# Coding and Modulation in Digital Communication Systems

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# Introduction

The wireless communication is characterized by the high bit error rate that degrades the performance of the system.

One challenge is to design the efficient coding that can combat the imperfect channel.

Convolutional codes has been widely used in the application of deep space and satellite communication, cellular mobile, digital broadcasting etc. However, it is not an efficient coding while the burst errors happen.

Concatenated interleaving and convolutional coding, Turbo Code with the distinguishing features of excellent performance has become a part of the mainstream of telecommunication theory and practice.

# Shannon Capacity Limit

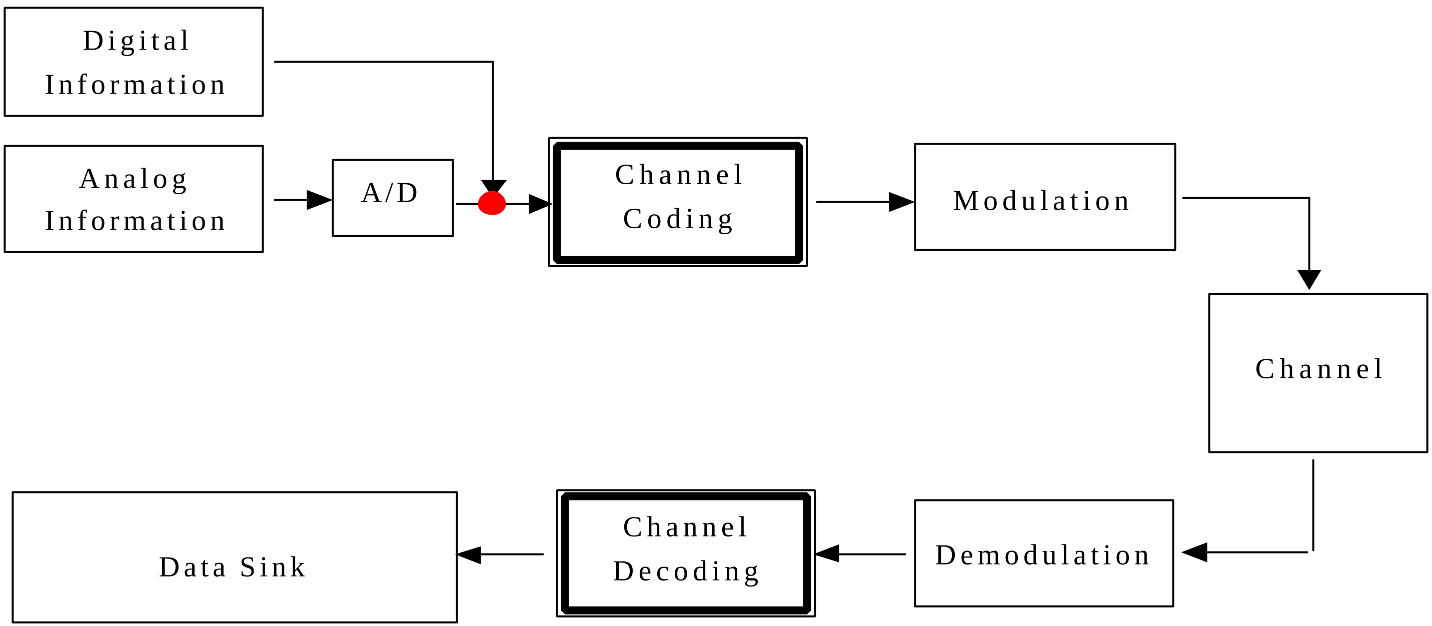


Fig.1 A Digital Communication System

Channel Encoder is incorporated to add redundancy which is used to minimize the transmission errors.

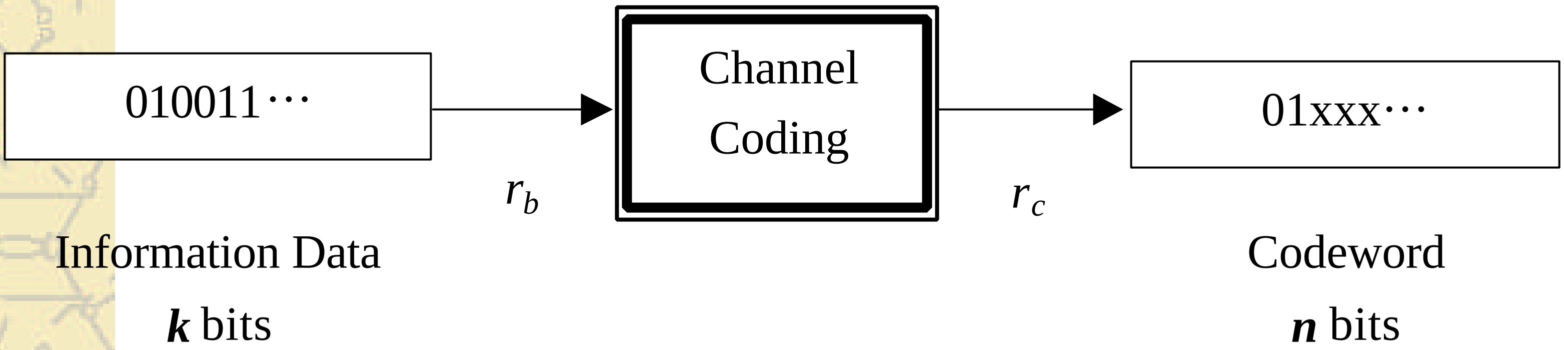


Fig. 2 The Input/Output of the channel encoder

The channel encoder assigns each information data of ***k*** bits to an ***n***-bit codeword, denoted (***k***,***n***) code.

The encoder is characterized by the ratio ***R=k/n***, called the *Coding rate*. Hence, the data rate at the output of the channel encoder is $r_c = r_b / \boldsymbol{R}$ bps.

