PV of each lump sum and add them together. It is important to note that the first cash flow of $1 million occurs today, so we do not need to discount that cash flow. The PV of the lottery winnings is:

PV = $1,000,000 + $1,500,000/1.09 + $2,000,000/1.09^2 + $2,500,000/1.09^3 + $3,000,000/1.09^4
+ $3,500,000/1.09^5 + $4,000,000/1.09^6 + $4,500,000/1.09^7 + $5,000,000/1.09^8
+ $5,500,000/1.09^9 + $6,000,000/1.09^{10}
PV = $22,812,873.40

45. Here we are finding interest rate for an annuity cash flow. We are given the PVA, number of periods, and the amount of the annuity. We should also note that the PV of the annuity is not the amount borrowed since we are making a down payment on the warehouse. The amount borrowed is:

Amount borrowed = 0.80($2,900,000) = $2,320,000

Using the PVA equation:

$PVA = \$2,320,000 = \$15,000[\{1 - [1 / (1 + r)]^{360}\} / r]$

Unfortunately this equation cannot be solved to find the interest rate using algebra. To find the interest rate, we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. If you use trial and error, remember that increasing the interest rate lowers the PVA, and decreasing the interest rate increases the PVA. Using a spreadsheet, we find:

$r = 0.560\%$

The APR is the monthly interest rate times the number of months in the year, so:

$APR = 12(0.560\%) = 6.72\%$

And the EAR is:

$EAR = (1 + .00560)^{12} - 1 = .0693$ or 6.93%

46. The profit the firm earns is just the PV of the sales price minus the cost to produce the asset. We find the PV of the sales price as the PV of a lump sum:

$PV = \$165,000 / 1.13^4 = \$101,197.59$

And the firm's profit is:

$Profit = \$101,197.59 - 94,000.00 = \$7,197.59$

To find the interest rate at which the firm will break even, we need to find the interest rate using the PV (or FV) of a lump sum. Using the PV equation for a lump sum, we get:

$\$94,000 = \$165,000 / (1 + r)^4$
$r = (\$165,000 / \$94,000)^{1/4} - 1 = .1510$ or 15.10%

47. We want to find the value of the cash flows today, so we will find the PV of the annuity, and then bring the lump sum PV back to today. The annuity has 18 payments, so the PV of the annuity is:

$PVA = \$4,000\{[1 - (1/1.10)^{18}] / .10\} = \$32,805.65$

Since this is an ordinary annuity equation, this is the PV one period before the first payment, so it is the PV at $t = 7$. To find the value today, we find the PV of this lump sum. The value today is:

$PV = \$32,805.65 / 1.10^7 = \$16,834.48$

48. This question is asking for the present value of an annuity, but the interest rate changes during the life of the annuity. We need to find the present value of the cash flows for the last eight years first. The PV of these cash flows is:

$PVA_2 = \$1,500 [\{1 - 1 / [1 + (.07/12)]^{96}\} / (.07/12)] = \$110,021.35$

Note that this is the PV of this annuity exactly seven years from today. Now we can discount this lump sum to today. The value of this cash flow today is:

$PV = \$110,021.35 / [1 + (.11/12)]^{84} = \$51,120.33$

Now we need to find the PV of the annuity for the first seven years. The value of these cash flows today is:

$PVA_1 = \$1,500 [\{1 – 1 / [1 + (.11/12)]^{84}\} / (.11/12)] = \$87,604.36$

The value of the cash flows today is the sum of these two cash flows, so:

$PV = \$51,120.33 + 87,604.36 = \$138,724.68$

49. Here we are trying to find the dollar amount invested today that will equal the FVA with a known interest rate, and payments. First we need to determine how much we would have in the annuity account. Finding the FV of the annuity, we get:

$FVA = \$1,200 [\{[1 + (.085/12)]^{180} – 1\} / (.085/12)] = \$434,143.62$

Now we need to find the PV of a lump sum that will give us the same FV. So, using the FV of a lump sum with continuous compounding, we get:

$FV = \$434,143.62 = PVe^{.08(15)}$
$PV = \$434,143.62e^{-1.20} = \$130,761.55$

50. To find the value of the perpetuity at $t = 7$, we first need to use the PV of a perpetuity equation. Using this equation we find:

$PV = \$3,500 / .062 = \$56,451.61$

Remember that the PV of a perpetuity (and annuity) equations give the PV one period before the first payment, so, this is the value of the perpetuity at $t = 14$. To find the value at $t = 7$, we find the PV of this lump sum as:

$PV = \$56,451.61 / 1.062^7 = \$37,051.41$

51. To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The interest rate for the cash flows of the loan is:

$PVA = \$25,000 = \$2,416.67\{(1 – [1 / (1 + r)]^{12}) / r\}$

Again, we cannot solve this equation for r, so we need to solve this equation on a financial calculator, using a spreadsheet, or by trial and error. Using a spreadsheet, we find:

$r = 2.361\%$ per month

So the APR is:

APR = 12(2.361%) = 28.33%

And the EAR is:

EAR = $(1.02361)^{12} - 1 = .3231$ or 32.31%

52. The cash flows in this problem are semiannual, so we need the effective semiannual rate. The interest rate given is the APR, so the monthly interest rate is:

Monthly rate = .10 / 12 = .00833

To get the semiannual interest rate, we can use the EAR equation, but instead of using 12 months as the exponent, we will use 6 months. The effective semiannual rate is:

Semiannual rate = $(1.00833)^6 - 1 = .0511$ or 5.11%

We can now use this rate to find the PV of the annuity. The PV of the annuity is:

PVA @ year 8: $\$7{,}000\{[1 - (1 / 1.0511)^{10}] / .0511\} = \$53{,}776.72$

Note, this is the value one period (six months) before the first payment, so it is the value at year 8. So, the value at the various times the questions asked for uses this value 8 years from now.

PV @ year 5: $\$53{,}776.72 / 1.0511^6 = \$39{,}888.33$

Note, you can also calculate this present value (as well as the remaining present values) using the number of years. To do this, you need the EAR. The EAR is:

EAR = $(1 + .0083)^{12} - 1 = .1047$ or 10.47%

So, we can find the PV at year 5 using the following method as well:

PV @ year 5: $\$53{,}776.72 / 1.1047^3 = \$39{,}888.33$

The value of the annuity at the other times in the problem is:

PV @ year 3: $\$53{,}776.72 / 1.0511^{10} = \$32{,}684.88$
PV @ year 3: $\$53{,}776.72 / 1.1047^5 = \$32{,}684.88$

PV @ year 0: $\$53{,}776.72 / 1.0511^{16} = \$24{,}243.67$
PV @ year 0: $\$53{,}776.72 / 1.1047^8 = \$24{,}243.67$

53. *a.* If the payments are in the form of an ordinary annuity, the present value will be:

$PVA = C(\{1 - [1/(1 + r)^t]\} / r)$
$PVA = \$10{,}000[\{1 - [1 / (1 + .11)]^5\} / .11]$
$PVA = \$36{,}958.97$

If the payments are an annuity due, the present value will be:

$PVA_{due} = (1 + r)\ PVA$
$PVA_{due} = (1 + .11)\$36,958.97$
$PVA_{due} = \$41,024.46$

b. We can find the future value of the ordinary annuity as:

$FVA = C\{[(1 + r)^t - 1] / r\}$
$FVA = \$10,000\{[(1 + .11)^5 - 1] / .11\}$
$FVA = \$62,278.01$

If the payments are an annuity due, the future value will be:

$FVA_{due} = (1 + r)\ FVA$
$FVA_{due} = (1 + .11)\$62,278.01$
$FVA_{due} = \$69,128.60$

c. Assuming a positive interest rate, the present value of an annuity due will always be larger than the present value of an ordinary annuity. Each cash flow in an annuity due is received one period earlier, which means there is one period less to discount each cash flow. Assuming a positive interest rate, the future value of an ordinary due will always higher than the future value of an ordinary annuity. Since each cash flow is made one period sooner, each cash flow receives one extra period of compounding.

54. We need to use the PVA due equation, that is:

$PVA_{due} = (1 + r)\ PVA$

Using this equation:

$PVA_{due} = \$68,000 = [1 + (.0785/12)] \times C[\{1 - 1 / [1 + (.0785/12)]^{60}\} / (.0785/12)$

$\$67,558.06 = \$C\{1 - [1 / (1 + .0785/12)^{60}]\} / (.0785/12)$

$C = \$1,364.99$

Notice, to find the payment for the PVA due we simply compound the payment for an ordinary annuity forward one period.

55. The payment for a loan repaid with equal payments is the annuity payment with the loan value as the PV of the annuity. So, the loan payment will be:

$PVA = \$42,000 = C\ \{[1 - 1 / (1 + .08)^5] / .08\}$

$C = \$10,519.17$

The interest payment is the beginning balance times the interest rate for the period, and the principal payment is the total payment minus the interest payment. The ending balance is the beginning balance minus the principal payment. The ending balance for a period is the beginning balance for the next period. The amortization table for an equal payment is:

Year	Beginning Balance	Total Payment	Interest Payment	Principal Payment	Ending Balance
1	$42,000.00	$10,519.17	$3,360.00	$7,159.17	$34,840.83
2	34,840.83	10,519.17	2,787.27	7,731.90	27,108.92
3	27,108.92	10,519.17	2,168.71	8,350.46	18,758.47
4	18,758.47	10,519.17	1,500.68	9,018.49	9,739.97
5	9,739.97	10,519.17	779.20	9,739.97	0.00

In the third year, $2,168.71 of interest is paid.

Total interest over life of the loan = $3,360 + 2,787.27 + 2,168.71 + 1,500.68 + 779.20
Total interest over life of the loan = $10,595.86

56. This amortization table calls for equal principal payments of $8,400 per year. The interest payment is the beginning balance times the interest rate for the period, and the total payment is the principal payment plus the interest payment. The ending balance for a period is the beginning balance for the next period. The amortization table for an equal principal reduction is:

Year	Beginning Balance	Total Payment	Interest Payment	Principal Payment	Ending Balance
1	$42,000.00	$11,760.00	$3,360.00	$8,400.00	$33,600.00
2	33,600.00	11,088.00	2,688.00	8,400.00	25,200.00
3	25,200.00	10,416.00	2,016.00	8,400.00	16,800.00
4	16,800.00	9,744.00	1,344.00	8,400.00	8,400.00
5	8,400.00	9,072.00	672.00	8,400.00	0.00

In the third year, $2,016 of interest is paid.

Total interest over life of the loan = $3,360 + 2,688 + 2,016 + 1,344 + 672 = $10,080

Notice that the total payments for the equal principal reduction loan are lower. This is because more principal is repaid early in the loan, which reduces the total interest expense over the life of the loan.

Challenge

57. The cash flows for this problem occur monthly, and the interest rate given is the EAR. Since the cash flows occur monthly, we must get the effective monthly rate. One way to do this is to find the APR based on monthly compounding, and then divide by 12. So, the pre-retirement APR is:

$EAR = .10 = [1 + (APR / 12)]^{12} - 1$; $APR = 12[(1.10)^{1/12} - 1] = .0957$ or 9.57%

And the post-retirement APR is:

$EAR = .07 = [1 + (APR / 12)]^{12} - 1$; $APR = 12[(1.07)^{1/12} - 1] = .0678$ or 6.78%

First, we will calculate how much he needs at retirement. The amount needed at retirement is the PV of the monthly spending plus the PV of the inheritance. The PV of these two cash flows is:

PVA = \$20,000{1 – [1 / (1 + .0678/12)$^{12(25)}$]} / (.0678/12) = \$2,885,496.45

PV = \$900,000 / [1 + (.0678/12)]^{300} = \$165,824.26

So, at retirement, he needs:

\$2,885,496.45 + 165,824.26 = \$3,051,320.71

He will be saving \$2,500 per month for the next 10 years until he purchases the cabin. The value of his savings after 10 years will be:

FVA = \$2,500[{[1 + (.0957/12)]$^{12(10)}$ – 1} / (.0957/12)] = \$499,659.64

After he purchases the cabin, the amount he will have left is:

\$499,659.64 – 380,000 = \$119,659.64

He still has 20 years until retirement. When he is ready to retire, this amount will have grown to:

FV = \$119,659.64[1 + (.0957/12)]$^{12(20)}$ = \$805,010.23

So, when he is ready to retire, based on his current savings, he will be short:

\$3,051,320.71 – 805,010.23 = \$2,246,310.48

This amount is the FV of the monthly savings he must make between years 10 and 30. So, finding the annuity payment using the FVA equation, we find his monthly savings will need to be:

FVA = \$2,246,310.48 = C[{[1 + (.1048/12)]$^{12(20)}$ – 1} / (.1048/12)]
C = \$3,127.44

58. To answer this question, we should find the PV of both options, and compare them. Since we are purchasing the car, the lowest PV is the best option. The PV of the leasing is simply the PV of the lease payments, plus the \$99. The interest rate we would use for the leasing option is the same as the interest rate of the loan. The PV of leasing is:

PV = \$99 + \$450{1 – [1 / (1 + .07/12)$^{12(3)}$]} / (.07/12) = \$14,672.91

The PV of purchasing the car is the current price of the car minus the PV of the resale price. The PV of the resale price is:

PV = \$23,000 / [1 + (.07/12)]$^{12(3)}$ = \$18,654.82

The PV of the decision to purchase is:

\$32,000 – 18,654.82 = \$13,345.18

In this case, it is cheaper to buy the car than leasing it since the PV of the purchase cash flows is lower. To find the breakeven resale price, we need to find the resale price that makes the PV of the two options the same. In other words, the PV of the decision to buy should be:

$32,000 – PV of resale price = $14,672.91
PV of resale price = $17,327.09

The resale price that would make the PV of the lease versus buy decision is the FV of this value, so:

Breakeven resale price = $17,327.09[1 + (.07/12)]$^{12(3)}$ = $21,363.01

59. To find the quarterly salary for the player, we first need to find the PV of the current contract. The cash flows for the contract are annual, and we are given a daily interest rate. We need to find the EAR so the interest compounding is the same as the timing of the cash flows. The EAR is:

EAR = [1 + (.055/365)]^{365} – 1 = 5.65%

The PV of the current contract offer is the sum of the PV of the cash flows. So, the PV is:

PV = $7,000,000 + $4,500,000/1.0565 + $5,000,000/1.0565^2 + $6,000,000/1.0565^3
+ $6,800,000/1.0565^4 + $7,900,000/1.0565^5 + $8,800,000/1.0565^6
PV = $38,610,482.57

The player wants the contract increased in value by $1,400,000, so the PV of the new contract will be:

PV = $38,610,482.57 + 1,400,000 = $40,010,482.57

The player has also requested a signing bonus payable today in the amount of $9 million. We can simply subtract this amount from the PV of the new contract. The remaining amount will be the PV of the future quarterly paychecks.

$40,010,482.57 – 9,000,000 = $31,010,482.57

To find the quarterly payments, first realize that the interest rate we need is the effective quarterly rate. Using the daily interest rate, we can find the quarterly interest rate using the EAR equation, with the number of days being 91.25, the number of days in a quarter (365 / 4). The effective quarterly rate is:

Effective quarterly rate = [1 + (.055/365)]$^{91.25}$ – 1 = .01384 or 1.384%

Now we have the interest rate, the length of the annuity, and the PV. Using the PVA equation and solving for the payment, we get:

PVA = $31,010,482.57 = C\{[1 – (1/1.01384)^{24}] / .01384\}
C = $1,527,463.76

60. To find the APR and EAR, we need to use the actual cash flows of the loan. In other words, the interest rate quoted in the problem is only relevant to determine the total interest under the terms given. The cash flows of the loan are the $25,000 you must repay in one year, and the $21,250 you borrow today. The interest rate of the loan is:

$\$25,000 = \$21,250(1 + r)$
$r = (\$25,000 / 21,250) - 1 = .1765$ or 17.65%

Because of the discount, you only get the use of $21,250, and the interest you pay on that amount is 17.65%, not 15%.

61. Here we have cash flows that would have occurred in the past and cash flows that would occur in the future. We need to bring both cash flows to today. Before we calculate the value of the cash flows today, we must adjust the interest rate so we have the effective monthly interest rate. Finding the APR with monthly compounding and dividing by 12 will give us the effective monthly rate. The APR with monthly compounding is:

$APR = 12[(1.08)^{1/12} - 1] = .0772$ or 7.72%

To find the value today of the back pay from two years ago, we will find the FV of the annuity, and then find the FV of the lump sum. Doing so gives us:

$FVA = (\$47,000/12) [\{[1 + (.0772/12)]^{12} - 1\} / (.0772/12)] = \$48,699.39$
$FV = \$48,699.39(1.08) = \$52,595.34$

Notice we found the FV of the annuity with the effective monthly rate, and then found the FV of the lump sum with the EAR. Alternatively, we could have found the FV of the lump sum with the effective monthly rate as long as we used 12 periods. The answer would be the same either way.

Now, we need to find the value today of last year's back pay:

$FVA = (\$50,000/12) [\{[1 + (.0772/12)]^{12} - 1\} / (.0772/12)] = \$51,807.86$

Next, we find the value today of the five year's future salary:

$PVA = (\$55,000/12)\{[\{1 - \{1 / [1 + (.0772/12)]^{12(5)}\}] / (.0772/12)\} = \$227,539.14$

The value today of the jury award is the sum of salaries, plus the compensation for pain and suffering, and court costs. The award should be for the amount of:

Award = $52,595.34 + 51,807.86 + 227,539.14 + 100,000 + 20,000 = $451,942.34

As the plaintiff, you would prefer a lower interest rate. In this problem, we are calculating both the PV and FV of annuities. A lower interest rate will decrease the FVA, but increase the PVA. So, by a lower interest rate, we are lowering the value of the back pay. But, we are also increasing the PV of the future salary. Since the future salary is larger and has a longer time, this is the more important cash flow to the plaintiff.

62. Again, to find the interest rate of a loan, we need to look at the cash flows of the loan. Since this loan is in the form of a lump sum, the amount you will repay is the FV of the principal amount, which will be:

Loan repayment amount = $10,000(1.08) = $10,800

The amount you will receive today is the principal amount of the loan times one minus the points.

Amount received = $10,000(1 – .03) = $9,700

Now, we simply find the interest rate for this PV and FV.

$10,800 = $9,700(1 + *r*)
r = ($10,800 / $9,700) – 1 = .1134 or 11.34%

63. This is the same question as before, with different values. So:

Loan repayment amount = $10,000(1.11) = $11,100

Amount received = $10,000(1 – .02) = $9,800

$11,100 = $9,800(1 + *r*)
r = ($11,100 / $9,800) – 1 = .1327 or 13.27%

The effective rate is not affected by the loan amount since it drops out when solving for *r*.

64. First we will find the APR and EAR for the loan with the refundable fee. Remember, we need to use the actual cash flows of the loan to find the interest rate. With the $2,300 application fee, you will need to borrow $242,300 to have $240,000 after deducting the fee. Solving for the payment under these circumstances, we get:

PVA = $242,300 = C {[1 – 1/(1.005667)^{360}]/.005667} where .005667 = .068/12
C = $1,579.61

We can now use this amount in the PVA equation with the original amount we wished to borrow, $240,000. Solving for *r*, we find:

PVA = $240,000 = $1,579.61[{1 – [1 / (1 + *r*)]^{360}}/ *r*]

Solving for *r* with a spreadsheet, on a financial calculator, or by trial and error, gives:

r = 0.5745% per month

APR = 12(0.5745%) = 6.89%

EAR = (1 + .005745)^{12} – 1 = 7.12%

With the nonrefundable fee, the APR of the loan is simply the quoted APR since the fee is not considered part of the loan. So:

APR = 6.80%

EAR = $[1 + (.068/12)]^{12} - 1 = 7.02\%$

65. Be careful of interest rate quotations. The actual interest rate of a loan is determined by the cash flows. Here, we are told that the PV of the loan is $1,000, and the payments are $41.15 per month for three years, so the interest rate on the loan is:

PVA = $\$1,000 = \$41.15[\{1 - [1 / (1 + r)]^{36} \} / r]$

Solving for r with a spreadsheet, on a financial calculator, or by trial and error, gives:

r = 2.30% per month

APR = 12(2.30%) = 27.61%

EAR = $(1 + .0230)^{12} - 1 = 31.39\%$

It's called add-on interest because the interest amount of the loan is added to the principal amount of the loan before the loan payments are calculated.

66. Here we are solving a two-step time value of money problem. Each question asks for a different possible cash flow to fund the same retirement plan. Each savings possibility has the same FV, that is, the PV of the retirement spending when your friend is ready to retire. The amount needed when your friend is ready to retire is:

PVA = $\$105,000\{[1 - (1/1.07)^{20}] / .07\} = \$1,112,371.50$

This amount is the same for all three parts of this question.

a. If your friend makes equal annual deposits into the account, this is an annuity with the FVA equal to the amount needed in retirement. The required savings each year will be:

FVA = $\$1,112,371.50 = C[(1.07^{30} - 1) / .07]$
$C = \$11,776.01$

b. Here we need to find a lump sum savings amount. Using the FV for a lump sum equation, we get:

FV = $\$1,112,371.50 = PV(1.07)^{30}$
PV = $146,129.04

c. In this problem, we have a lump sum savings in addition to an annual deposit. Since we already know the value needed at retirement, we can subtract the value of the lump sum savings at retirement to find out how much your friend is short. Doing so gives us:

FV of trust fund deposit = $\$150{,}000(1.07)^{10} = \$295{,}072.70$

So, the amount your friend still needs at retirement is:

FV = $\$1{,}112{,}371.50 – 295{,}072.70 = \$817{,}298.80$

Using the FVA equation, and solving for the payment, we get:

$\$817{,}298.80 = C[(1.07^{30} – 1) / .07]$

$C = \$8{,}652.25$

This is the total annual contribution, but your friend's employer will contribute $1,500 per year, so your friend must contribute:

Friend's contribution = $\$8{,}652.25 – 1{,}500 = \$7{,}152.25$

67. We will calculate the number of periods necessary to repay the balance with no fee first. We simply need to use the PVA equation and solve for the number of payments.

Without fee and annual rate = 19.80%:

PVA = $\$10{,}000 = \$200\{[1 – (1/1.0165)^t] / .0165\}$ where $.0165 = .198/12$

Solving for t, we get:

$1/1.0165^t = 1 – (\$10{,}000/\$200)(.0165)$
$1/1.0165^t = .175$
$t = \ln (1/.175) / \ln 1.0165$
$t = 106.50$ months

Without fee and annual rate = 6.20%:

PVA = $\$10{,}000 = \$200\{[1 – (1/1.005167)^t] / .005167\}$ where $.005167 = .062/12$

Solving for t, we get:

$1/1.005167^t = 1 – (\$10{,}000/\$200)(.005167)$
$1/1.005167^t = .7417$
$t = \ln (1/.7417) / \ln 1.005167$
$t = 57.99$ months

Note that we do not need to calculate the time necessary to repay your current credit card with a fee since no fee will be incurred. The time to repay the new card with a transfer fee is:

With fee and annual rate = 6.20%:

$PVA = \$10,200 = \$200\{ [1 - (1/1.005167)^t] / .005167 \}$ where .005167 = .082/12

Solving for *t*, we get:

$1/1.005167^t = 1 - (\$10,200/\$200)(.005167)$
$1/1.005167^t = .7365$
$t = \ln (1/.7365) / \ln 1.005167$
$t = 59.35$ months

68. We need to find the FV of the premiums to compare with the cash payment promised at age 65. We have to find the value of the premiums at year 6 first since the interest rate changes at that time. So:

$FV_1 = \$900(1.12)^5 = \$1,586.11$

$FV_2 = \$900(1.12)^4 = \$1,416.17$

$FV_3 = \$1,000(1.12)^3 = \$1,404.93$

$FV_4 = \$1,000(1.12)^2 = \$1,254.40$

$FV_5 = \$1,100(1.12)^1 = \$1,232.00$

Value at year six = \$1,586.11 + 1,416.17 + 1,404.93 + 1,254.40 + 1,232.00 + 1,100
Value at year six = \$7,993.60

Finding the FV of this lump sum at the child's 65th birthday:

$FV = \$7,993.60(1.08)^{59} = \$749,452.56$

The policy is not worth buying; the future value of the deposits is \$749,452.56, but the policy contract will pay off \$500,000. The premiums are worth \$249,452.56 more than the policy payoff.

Note, we could also compare the PV of the two cash flows. The PV of the premiums is:

$PV = \$900/1.12 + \$900/1.12^2 + \$1,000/1.12^3 + \$1,000/1.12^4 + \$1,100/1.12^5 + \$1,100/1.12^6$
PV = \$4,049.81

And the value today of the \$500,000 at age 65 is:

$PV = \$500,000/1.08^{59} = \$5,332.96$

$PV = \$5,332.96/1.12^6 = \$2,701.84$

The premiums still have the higher cash flow. At time zero, the difference is \$1,347.97. Whenever you are comparing two or more cash flow streams, the cash flow with the highest value at one time will have the highest value at any other time.

Here is a question for you: Suppose you invest \$1,347.97, the difference in the cash flows at time zero, for six years at a 12 percent interest rate, and then for 59 years at an 8 percent interest rate. How much will it be

worth? Without doing calculations, you know it will be worth $249,452.56, the difference in the cash flows at time 65!

69. The monthly payments with a balloon payment loan are calculated assuming a longer amortization schedule, in this case, 30 years. The payments based on a 30-year repayment schedule would be:

PVA = $750,000 = $C(\{1 - [1 / (1 + .081/12)]^{360}\} / (.081/12))$
C = $5,555.61

Now, at time = 8, we need to find the PV of the payments which have not been made. The balloon payment will be:

PVA = $5,555.61$(\{1 - [1 / (1 + .081/12)]^{12(22)}\} / (.081/12))$
PVA = $683,700.32

70. Here we need to find the interest rate that makes the PVA, the college costs, equal to the FVA, the savings. The PV of the college costs are:

PVA = $20,000$[\{1 - [1 / (1 + r)^4]\} / r]$

And the FV of the savings is:

FVA = $9,000$\{[(1 + r)^6 - 1] / r \}$

Setting these two equations equal to each other, we get:

$20,000$[\{1 - [1 / (1 + r)]^4 \} / r]$ = $9,000$\{[(1 + r)^6 - 1] / r \}$

Reducing the equation gives us:

$(1 + r)^6 - 11{,}000(1 + r)^4 + 29{,}000 = 0$

Now we need to find the roots of this equation. We can solve using trial and error, a root-solving calculator routine, or a spreadsheet. Using a spreadsheet, we find:

r = 8.07%

71. Here we need to find the interest rate that makes us indifferent between an annuity and a perpetuity. To solve this problem, we need to find the PV of the two options and set them equal to each other. The PV of the perpetuity is:

PV = $20,000 / r$

And the PV of the annuity is:

PVA = $28,000$[\{1 - [1 / (1 + r)]^{20} \} / r]$

Setting them equal and solving for *r*, we get:

$\$20{,}000 / r = \$28{,}000[\ \{1 - [1 / (1 + r)]^{20}\ \} / r\ ]$
$\$20{,}000 / \$28{,}000 = 1 - [1 / (1 + r)]^{20}$
$.2857^{1/20} = 1 / (1 + r)$
$r = .0646$ or 6.46%

72. The cash flows in this problem occur every two years, so we need to find the effective two year rate. One way to find the effective two year rate is to use an equation similar to the EAR, except use the number of days in two years as the exponent. (We use the number of days in two years since it is daily compounding; if monthly compounding was assumed, we would use the number of months in two years.) So, the effective two-year interest rate is:

Effective 2-year rate = $[1 + (.10/365)]^{365(2)} - 1 = .2214$ or 22.14%

We can use this interest rate to find the PV of the perpetuity. Doing so, we find:

PV = \$15,000 /.2214 = \$67,760.07

This is an important point: Remember that the PV equation for a perpetuity (and an ordinary annuity) tells you the PV one period before the first cash flow. In this problem, since the cash flows are two years apart, we have found the value of the perpetuity one period (two years) before the first payment, which is one year ago. We need to compound this value for one year to find the value today. The value of the cash flows today is:

PV = $\$67{,}760.07(1 + .10/365)^{365} = \$74{,}885.44$

The second part of the question assumes the perpetuity cash flows begin in four years. In this case, when we use the PV of a perpetuity equation, we find the value of the perpetuity two years from today. So, the value of these cash flows today is:

PV = \$67,760.07 / (1 + .2214) = \$55,478.78

73. To solve for the PVA due:

$$\text{PVA} = \frac{C}{(1+r)} + \frac{C}{(1+r)^2} + ... + \frac{C}{(1+r)^t}$$

$$\text{PVA}_{\text{due}} = C + \frac{C}{(1+r)} + ... + \frac{C}{(1+r)^{t-1}}$$

$$\text{PVA}_{\text{due}} = (1+r)\left(\frac{C}{(1+r)} + \frac{C}{(1+r)^2} + ... + \frac{C}{(1+r)^t} \right)$$

$\text{PVA}_{\text{due}} = (1 + r)\ \text{PVA}$

And the FVA due is:

$\text{FVA} = C + C(1 + r) + C(1 + r)^2 + \ldots. + C(1 + r)^{t-1}$
$\text{FVA}_{\text{due}} = C(1 + r) + C(1 + r)^2 + \ldots. + C(1 + r)^t$
$\text{FVA}_{\text{due}} = (1 + r)[C + C(1 + r) + \ldots. + C(1 + r)^{t-1}]$
$\text{FVA}_{\text{due}} = (1 + r)\text{FVA}$

74. We need to find the lump sum payment into the retirement account. The present value of the desired amount at retirement is:

$PV = FV/(1 + r)^t$
$PV = \$2{,}000{,}000/(1 + .11)^{40}$
$PV = \$30{,}768.82$

This is the value today. Since the savings are in the form of a growing annuity, we can use the growing annuity equation and solve for the payment. Doing so, we get:

$PV = C\{[1 - ((1 + g)/(1 + r))^t] / (r - g)\}$
$\$30{,}768.82 = C\{[1 - ((1 + .03)/(1 + .11))^{40}] / (.11 - .03)\}$
$C = \$2{,}591.56$

This is the amount you need to save next year. So, the percentage of your salary is:

Percentage of salary = \$2,591.56/\$40,000
Percentage of salary = .0648 or 6.48%

Note that this is the percentage of your salary you must save each year. Since your salary is increasing at 3 percent, and the savings are increasing at 3 percent, the percentage of salary will remain constant.

75. *a.* The APR is the interest rate per week times 52 weeks in a year, so:

APR = 52(7%) = 364%

$EAR = (1 + .07)^{52} - 1 = 32.7253$ or 3,273.53%

b. In a discount loan, the amount you receive is lowered by the discount, and you repay the full principal. With a 7 percent discount, you would receive \$9.30 for every \$10 in principal, so the weekly interest rate would be:

$\$10 = \$9.30(1 + r)$
$r = (\$10 / \$9.30) - 1 = .0753$ or 7.53%

Note the dollar amount we use is irrelevant. In other words, we could use \$0.93 and \$1, \$93 and \$100, or any other combination and we would get the same interest rate. Now we can find the APR and the EAR:

APR = 52(7.53%) = 391.40%

$EAR = (1 + .0753)^{52} - 1 = 42.5398$ or 4,253.98%

c. Using the cash flows from the loan, we have the PVA and the annuity payments and need to find the interest rate, so:

PVA = \$68.92 = \$25[{1 – [1 / (1 + *r*)]^4}/ *r*]

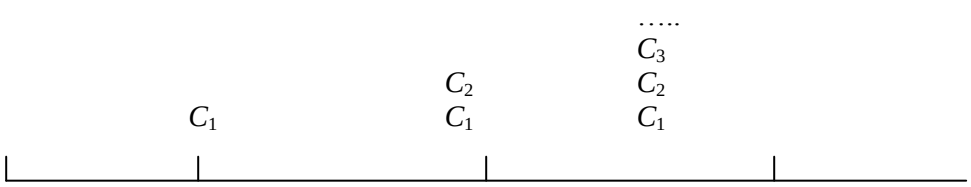
Using a spreadsheet, trial and error, or a financial calculator, we find:

r = 16.75% per week

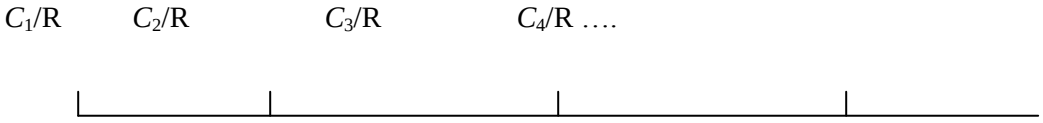
APR = 52(16.75%) = 870.99%

EAR = 1.1675^{52} – 1 = 3141.7472 or 314,174.72%

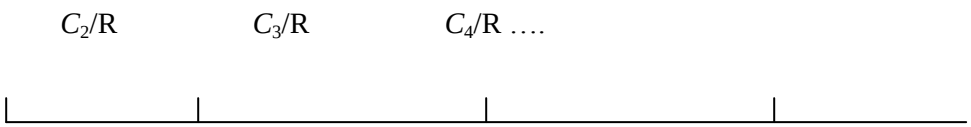
76. To answer this, we need to diagram the perpetuity cash flows, which are: (Note, the subscripts are only to differentiate when the cash flows begin. The cash flows are all the same amount.)



Thus, each of the increased cash flows is a perpetuity in itself. So, we can write the cash flows stream as:



So, we can write the cash flows as the present value of a perpetuity, and a perpetuity of:



The present value of this perpetuity is:

PV = (*C*/R) / R = *C*/R^2

So, the present value equation of a perpetuity that increases by *C* each period is:

PV = *C*/R + *C*/R^2

77. We are only concerned with the time it takes money to double, so the dollar amounts are irrelevant. So, we can write the future value of a lump sum as:

$FV = PV(1 + R)^t$
$\$2 = \$1(1 + R)^t$

Solving for t, we find:

$\ln(2) = t[\ln(1 + R)]$
$t = \ln(2) / \ln(1 + R)$

Since R is expressed as a percentage in this case, we can write the expression as:

$t = \ln(2) / \ln(1 + R/100)$

To simplify the equation, we can make use of a Taylor Series expansion:

$\ln(1 + R) = R – R^2/2 + R^3/3 – ...$

Since R is small, we can truncate the series after the first term:

$\ln(1 + R) = R$

Combine this with the solution for the doubling expression:

$t = \ln(2) / (R/100)$
$t = 100\ln(2) / R$
$t = 69.3147 / R$

This is the exact (approximate) expression, Since 69.3147 is not easily divisible, and we are only concerned with an approximation, 72 is substituted.

78. We are only concerned with the time it takes money to double, so the dollar amounts are irrelevant. So, we can write the future value of a lump sum with continuously compounded interest as:

$\$2 = \$1e^{Rt}$
$2 = e^{Rt}$
$Rt = \ln(2)$
$Rt = .693147$
$t = .693147 / R$

Since we are using interest rates while the equation uses decimal form, to make the equation correct with percentages, we can multiply by 100:

$t = 69.3147 / R$

Calculator Solutions

1.

CFo	$0	**CFo**	$0	**CFo**	$0
C01	$950	**C01**	$950	**C01**	$950
F01	1	**F01**	1	**F01**	1
C02	$1,040	**C02**	$1,040	**C02**	$1,040
F02	1	**F02**	1	**F02**	1
C03	$1,130	**C03**	$1,130	**C03**	$1,130
F03	1	**F03**	1	**F03**	1
C04	$1,075	**C04**	$1,075	**C04**	$1,075
F04	1	**F04**	1	**F04**	1
I = 10		I = 18		I = 24	
NPV CPT		NPV CPT		NPV CPT	
$3,306.37		$2,794.22		$2,489.88	

2.

Enter	9	5%		$6,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$42,646.93		

Enter	6	5%		$8,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$40,605.54		

Enter	9	15%		$6,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$28,629.50		

Enter	5	15%		$8,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$30,275.86		

3.

Enter	3	8%	$940		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,184.13

Enter	2	8%	$1,090		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,271.38

Enter	1	8%	$1,340		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$1,447.20

FV = $1,184.13 + 1,271.38 + 1,447.20 + 1,405 = $5,307.71

	N	I/Y	PV	PMT	FV
Enter	3	11%	$940		
Solve for					$1,285.57

	N	I/Y	PV	PMT	FV
Enter	2	11%	$1,090		
Solve for					$1,342.99

	N	I/Y	PV	PMT	FV
Enter	1	11%	$1,340		
Solve for					$1,487.40

FV = $1,285.57 + 1,342.99 + 1,487.40 + 1,405 = $5,520.96

	N	I/Y	PV	PMT	FV
Enter	3	24%	$940		
Solve for					$1,792.23

	N	I/Y	PV	PMT	FV
Enter	2	24%	$1,090		
Solve for					$1,675.98

	N	I/Y	PV	PMT	FV
Enter	1	24%	$1,340		
Solve for					$1,661.60

FV = $1,792.23 + 1,675.98 + 1,661.60 + 1,405 = $6,534.81

4.

	N	I/Y	PV	PMT	FV
Enter	15	7%		$5,300	
Solve for			$48,271.94		

	N	I/Y	PV	PMT	FV
Enter	40	7%		$5,300	
Solve for			$70,658.06		

	N	I/Y	PV	PMT	FV
Enter	75	7%		$5,300	
Solve for			$75,240.70		

5.

	N	I/Y	PV	PMT	FV
Enter	15	7.65%	$34,000		
Solve for				$3,887.72	

6.

	N	I/Y	PV	PMT	FV
Enter	8	8.5%		$73,000	
Solve for			$411,660.36		

7.

	N	I/Y	PV	PMT	FV
Enter	20	11.2%		$4,000	
Solve for					$262,781.16
Enter	40	11.2%		$4,000	
Solve for					$2,459,072.63

8.

	N	I/Y	PV	PMT	FV
Enter	10	6.8%			$90,000
Solve for				$6,575.77	

9.

	N	I/Y	PV	PMT	FV
Enter	7	7.5%	$70,000		
Solve for				$9,440.02	

12.

	NOM	EFF	C/Y
Enter	8%		4
Solve for		8.24%	
Enter	16%		12
Solve for		17.23%	
Enter	12%		365
Solve for		12.75%	

13.

	NOM	EFF	C/Y
Enter		8.6%	2
Solve for	8.24%		

Enter		19.8%	12		
	NOM	**EFF**	**C/Y**		
Solve for	18.20%				
Enter		9.40%	52		
	NOM	**EFF**	**C/Y**		
Solve for	8.99%				

14.

Enter	14.2%		12		
	NOM	**EFF**	**C/Y**		
Solve for		15.16%			
Enter	14.5%		2		
	NOM	**EFF**	**C/Y**		
Solve for		15.03%			

15.

Enter		16%	365		
	NOM	**EFF**	**C/Y**		
Solve for	14.85%				

16.

Enter	17 × 2	8.4%/2	$2,100		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$8,505.93

17.

Enter	5 × 365	9.3% / 365	$4,500		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$7,163.64
Enter	10 × 365	9.3% / 365	$4,500		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$11,403.94
Enter	20 × 365	9.3% / 365	$4,500		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$28,899.97

18.

Enter	7 × 365	10% / 365			$58,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$28,804.71		

19.

Enter	360%		12		
	NOM	**EFF**	**C/Y**		
Solve for		2,229.81%			

20.

Enter	60	6.9% / 12	$68,500		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$1,353.15	

Enter	6.9%		12		
	NOM	**EFF**	**C/Y**		
Solve for		7.12%			

21.

Enter		1.3%	$18,000	±$500	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for	48.86				

22.

Enter	1,733.33%		52		
	NOM	**EFF**	**C/Y**		
Solve for		313,916,515.69%			

23.

Enter	22.74%		12		
	NOM	**EFF**	**C/Y**		
Solve for		25.26%			

24.

Enter	30 × 12	10% / 12		$300	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$678,146.38

25.

Enter	10.00%		12		
	NOM	**EFF**	**C/Y**		
Solve for		10.47%			

Enter	30	10.47%		$3,600	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$647,623.45

26.

Enter	4 × 4	0.65%		$2,300	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$34,843.71		

27.

Enter	11.00%		4
	NOM	**EFF**	**C/Y**
Solve for		11.46%	

CF0	$0
C01	$725
F01	1
C02	$980
F02	1
C03	$0
F03	1
C04	$1,360
F04	1

I = 11.46%
NPV CPT
$2,320.36

28.

CF0	$0
C01	$1,650
F01	1
C02	$0
F02	1
C03	$4,200
F03	1
C04	$2,430
F04	1

I = 8.45%
NPV CPT
$6,570.86

30.

Enter		17%	2
	NOM	**EFF**	**C/Y**
Solve for	16.33%		

16.33% / 2 = 8.17%

Enter		17%	4
	NOM	**EFF**	**C/Y**
Solve for	16.01%		

16.01% / 4 = 4.00%

Enter		17%	12
	NOM	**EFF**	**C/Y**
Solve for	15.80%		

15.80% / 12 = 1.32%

31.

	N	I/Y	PV	PMT	FV
Enter	6	1.50% / 12	$5,000		
Solve for					$5,037.62

	N	I/Y	PV	PMT	FV
Enter	6	18% / 12	$5,037.62		
Solve for					$5,508.35

$5,508.35 – 5,000 = $508.35

32. Stock account:

	N	I/Y	PV	PMT	FV
Enter	360	11% / 12		$700	
Solve for					$1,963,163.82

Bond account:

	N	I/Y	PV	PMT	FV
Enter	360	6% / 12		$300	
Solve for					$301,354.51

Savings at retirement = $1,963,163.82 + 301,354.51 = $2,264,518.33

	N	I/Y	PV	PMT	FV
Enter	300	9% / 12	$2,264,518.33		
Solve for				$19,003.76	

33.

	N	I/Y	PV	PMT	FV
Enter	12	1.17%	$1		
Solve for					$1.15

	N	I/Y	PV	PMT	FV
Enter	24	1.17%	$1		
Solve for					$1.32

34.

	N	I/Y	PV	PMT	FV
Enter	480	12% / 12			$1,000,000
Solve for				$85.00	

	N	I/Y	PV	PMT	FV
Enter	360	12% / 12			$1,000,000
Solve for				$286.13	

Enter	240	12% / 12			$1,000,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$1,010.86	

35.

Enter	12 / 3		±$1		$3
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		31.61%			

36.

Enter	10.00%		12
	NOM	**EFF**	**C/Y**
Solve for		10.47%	

Enter	2	10.47%		$95,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$163,839.09		

CF$_0$	$45,000
C01	$70,000
F01	2

I = 10.47%
NPV CPT
$165,723.94

39.

Enter	15	10%		$9,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$68,454.72		

Enter	15	5%		$9,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$93,426.92		

Enter	15	15%		$9,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$52,626.33		

40.

Enter		6% / 12		±$340	$20,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for	51.69				

41.

	N	I/Y	PV	PMT	FV
Enter	60		$73,000	±$1,450	
Solve for		0.594%			

0.594% × 12 = 7.13%

42.

	N	I/Y	PV	PMT	FV
Enter	360	6.35% / 12		$1,150	
Solve for			$184,817.42		

$240,000 – 184,817.42 = $55,182.58

	N	I/Y	PV	PMT	FV
Enter	360	6.35% / 12	$55,182.58		
Solve for					$368,936.54

43.

CF0	$0
C01	$1,700
F01	1
C02	$0
F02	1
C03	$2,100
F03	1
C04	$2,800
F04	1

I = 10%
NPV CPT
$5,035.65

PV of missing CF = $6,550 – 5,035.65 = $1,514.35
Value of missing CF:

	N	I/Y	PV	PMT	FV
Enter	2	10%	$1,514.35		
Solve for					$1,832.36

44.

CFo	$1,000,000
C01	$1,500,000
F01	1
C02	$2,500,000
F02	1
C03	$2,800,000
F03	1
C04	$3,000,000
F04	1
C05	$3,500,000
F05	1
C06	$4,000,000
F06	1
C07	$4,500,000
F07	1
C08	$5,000,000
F08	1
C09	$5,500,000
F09	1
C010	$6,000,000

I = 9%
NPV CPT
$22,812,873

45.

	N	I/Y	PV	PMT	FV
Enter	360		.80($2,900,000)	±$15,000	
Solve for		0.560%			

APR = 0.560% × 12 = 6.72%

	NOM	EFF	C/Y
Enter	6.72%		12
Solve for		6.93%	

46.

	N	I/Y	PV	PMT	FV
Enter	4	13%			$165,000
Solve for			$101,197.59		

Profit = $101,197.59 – 94,000 = $7,197.59

	N	I/Y	PV	PMT	FV
Enter	4		±$94,000		$165,000
Solve for		15.10%			

47.

	N	I/Y	PV	PMT	FV
Enter	18	10%		$4,000	
Solve for			$32,805.65		

	N	I/Y	PV	PMT	FV
Enter	7	10%			$32,805.65
Solve for			$16,834.48		

48.

	N	I/Y	PV	PMT	FV
Enter	84	7% / 12		$1,500	
Solve for			$87,604.36		

	N	I/Y	PV	PMT	FV
Enter	96	11% / 12		$1,500	
Solve for			$110,021.35		

	N	I/Y	PV	PMT	FV
Enter	84	11% / 12			$110,021.35
Solve for			$51,120.33		

$87,604.36 + 51,120.33 = $138,724.68

49.

	N	I/Y	PV	PMT	FV
Enter	15 × 12	8.5%/12		$1,200	
Solve for					$434,143.62

FV = $434,143.62 = PV $e^{.08(15)}$; PV = $434,143.62$e^{-1.20}$ = $130,761.55

50. PV@ t = 14: $3,500 / 0.062 = $56,451.61

	N	I/Y	PV	PMT	FV
Enter	7	6.2%			$56,451.61
Solve for			$37,051.41		

51.

	N	I/Y	PV	PMT	FV
Enter	12		$25,000	±$2,416.67	
Solve for		2.361%			

APR = 2.361% × 12 = 28.33%

	NOM	EFF	C/Y
Enter	28.33%		12
Solve for		32.31%	

52. Monthly rate = .10 / 12 = .0083; semiannual rate = $(1.0083)^6 - 1 = 5.11\%$

	N	I/Y	PV	PMT	FV
Enter	10	5.11%		$7,000	
Solve for			$53,776.72		

	N	I/Y	PV	PMT	FV
Enter	6	5.11%			$53,776.72
Solve for			$39,888.33		

	N	I/Y	PV	PMT	FV
Enter	10	5.11%			$53,776.72
Solve for			$32,684.88		

	N	I/Y	PV	PMT	FV
Enter	16	5.11%			$53,776.72
Solve for			$24,243.67		

53.

a.

	N	I/Y	PV	PMT	FV
Enter	5	11%		±$10,000	
Solve for			$36,958.97		

2^{nd} BGN 2^{nd} SET

	N	I/Y	PV	PMT	FV
Enter	5	11%		±$10,000	
Solve for			$41,024.46		

b.

	N	I/Y	PV	PMT	FV
Enter	5	11%		±$10,000	
Solve for					$62,278.01

2^{nd} BGN 2^{nd} SET

	N	I/Y	PV	PMT	FV
Enter	5	11%		±$10,000	
Solve for					$69,128.60

54. 2^{nd} BGN 2^{nd} SET

Enter	60	7.85% / 12	$68,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$1,364.99	

57. Pre-retirement APR:

Enter		10%	12
	NOM	**EFF**	**C/Y**
Solve for	9.57%		

Post-retirement APR:

Enter		7%	12
	NOM	**EFF**	**C/Y**
Solve for	6.78%		

At retirement, he needs:

Enter	300	6.78% / 12		$20,000	$900,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$3,051,320.71		

In 10 years, his savings will be worth:

Enter	120	7.72% / 12		$2,500	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$499,659.64

After purchasing the cabin, he will have: $499,659.64 – 380,000 = $119,659.64

Each month between years 10 and 30, he needs to save:

Enter	240	9.57% / 12	$119,659.64		$3,051,320.71±
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$3,127.44	

58. PV of purchase:

Enter	36	7% / 12			$23,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$18,654.82		

$32,000 – 18,654.82 = $13,345.18

PV of lease:

	N	I/Y	PV	PMT	FV
Enter	36	7% / 12		$450	
Solve for			$14,573.99		

$14,573.91 + 99 = $14,672.91
Buy the car.

You would be indifferent when the PV of the two cash flows are equal. The present value of the purchase decision must be $14,672.91. Since the difference in the two cash flows is $32,000 – 14,672.91 = $17,327.09, this must be the present value of the future resale price of the car. The break-even resale price of the car is:

	N	I/Y	PV	PMT	FV
Enter	36	7% / 12	$17,327.09		
Solve for					$21,363.01

59.

	NOM	EFF	C/Y
Enter	5.50%		365
Solve for		5.65%	

CF0	$7,000,000
C01	$4,500,000
F01	1
C02	$5,000,000
F02	1
C03	$6,000,000
F03	1
C04	$6,800,000
F04	1
C05	$7,900,000
F05	1
C06	$8,800,000
F06	1

I = 5.65%
NPV CPT
$38,610,482.57

New contract value = $38,610,482.57 + 1,400,000 = $40,010,482.57

PV of payments = $40,010,482.57 – 9,000,000 = $31,010,482.57
Effective quarterly rate = $[1 + (.055/365)]^{91.25} – 1 = .01384$ or 1.384%

	N	I/Y	PV	PMT	FV
Enter	24	1.384%	$31,010,482.57		
Solve for				$1,527,463.76	

60.

	N	I/Y	PV	PMT	FV
Enter	1		$21,250		±$25,000
Solve for		17.65%			

61.

	NOM	EFF	C/Y
Enter		8%	12
Solve for	7.72%		

	N	I/Y	PV	PMT	FV
Enter	12	7.72% / 12		$47,000 / 12	
Solve for					$48,699.39

	N	I/Y	PV	PMT	FV
Enter	1	8%	$48,699.39		
Solve for					$52,595.34

	N	I/Y	PV	PMT	FV
Enter	12	7.72% / 12		$50,000 / 12	
Solve for					$51,807.86

	N	I/Y	PV	PMT	FV
Enter	60	7.72% / 12		$55,000 / 12	
Solve for			$227,539.14		

Award = $52,595.34 + 51,807.86 + 227,539.14 + 100,000 + 20,000 = $451,942.34

62.

	N	I/Y	PV	PMT	FV
Enter	1		$9,700		±$10,800
Solve for		11.34%			

63.

	N	I/Y	PV	PMT	FV
Enter	1		$9,800		±$11,200
Solve for		14.29%			

64. Refundable fee: With the $2,300 application fee, you will need to borrow $242,300 to have $240,000 after deducting the fee. Solve for the payment under these circumstances.

	N	I/Y	PV	PMT	FV
Enter	30 × 12	6.80% / 12	$242,300		
Solve for				$1,579.61	

Enter	30 × 12		$240,000	±$1,579.61	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		0.5745%			

APR = 0.5745% × 12 = 6.89%

Enter	6.89%		12
	NOM	**EFF**	**C/Y**
Solve for		7.12%	

Without refundable fee: APR = 6.80%

Enter	6.80%		12
	NOM	**EFF**	**C/Y**
Solve for		7.02%	

65.

Enter	36		$1,000	±$41.15	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		2.30%			

APR = 2.30% × 12 = 27.61%

Enter	27.61%		12
	NOM	**EFF**	**C/Y**
Solve for		31.39%	

66. What she needs at age 65:

Enter	20	7%		$105,000	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,112,371.50		

a.

Enter	30	7%			$1,112,371.50
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for				$11,776.01	

b.

Enter	30	7%			$1,112,371.50
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$146,129.04		

c.

Enter	10	7%	$150,000		
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for					$295,072.70

At 65, she is short: $1,112,371.50 – 295,072.50 = $817,298.80

	N	I/Y	PV	PMT	FV
Enter	30	7%			±$817,298.80
Solve for				$8,652.25	

Her employer will contribute $1,500 per year, so she must contribute:

$8,652.25 – 1,500 = $7,152.25 per year

67. Without fee:

	N	I/Y	PV	PMT	FV
Enter		19.8% / 12	$10,000	±$200	
Solve for	106.50				

	N	I/Y	PV	PMT	FV
Enter		6.8% / 12	$10,000	±$200	
Solve for	57.99				

With fee:

	N	I/Y	PV	PMT	FV
Enter		6.8% / 12	$10,200	±$200	
Solve for	59.35				

68. Value at Year 6:

	N	I/Y	PV	PMT	FV
Enter	5	12%	$900		
Solve for					$1,586.11

	N	I/Y	PV	PMT	FV
Enter	4	12%	$900		
Solve for					$1,416.17

	N	I/Y	PV	PMT	FV
Enter	3	12%	$1,000		
Solve for					$1,404.93

	N	I/Y	PV	PMT	FV
Enter	2	12%	$1,000		
Solve for					$1,254.40

	N	I/Y	PV	PMT	FV
Enter	1	12%	$1,100		
Solve for					$1,232

So, at Year 5, the value is: $1,586.11 + 1,416.17 + 1,404.93 + 1,254.40 + 1,232 + 1,100 = $7,993.60

At Year 65, the value is:

	N	I/Y	PV	PMT	FV
Enter	59	8%	$7,993.60		
Solve for					$749,452.56

The policy is not worth buying; the future value of the deposits is $749,452.56 but the policy contract will pay off $500,000.

69.

	N	I/Y	PV	PMT	FV
Enter	30 × 12	8.1% / 12	$750,000		
Solve for				$5,555.61	

	N	I/Y	PV	PMT	FV
Enter	22 × 12	8.1% / 12		$5,555.61	
Solve for			$683,700.32		

70.

CFo	±$9,000
C01	±$9,000
F01	5
C02	$20,000
F02	4

IRR CPT
8.07%

75.

a. APR = 7% × 52 = 364%

	NOM	EFF	C/Y
Enter	364%		52
Solve for		3,272.53%	

b.

	N	I/Y	PV	PMT	FV
Enter	1		$9.30		±$10.00
Solve for		7.53%			

APR = 7.53% × 52 = 391.40%

Enter	391.40%		52		
	NOM	**EFF**	**C/Y**		
Solve for		4,253.98%			

c.

Enter	4		$68.92	±$25	
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		16.75%			

APR = 16.75% × 52 = 870.99%

Enter	870.99%		52
	NOM	**EFF**	**C/Y**
Solve for		314,174.72%	

CHAPTER 7
INTEREST RATES AND BOND VALUATION

Answers to Concepts Review and Critical Thinking Questions

1. No. As interest rates fluctuate, the value of a Treasury security will fluctuate. Long-term Treasury securities have substantial interest rate risk.

2. All else the same, the Treasury security will have lower coupons because of its lower default risk, so it will have greater interest rate risk.

3. No. If the bid price were higher than the ask price, the implication would be that a dealer was willing to sell a bond and immediately buy it back at a higher price. How many such transactions would you like to do?

4. Prices and yields move in opposite directions. Since the bid price must be lower, the bid yield must be higher.

5. There are two benefits. First, the company can take advantage of interest rate declines by calling in an issue and replacing it with a lower coupon issue. Second, a company might wish to eliminate a covenant for some reason. Calling the issue does this. The cost to the company is a higher coupon. A put provision is desirable from an investor's standpoint, so it helps the company by reducing the coupon rate on the bond. The cost to the company is that it may have to buy back the bond at an unattractive price.

6. Bond issuers look at outstanding bonds of similar maturity and risk. The yields on such bonds are used to establish the coupon rate necessary for a particular issue to initially sell for par value. Bond issuers also simply ask potential purchasers what coupon rate would be necessary to attract them. The coupon rate is fixed and simply determines what the bond's coupon payments will be. The required return is what investors actually demand on the issue, and it will fluctuate through time. The coupon rate and required return are equal only if the bond sells for exactly at par.

7. Yes. Some investors have obligations that are denominated in dollars; i.e., they are nominal. Their primary concern is that an investment provide the needed nominal dollar amounts. Pension funds, for example, often must plan for pension payments many years in the future. If those payments are fixed in dollar terms, then it is the nominal return on an investment that is important.

8. Companies pay to have their bonds rated simply because unrated bonds can be difficult to sell; many large investors are prohibited from investing in unrated issues.

9. Treasury bonds have no credit risk since it is backed by the U.S. government, so a rating is not necessary. Junk bonds often are not rated because there would be no point in an issuer paying a rating agency to assign its bonds a low rating (it's like paying someone to kick you!).

10. The term structure is based on pure discount bonds. The yield curve is based on coupon-bearing issues.

11. Bond ratings have a subjective factor to them. Split ratings reflect a difference of opinion among credit agencies.

12. As a general constitutional principle, the federal government cannot tax the states without their consent if doing so would interfere with state government functions. At one time, this principle was thought to provide for the tax-exempt status of municipal interest payments. However, modern court rulings make it clear that Congress can revoke the municipal exemption, so the only basis now appears to be historical precedent. The fact that the states and the federal government do not tax each other's securities is referred to as "reciprocal immunity."

13. Lack of transparency means that a buyer or seller can't see recent transactions, so it is much harder to determine what the best bid and ask prices are at any point in time.

14. Companies charge that bond rating agencies are pressuring them to pay for bond ratings. When a company pays for a rating, it has the opportunity to make its case for a particular rating. With an unsolicited rating, the company has no input.

15. A 100-year bond looks like a share of preferred stock. In particular, it is a loan with a life that almost certainly exceeds the life of the lender, assuming that the lender is an individual. With a junk bond, the credit risk can be so high that the borrower is almost certain to default, meaning that the creditors are very likely to end up as part owners of the business. In both cases, the "equity in disguise" has a significant tax advantage.

Solutions to Questions and Problems

NOTE: All end of chapter problems were solved using a spreadsheet. Many problems require multiple steps. Due to space and readability constraints, when these intermediate steps are included in this solutions manual, rounding may appear to have occurred. However, the final answer for each problem is found without rounding during any step in the problem.

Basic

1. The yield to maturity is the required rate of return on a bond expressed as a nominal annual interest rate. For noncallable bonds, the yield to maturity and required rate of return are

interchangeable terms. Unlike YTM and required return, the coupon rate is not a return used as the interest rate in bond cash flow valuation, but is a fixed percentage of par over the life of the bond used to set the coupon payment amount. For the example given, the coupon rate on the bond is still 10 percent, and the YTM is 8 percent.

2. Price and yield move in opposite directions; if interest rates rise, the price of the bond will fall. This is because the fixed coupon payments determined by the fixed coupon rate are not as valuable when interest rates rise—hence, the price of the bond decreases.

NOTE: Most problems do not explicitly list a par value for bonds. Even though a bond can have any par value, in general, corporate bonds in the United States will have a par value of $1,000. We will use this par value in all problems unless a different par value is explicitly stated.

3. The price of any bond is the PV of the interest payment, plus the PV of the par value. Notice this problem assumes an annual coupon. The price of the bond will be:

$P = \$75(\{1 - [1/(1 + .0875)]^{10} \} / .0875) + \$1{,}000[1 / (1 + .0875)^{10}] = \$918.89$

We would like to introduce shorthand notation here. Rather than write (or type, as the case may be) the entire equation for the PV of a lump sum, or the PVA equation, it is common to abbreviate the equations as:

$PVIF_{R,t} = 1 / (1 + r)^t$

which stands for Present Value Interest Factor

$PVIFA_{R,t} = (\{1 - [1/(1 + r)]^t \} / r)$

which stands for Present Value Interest Factor of an Annuity

These abbreviations are short hand notation for the equations in which the interest rate and the number of periods are substituted into the equation and solved. We will use this shorthand notation in remainder of the solutions key.

4. Here we need to find the YTM of a bond. The equation for the bond price is:

$P = \$934 = \$90(PVIFA_{R\%,9}) + \$1{,}000(PVIF_{R\%,9})$

Notice the equation cannot be solved directly for *R*. Using a spreadsheet, a financial calculator, or trial and error, we find:

R = YTM = 10.15%

If you are using trial and error to find the YTM of the bond, you might be wondering how to pick an interest rate to start the process. First, we know the YTM has to be higher than the coupon rate since the bond is a discount bond. That still leaves a lot of interest rates to

check. One way to get a starting point is to use the following equation, which will give you an approximation of the YTM:

Approximate YTM = [Annual interest payment + (Price difference from par / Years to maturity)] /
[(Price + Par value) / 2]

Solving for this problem, we get:

Approximate YTM = [$90 + ($64 / 9] / [($934 + 1,000) / 2] = 10.04%

This is not the exact YTM, but it is close, and it will give you a place to start.

5. Here we need to find the coupon rate of the bond. All we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:

$P = \$1{,}045 = C(\text{PVIFA}_{7.5\%,13}) + \$1{,}000(\text{PVIF}_{7.5\%,36})$

Solving for the coupon payment, we get:

$C = \$80.54$

The coupon payment is the coupon rate times par value. Using this relationship, we get:

Coupon rate = \$80.54 / \$1,000 = .0805 or 8.05%

6. To find the price of this bond, we need to realize that the maturity of the bond is 10 years. The bond was issued one year ago, with 11 years to maturity, so there are 10 years left on the bond. Also, the coupons are semiannual, so we need to use the semiannual interest rate and the number of semiannual periods. The price of the bond is:

$P = \$34.50(\text{PVIFA}_{3.7\%,20}) + \$1{,}000(\text{PVIF}_{3.7\%,20}) = \$965.10$

7. Here we are finding the YTM of a semiannual coupon bond. The bond price equation is:

$P = \$1{,}050 = \$42(\text{PVIFA}_{R\%,20}) + \$1{,}000(\text{PVIF}_{R\%,20})$

Since we cannot solve the equation directly for *R*, using a spreadsheet, a financial calculator, or trial and error, we find:

$R = 3.837\%$

Since the coupon payments are semiannual, this is the semiannual interest rate. The YTM is the APR of the bond, so:

$\text{YTM} = 2 \times 3.837\% = 7.67\%$

8. Here we need to find the coupon rate of the bond. All we need to do is to set up the bond pricing equation and solve for the coupon payment as follows:

$P = \$924 = C(\text{PVIFA}_{3.4\%,29}) + \$1{,}000(\text{PVIF}_{3.4\%,29})$

Solving for the coupon payment, we get:

C = \$29.84

Since this is the semiannual payment, the annual coupon payment is:

2 × \$29.84 = \$59.68

And the coupon rate is the annual coupon payment divided by par value, so:

Coupon rate = \$59.68 / \$1,000
Coupon rate = .0597 or 5.97%

9. The approximate relationship between nominal interest rates (R), real interest rates (r), and inflation (h) is:

$R = r + h$

Approximate $r = .07 – .038 = .032$ or 3.20%

The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation is:

$(1 + R) = (1 + r)(1 + h)$

$(1 + .07) = (1 + r)(1 + .038)$

Exact $r = [(1 + .07) / (1 + .038)] – 1 = .0308$ or 3.08%

10. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation is:

$(1 + R) = (1 + r)(1 + h)$

$R = (1 + .047)(1 + .03) – 1 = .0784$ or 7.84%

11. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation is:

$(1 + R) = (1 + r)(1 + h)$

$h = [(1 + .14) / (1 + .09)] – 1 = .0459$ or 4.59%

12. The Fisher equation, which shows the exact relationship between nominal interest rates, real interest rates, and inflation is:

$(1 + R) = (1 + r)(1 + h)$

$r = [(1 + .114) / (1.048)] – 1 = .0630$ or 6.30%

13. This is a bond since the maturity is greater than 10 years. The coupon rate, located in the first column of the quote is 6.125%. The bid price is:

Bid price = 120:07 = 120 7/32 = 120.21875% × $1,000 = $1,202.1875

The previous day's ask price is found by:

Previous day's asked price = Today's asked price – Change = 120 8/32 – (5/32) = 120 3/32

The previous day's price in dollars was:

Previous day's dollar price = 120.09375% × $1,000 = $1,200.94

14. This is a premium bond because it sells for more than 100% of face value. The current yield is:

Current yield = Annual coupon payment / Price = \$75/\$1,351.875 = .05548 or 5.548%

The YTM is located under the "Asked Yield" column, so the YTM is 4.47%.

The bid-ask spread is the difference between the bid price and the ask price, so:

Bid-Ask spread = 135:06 – 135:05 = 1/32

Intermediate

15. Here we are finding the YTM of semiannual coupon bonds for various maturity lengths. The bond price equation is:

$P = C(\text{PVIFA}_{R\%,t}) + \$1{,}000(\text{PVIF}_{R\%,t})$

X: $P_0 = \$80(\text{PVIFA}_{6\%,13}) + \$1{,}000(\text{PVIF}_{6\%,13}) = \$1{,}177.05$
$P_1 = \$80(\text{PVIFA}_{6\%,12}) + \$1{,}000(\text{PVIF}_{6\%,12}) = \$1{,}167.68$
$P_3 = \$80(\text{PVIFA}_{6\%,10}) + \$1{,}000(\text{PVIF}_{6\%,10}) = \$1{,}147.20$
$P_8 = \$80(\text{PVIFA}_{6\%,5}) + \$1{,}000(\text{PVIF}_{6\%,5}) = \$1{,}084.25$
$P_{12} = \$80(\text{PVIFA}_{6\%,1}) + \$1{,}000(\text{PVIF}_{6\%,1}) = \$1{,}018.87$
$P_{13} = \$1{,}000$

Y: $P_0 = \$60(\text{PVIFA}_{8\%,13}) + \$1{,}000(\text{PVIF}_{8\%,13}) = \$841.92$
$P_1 = \$60(\text{PVIFA}_{8\%,12}) + \$1{,}000(\text{PVIF}_{8\%,12}) = \$849.28$
$P_3 = \$60(\text{PVIFA}_{8\%,10}) + \$1{,}000(\text{PVIF}_{8\%,10}) = \$865.80$
$P_8 = \$60(\text{PVIFA}_{8\%,5}) + \$1{,}000(\text{PVIF}_{8\%,5}) = \$920.15$
$P_{12} = \$60(\text{PVIFA}_{8\%,1}) + \$1{,}000(\text{PVIF}_{8\%,1}) = \$981.48$
$P_{13} = \$1{,}000$

All else held equal, the premium over par value for a premium bond declines as maturity approaches, and the discount from par value for a discount bond declines as maturity approaches. This is called "pull to par." In both cases, the largest percentage price changes occur at the shortest maturity lengths.

Also, notice that the price of each bond when no time is left to maturity is the par value, even though the purchaser would receive the par value plus the coupon payment immediately. This is because we calculate the clean price of the bond.

16. Any bond that sells at par has a YTM equal to the coupon rate. Both bonds sell at par, so the initial YTM on both bonds is the coupon rate, 9 percent. If the YTM suddenly rises to 11 percent:

P_{Sam} = \$45(PVIFA$_{5.5\%,6}$) + \$1,000(PVIF$_{5.5\%,6}$) = \$950.04

P_{Dave} = \$45(PVIFA$_{5.5\%,40}$) + \$1,000(PVIF$_{5.5\%,40}$) = \$839.54

The percentage change in price is calculated as:

Percentage change in price = (New price – Original price) / Original price

$\Delta P_{Sam}\%$ = (\$950.04 – 1,000) / \$1,000 = – 5.00%

$\Delta P_{Dave}\%$ = (\$839.54 – 1,000) / \$1,000 = – 16.05%

If the YTM suddenly falls to 7 percent:

P_{Sam} = \$45(PVIFA$_{3.5\%,6}$) + \$1,000(PVIF$_{3.5\%,6}$) = \$1,053.29

P_{Dave} = \$45(PVIFA$_{3.5\%,40}$) + \$1,000(PVIF$_{3.5\%,40}$) = \$1,213.55

$\Delta P_{Sam}\%$ = (\$1,053.29 – 1,000) / \$1,000 = + 5.33%

$\Delta P_{Dave}\%$ = (\$1,213.55 – 1,000) / \$1,000 = + 21.36%

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes in interest rates.

17. Initially, at a YTM of 8 percent, the prices of the two bonds are:

P_J = \$20(PVIFA$_{4\%,18}$) + \$1,000(PVIF$_{4\%,18}$) = \$746.81

P_K = \$60(PVIFA$_{4\%,18}$) + \$1,000(PVIF$_{4\%,18}$) = \$1,253.19

If the YTM rises from 8 percent to 10 percent:

P_J = \$20(PVIFA$_{5\%,18}$) + \$1,000(PVIF$_{5\%,18}$) = \$649.31

P_K = \$60($PVIFA_{5\%,18}$) + \$1,000($PVIF_{5\%,18}$) = \$1,116.90

The percentage change in price is calculated as:

Percentage change in price = (New price – Original price) / Original price

$\Delta P_J\%$ = (\$649.31 – 746.81) / \$746.81 = – 13.06%
$\Delta P_K\%$ = (\$1,116.90 – 1,253.19) / \$1,253.19 = – 10.88%

If the YTM declines from 8 percent to 6 percent:

P_J = \$20(PVIFA$_{3\%,18}$) + \$1,000(PVIF$_{3\%,18}$) = \$862.46

P_K = \$60(PVIFA$_{3\%,18}$) + \$1,000(PVIF$_{3\%,18}$) = \$1,412.61

$\Delta P_J\%$ = (\$862.46 – 746.81) / \$746.81 = + 15.49%

$\Delta P_K\%$ = (\$1,412.61 – 1,253.19) / \$1,253.19 = + 12.72%

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

18. The bond price equation for this bond is:

P_0 = \$1,068 = \$46(PVIFA$_{R\%,18}$) + \$1,000(PVIF$_{R\%,18}$)

Using a spreadsheet, financial calculator, or trial and error we find:

R = 4.06%

This is the semiannual interest rate, so the YTM is:

YTM = 2 × 4.06% = 8.12%

The current yield is:

Current yield = Annual coupon payment / Price = \$92 / \$1,068 = .0861 or 8.61%

The effective annual yield is the same as the EAR, so using the EAR equation from the previous chapter:

Effective annual yield = $(1 + 0.0406)^2 – 1$ = .0829 or 8.29%

19. The company should set the coupon rate on its new bonds equal to the required return. The required return can be observed in the market by finding the YTM on the outstanding bonds of the company. So, the YTM on the bonds currently sold in the market is:

$P = \$930 = \$40(PVIFA_{R\%,40}) + \$1{,}000(PVIF_{R\%,40})$

Using a spreadsheet, financial calculator, or trial and error we find:

$R = 4.373\%$

This is the semiannual interest rate, so the YTM is:

$YTM = 2 \times 4.373\% = 8.75\%$

20. Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are four months until the next coupon payment, so two months have passed since the last coupon payment. The accrued interest for the bond is:

Accrued interest = \$74/2 × 2/6 = \$12.33

And we calculate the clean price as:

Clean price = Dirty price – Accrued interest = \$968 – 12.33 = \$955.67

21. Accrued interest is the coupon payment for the period times the fraction of the period that has passed since the last coupon payment. Since we have a semiannual coupon bond, the coupon payment per six months is one-half of the annual coupon payment. There are two months until the next coupon payment, so four months have passed since the last coupon payment. The accrued interest for the bond is:

Accrued interest = \$68/2 × 4/6 = \$22.67

And we calculate the dirty price as:

Dirty price = Clean price + Accrued interest = \$1,073 + 22.67 = \$1,095.67

22. To find the number of years to maturity for the bond, we need to find the price of the bond. Since we already have the coupon rate, we can use the bond price equation, and solve for the number of years to maturity. We are given the current yield of the bond, so we can calculate the price as:

Current yield = .0755 = \$80/P_0
P_0 = \$80/.0755 = \$1,059.60

Now that we have the price of the bond, the bond price equation is:

$P = \$1{,}059.60 = \$80[(1 - (1/1.072)^t) / .072] + \$1{,}000/1.072^t$

We can solve this equation for *t* as follows:

$\$1{,}059.60(1.072)^t = \$1{,}111.11(1.072)^t - 1{,}111.11 + 1{,}000$

$111.11 = 51.51(1.072)^t$

$2.1570 = 1.072^t$

$t = \log 2.1570 / \log 1.072 = 11.06 \approx 11$ years

The bond has 11 years to maturity.

23. The bond has 14 years to maturity, so the bond price equation is:

$P = \$1{,}089.60 = \$36(PVIFA_{R\%,28}) + \$1{,}000(PVIF_{R\%,28})$

Using a spreadsheet, financial calculator, or trial and error we find:

$R = 3.116\%$

This is the semiannual interest rate, so the YTM is:

$YTM = 2 \times 3.116\% = 6.23\%$

The current yield is the annual coupon payment divided by the bond price, so:

Current yield = \$72 / \$1,089.60 = .0661 or 6.61%

24. *a*. The bond price is the present value of the cash flows from a bond. The YTM is the interest rate used in valuing the cash flows from a bond.

b. If the coupon rate is higher than the required return on a bond, the bond will sell at a premium, since it provides periodic income in the form of coupon payments in excess of that required by investors on other similar bonds. If the coupon rate is lower than the required return on a bond, the bond will sell at a discount since it provides insufficient coupon payments compared to that required by investors on other similar bonds. For premium bonds, the coupon rate exceeds the YTM; for discount bonds, the YTM exceeds the coupon rate, and for bonds selling at par, the YTM is equal to the coupon rate.

c. Current yield is defined as the annual coupon payment divided by the current bond price. For

premium bonds, the current yield exceeds the YTM, for discount bonds the current yield is less than the YTM, and for bonds selling at par value, the current yield is equal to the YTM. In all cases, the current yield plus the expected one-period capital gains yield of the bond must be equal to the required return.

25. The price of a zero coupon bond is the PV of the par, so:

a. $P_0 = \$1{,}000/1.045^{50} = \$110.71$

b. In one year, the bond will have 24 years to maturity, so the price will be:

$P_1 = \$1{,}000/1.045^{48} = \$120.90$

The interest deduction is the price of the bond at the end of the year, minus the price at the beginning of the year, so:

Year 1 interest deduction = $120.90 – 110.71 = $10.19

The price of the bond when it has one year left to maturity will be:

$P_{24} = \$1{,}000/1.045^2 = \$915.73$

Year 24 interest deduction = $1,000 – 915.73 = $84.27

c. Previous IRS regulations required a straight-line calculation of interest. The total interest received by the bondholder is:

Total interest = $1,000 – 110.71 = $889.29

The annual interest deduction is simply the total interest divided by the maturity of the bond, so the straight-line deduction is:

Annual interest deduction = $889.29 / 25 = $35.57

d. The company will prefer straight-line methods when allowed because the valuable interest deductions occur earlier in the life of the bond.

26. *a*. The coupon bonds have an 8% coupon which matches the 8% required return, so they will sell at par. The number of bonds that must be sold is the amount needed divided by the bond price, so:

Number of coupon bonds to sell = $30,000,000 / $1,000 = 30,000

The number of zero coupon bonds to sell would be:

Price of zero coupon bonds = $1,000/1.04^60 = $95.06

Number of zero coupon bonds to sell = $30,000,000 / $95.06 = 315,589

b. The repayment of the coupon bond will be the par value plus the last coupon payment times the number of bonds issued. So:

Coupon bonds repayment = 30,000($1,040) = $31,200,000

The repayment of the zero coupon bond will be the par value times the number of bonds issued, so:

Zeroes: repayment = 315,589($1,000) = $315,588,822

c. The total coupon payment for the coupon bonds will be the number bonds times the coupon payment. For the cash flow of the coupon bonds, we need to account for the tax deductibility of the interest payments. To do this, we will multiply the total coupon payment times one minus the tax rate. So:

Coupon bonds: (30,000)($80)(1–.35) = $1,560,000 cash outflow

Note that this is cash outflow since the company is making the interest payment.

For the zero coupon bonds, the first year interest payment is the difference in the price of the zero at the end of the year and the beginning of the year. The price of the zeroes in one year will be:

$P_1 = \$1{,}000/1.04^{58} = \$102.82$

The year 1 interest deduction per bond will be this price minus the price at the beginning of the year, which we found in part *b*, so:

Year 1 interest deduction per bond = $102.82 – 95.06 = $7.76

The total cash flow for the zeroes will be the interest deduction for the year times the number of zeroes sold, times the tax rate. The cash flow for the zeroes in year 1 will be:

Cash flows for zeroes in Year 1 = (315,589)($7.76)(.35) = $856,800.00

Notice the cash flow for the zeroes is a cash inflow. This is because of the tax deductibility of the imputed interest expense. That is, the company gets to write off the interest expense for the year even though the company did not have a cash flow for the interest expense. This reduces the company's tax liability, which is a cash inflow.

During the life of the bond, the zero generates cash inflows to the firm in the form of the interest tax shield of debt. We should note an important point here: If you find the PV of the cash flows from the coupon bond and the zero coupon bond, they will be the same. This is because of the much larger repayment amount for the zeroes.

27. We found the maturity of a bond in Problem 22. However, in this case, the maturity is indeterminate. A bond selling at par can have any length of maturity. In other words, when we solve the bond pricing equation as we did in Problem 22, the number of periods can be any positive number.

28. We first need to find the real interest rate on the savings. Using the Fisher equation, the real interest rate is:

$(1 + R) = (1 + r)(1 + h)$
$1 + .11 = (1 + r)(1 + .038)$
$r = .0694$ or 6.94%

Now we can use the future value of an annuity equation to find the annual deposit. Doing so, we find:

FVA = $C\{[(1 + r)^t - 1] / r\}$
$\$1{,}500{,}000 = \$C[(1.0694^{40} - 1) / .0694]$
$C = \$7{,}637.76$

Challenge

29. To find the capital gains yield and the current yield, we need to find the price of the bond. The current price of Bond P and the price of Bond P in one year is:

P: $P_0 = \$120(PVIFA_{7\%,5}) + \$1{,}000(PVIF_{7\%,5}) = \$1{,}116.69$

$P_1 = \$120(PVIFA_{7\%,4}) + \$1{,}000(PVIF_{7\%,4}) = \$1{,}097.19$

Current yield = \$120 / \$1,116.69 = .1075 or 10.75%

The capital gains yield is:

Capital gains yield = (New price – Original price) / Original price

Capital gains yield = (\$1,097.19 – 1,111.69) / \$1,116.69 = –.0175 or –1.75%

The current price of Bond D and the price of Bond D in one year is:

D: $P_0 = \$60(PVIFA_{7\%,5}) + \$1{,}000(PVIF_{7\%,5}) = \$883.31$

$P_1 = \$60(PVIFA_{7\%,4}) + \$1{,}000(PVIF_{7\%,4}) = \$902.81$

Current yield = \$60 / \$883.81 = .0679 or 6.79%

Capital gains yield = (\$902.81 – 883.31) / \$883.31 = +.0221 or +2.21%

All else held constant, premium bonds pay high current income while having price depreciation as maturity nears; discount bonds do not pay high current income but have price appreciation as maturity nears. For either bond, the total return is still 9%, but this return is distributed differently between current income and capital gains.

30. *a*. The rate of return you expect to earn if you purchase a bond and hold it until maturity is the YTM. The bond price equation for this bond is:

$P_0 = \$1{,}060 = \$70(\text{PVIFA}_{R\%,10}) + \$1{,}000(\text{PVIF}_{R\%,10})$

Using a spreadsheet, financial calculator, or trial and error we find:

R = YTM = 6.18%

b. To find our HPY, we need to find the price of the bond in two years. The price of the bond in two years, at the new interest rate, will be:

$P_2 = \$70(PVIFA_{5.18\%,8}) + \$1{,}000(PVIF_{5.18\%,8}) = \$1{,}116.92$

To calculate the HPY, we need to find the interest rate that equates the price we paid for the bond with the cash flows we received. The cash flows we received were $70 each year for two years, and the price of the bond when we sold it. The equation to find our HPY is:

$P_0 = \$1{,}060 = \$70(PVIFA_{R\%,2}) + \$1{,}116.92(PVIF_{R\%,2})$

Solving for *R*, we get:

R = HPY = 9.17%

The realized HPY is greater than the expected YTM when the bond was bought because interest rates dropped by 1 percent; bond prices rise when yields fall.

31. The price of any bond (or financial instrument) is the PV of the future cash flows. Even though Bond M makes different coupons payments, to find the price of the bond, we just find the PV of the cash flows. The PV of the cash flows for Bond M is:

$P_M = \$1{,}100(PVIFA_{3.5\%,16})(PVIF_{3.5\%,12}) + \$1{,}400(PVIFA_{3.5\%,12})(PVIF_{3.5\%,28}) + \$20{,}000(PVIF_{3.5\%,40})$

$P_M = \$19{,}018.78$

Notice that for the coupon payments of $1,400, we found the PVA for the coupon payments, and then discounted the lump sum back to today.

Bond N is a zero coupon bond with a $20,000 par value, therefore, the price of the bond is the PV of the par, or:

$P_N = \$20{,}000(PVIF_{3.5\%,40}) = \$5{,}051.45$

32. To calculate this, we need to set up an equation with the callable bond equal to a weighted average of the noncallable bonds. We will invest X percent of our money in the first

noncallable bond, which means our investment in Bond 3 (the other noncallable bond) will be (1 – X). The equation is:

$$
\begin{aligned}
C_2 &= C_1 X + C_3(1 - X) \\
8.25 &= 6.50 X + 12(1 - X) \\
8.25 &= 6.50 X + 12 - 12 X \\
X &= 0.68181
\end{aligned}
$$

So, we invest about 68 percent of our money in Bond 1, and about 32 percent in Bond 3. This combination of bonds should have the same value as the callable bond, excluding the value of the call. So:

$$
\begin{aligned}
P_2 &= 0.68181P_1 + 0.31819P_3 \\
P_2 &= 0.68181(106.375) + 0.31819(134.96875) \\
P_2 &= 115.4730
\end{aligned}
$$

The call value is the difference between this implied bond value and the actual bond price. So, the call value is:

Call value = 115.4730 – 103.50 = 11.9730

Assuming $1,000 par value, the call value is $119.73.

33. In general, this is not likely to happen, although it can (and did). The reason this bond has a negative YTM is that it is a callable U.S. Treasury bond. Market participants know this. Given the high coupon rate of the bond, it is extremely likely to be called, which means the bondholder will not receive all the cash flows promised. A better measure of the return on a callable bond is the yield to call (YTC). The YTC calculation is the basically the same as the YTM calculation, but the number of periods is the number of periods until the call date. If the YTC were calculated on this bond, it would be positive.

34. To find the present value, we need to find the real weekly interest rate. To find the real return, we need to use the effective annual rates in the Fisher equation. So, we find the real EAR is:

$(1 + R) = (1 + r)(1 + h)$
$1 + .084 = (1 + r)(1 + .037)$
$r = .0453$ or 4.53%

Now, to find the weekly interest rate, we need to find the APR. Using the equation for discrete compounding:

$EAR = [1 + (APR / m)]^m - 1$

We can solve for the APR. Doing so, we get:

$APR = m[(1 + EAR)^{1/m} - 1]$
$APR = 52[(1 + .0453)^{1/52} - 1]$
APR = .0443 or 4.43%

So, the weekly interest rate is:

Weekly rate = APR / 52
Weekly rate = .0443 / 52

Weekly rate = .0009 or 0.09%

Now we can find the present value of the cost of the roses. The real cash flows are an ordinary annuity, discounted at the real interest rate. So, the present value of the cost of the roses is:

$PVA = C(\{1 - [1/(1 + r)]^t\} / r)$
$PVA = \$5(\{1 - [1/(1 + .0009)]^{30(52)}\} / .0009)$
$PVA = \$4{,}312.13$

35. To answer this question, we need to find the monthly interest rate, which is the APR divided by 12. We also must be careful to use the real interest rate. The Fisher equation uses the effective annual rate, so, the real effective annual interest rates, and the monthly interest rates for each account are:

Stock account:
$(1 + R) = (1 + r)(1 + h)$
$1 + .11 = (1 + r)(1 + .04)$
$r = .0673$ or 6.73%

$APR = m[(1 + EAR)^{1/m} - 1]$
$APR = 12[(1 + .0673)^{1/12} - 1]$
$APR = .0653$ or 6.53%

Monthly rate = APR / 12
Monthly rate = .0653 / 12
Monthly rate = .0054 or 0.54%

Bond account:
$(1 + R) = (1 + r)(1 + h)$
$1 + .07 = (1 + r)(1 + .04)$
$r = .0288$ or 2.88%

$APR = m[(1 + EAR)^{1/m} - 1]$
$APR = 12[(1 + .0288)^{1/12} - 1]$
$APR = .0285$ or 2.85%

Monthly rate = APR / 12
Monthly rate = .0285 / 12
Monthly rate = .0024 or 0.24%

Now we can find the future value of the retirement account in real terms. The future value of each account will be:

Stock account:
$FVA = C\{(1 + r)^t - 1] / r\}$
$FVA = \$900\{[(1 + .0054)^{360} - 1] / .0054]\}$
FVA = $1,001,704.05

Bond account:

FVA = $C\ \{(1 + r)^t - 1] / r\}$

FVA = $\$450\{[(1 + .0024)^{360} - 1] / .0024]\}$

FVA = $255,475.17

The total future value of the retirement account will be the sum of the two accounts, or:

Account value = $1,001,704.05 + 255,475.17

Account value = $1,257,179.22

Now we need to find the monthly interest rate in retirement. We can use the same procedure that we used to find the monthly interest rates for the stock and bond accounts, so:

$(1 + R) = (1 + r)(1 + h)$
$1 + .09 = (1 + r)(1 + .04)$
$r = .0481$ or 4.81%

$APR = m[(1 + EAR)^{1/m} - 1]$
$APR = 12[(1 + .0481)^{1/12} - 1]$
$APR = .0470$ or 4.70%

Monthly rate = APR / 12
Monthly rate = .0470 / 12
Monthly rate = .0039 or 0.39%

Now we can find the real monthly withdrawal in retirement. Using the present value of an annuity equation and solving for the payment, we find:

$PVA = C(\{1 - [1/(1 + r)]^t\} / r)$
$\$1,257,179.22 = C(\{1 - [1/(1 + .0039)]^{300}\} / .0039)$
$C = \$7,134.82$

This is the real dollar amount of the monthly withdrawals. The nominal monthly withdrawals will increase by the inflation rate each month. To find the nominal dollar amount of the last withdrawal, we can increase the real dollar withdrawal by the inflation rate. We can increase the real withdrawal by the effective annual inflation rate since we are only interested in the nominal amount of the last withdrawal. So, the last withdrawal in nominal terms will be:

$FV = PV(1 + r)^t$
$FV = \$7,134.82(1 + .04)^{(30 + 25)}$
$FV = \$61,690.29$

Calculator Solutions

3.

Enter	10	8.75%		\$75	\$1,000

	N	I/Y	PV	PMT	FV
Solve for			$918.89		

4.

	N	I/Y	PV	PMT	FV
Enter	9		±$934	$90	$1,000
Solve for		10.15%			

5.

	N	I/Y	PV	PMT	FV
Enter	13	7.5%	±$1,045		$1,000
Solve for				$80.54	

Coupon rate = $80.54 / $1,000 = 8.05%

6.

	N	I/Y	PV	PMT	FV
Enter	20	3.70%		$34.50	$1,000
Solve for			$965.10		

7.

	N	I/Y	PV	PMT	FV
Enter	20		±$1,050	$42	$1,000
Solve for		3.837%			

3.837% × 2 = 7.67%

8.

	N	I/Y	PV	PMT	FV
Enter	29	3.40%	±$924		$1,000
Solve for				$29.84	

($29.84 / $1,000)(2) = 5.97%

15. Bond X

P_0

	N	I/Y	PV	PMT	FV
Enter	13	6%		$80	$1,000
Solve for			$1,177.05		

P_1

	N	I/Y	PV	PMT	FV
Enter	12	6%		$80	$1,000
Solve for			$1,167.68		

P_3

	N	I/Y	PV	PMT	FV
Enter	10	6%		$80	$1,000
Solve for			$1,147.20		

P_8

	N	I/Y	PV	PMT	FV
Enter	5	6%		$80	$1,000
Solve for			$1,084.25		

P_{12}

Enter	1	6%		$80	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,018.87		

Bond Y

P_0

Enter	13	8%		$60	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$841.92		

P_1

Enter	12	8%		$60	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$849.28		

P_3

Enter	10	8%		$60	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$865.80		

P_8

Enter	5	8%		$60	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$920.15		

P_{12}

Enter	1	8%		$60	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$981.48		

16. If both bonds sell at par, the initial YTM on both bonds is the coupon rate, 9 percent. If the YTM suddenly rises to 11 percent:

P_{Sam}

Enter	6	5.5%		$45	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$950.04		

$\Delta P_{Sam}\% = (\$950.04 - 1{,}000) / \$1{,}000 = -5.00\%$

P_{Dave}

Enter	40	5.5%		$45	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**

Solve for $839.54

$\Delta P_{Dave}\% = (\$839.54 - 1{,}000) / \$1{,}000 = -16.05\%$

If the YTM suddenly falls to 7 percent:

P_{Sam}

Enter	6	3.5%		$45	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**

Solve for $1,053.29

$\Delta P_{Sam}\% = (\$1{,}053.29 - 1{,}000) / \$1{,}000 = +5.33\%$

P_{Dave}

Enter	40	3.5%		$45	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,1213.55		

$\Delta P_{Dave}\% = (\$1,213.55 – 1,000) / \$1,000 = + 21.36\%$

All else the same, the longer the maturity of a bond, the greater is its price sensitivity to changes
in interest rates.

17. Initially, at a YTM of 8 percent, the prices of the two bonds are:

P_J

Enter	18	4%		$20	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$746.81		

P_K

Enter	18	4%		$60	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,253.19		

If the YTM rises from 8 percent to 10 percent:

P_J

Enter	18	5%		$20	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$649.31		

$\Delta P_J\% = (\$649.31 – 746.81) / \$746.81 = – 13.06\%$

P_K

Enter	18	5%		$60	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,116.90		

$\Delta P_K\% = (\$1,116.90 – 1,253.19) / \$1,253.19 = – 10.88\%$

If the YTM declines from 8 percent to 6 percent:

P_J

Enter	18	3%		$20	$1,000

	N	I/Y	PV	PMT	FV
Solve for			$862.46		

$\Delta P_J\% = (\$862.46 - 746.81) / \$746.81 = + 15.49\%$

P_K

Enter	18	3%		$60	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,412.61		

$\Delta P_K\% = (\$1,412.61 - 1,253.19) / \$1,253.19 = + 12.72\%$

All else the same, the lower the coupon rate on a bond, the greater is its price sensitivity to changes in interest rates.

18.

Enter	18		±$1,068	$46	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		4.06%			

4.06% × 2 = 8.12%

Enter	8.12 %		2
	NOM	**EFF**	**C/Y**
Solve for		8.29%	

19. The company should set the coupon rate on its new bonds equal to the required return; the required return can be observed in the market by finding the YTM on outstanding bonds of the company.

Enter	40		±$930	$35	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		4.373%			

4.373% × 2 = 8.75%

22. Current yield = .0755 = \$90/P_0 ; P_0 = \$90/.0755 = \$1,059.60

Enter		7.2%	±$1,059.60	$80	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for	11.06				

11.06 or ≈ 11 years

23.

Enter	28		±$1,089.60	$36	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		3.116%			

3.116% × 2 = 6.23%

25.

a. P_o

Enter	50	4.5%			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$110.71		

b. P_1

Enter	48	4.5%			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$120.90		

year 1 interest deduction = $120.90 – 110.71 = $10.19

P_{19}

Enter	2	4.5%			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$915.73		

year 25 interest deduction = $1,000 – 915.73 = $84.27

c. Total interest = $1,000 – 110.71 = $889.29
Annual interest deduction = $889.29 / 25 = $35.57

d. The company will prefer straight-line method when allowed because the valuable interest deductions occur earlier in the life of the bond.

26. *a*. The coupon bonds have an 8% coupon rate, which matches the 8% required return, so they will sell at par; # of bonds = $30,000,000/$1,000 = 30,000.

For the zeroes:

Enter	60	4%			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$95.06		

$30,000,000/$95.06 = 315,589 will be issued.

b. Coupon bonds: repayment = 30,000($1,080) = $32,400,000
Zeroes: repayment = 315,589($1,000) = $315,588,822

c. Coupon bonds: (30,000)($80)(1 –.35) = $1,560,000 cash outflow
Zeroes:

Enter	58	4%			$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$102.82		

year 1 interest deduction = $102.82 – 95.06 = $7.76
(315,589)($7.76)(.35) = $856,800.00 cash inflow

During the life of the bond, the zero generates cash inflows to the firm in the form of the interest tax shield of debt.

29.

Bond P

P_0

Enter	5	9%		$120	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,116.69		

P_1

Enter	4	9%		$120	$1,000

	N	I/Y	PV	PMT	FV
Solve for			$1,097.19		

Current yield = $120 / $1,116.69 = 10.75%

Capital gains yield = ($1,097.19 – 1,116.69) / $1,116.69 = –1.75%

Bond D

P_0

	N	I/Y	PV	PMT	FV
Enter	5	9%		$60	$1,000
Solve for			$883.31		

P_1

Enter	4	9%		$60	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$902.81		

Current yield = $60 / $883.31 = 6.79%

Capital gains yield = ($902.81 – 883.31) / $883.31 = 2.21%

All else held constant, premium bonds pay high current income while having price depreciation as maturity nears; discount bonds do not pay high current income but have price appreciation as maturity nears. For either bond, the total return is still 9%, but this return is distributed differently between current income and capital gains.

30.

a.

Enter	10		±$1,060	$70	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		6.18%			

This is the rate of return you expect to earn on your investment when you purchase the bond.

b.

Enter	8	5.18%		$70	$1,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$1,116.92		

The HPY is:

Enter	2		±$1,060	$70	$1,116.92
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for		9.17%			

The realized HPY is greater than the expected YTM when the bond was bought because interest rates dropped by 1 percent; bond prices rise when yields fall.

31.

P_M

CF0	$0
C01	$0
F01	12
C02	$1,100
F02	16

C03	$1,400
F03	11
C04	$21,400
F04	1

I = 3.5%

NPV CPT

$19,018.78

P_N

Enter	40	3.5%			$20,000
	N	**I/Y**	**PV**	**PMT**	**FV**
Solve for			$5,051.45		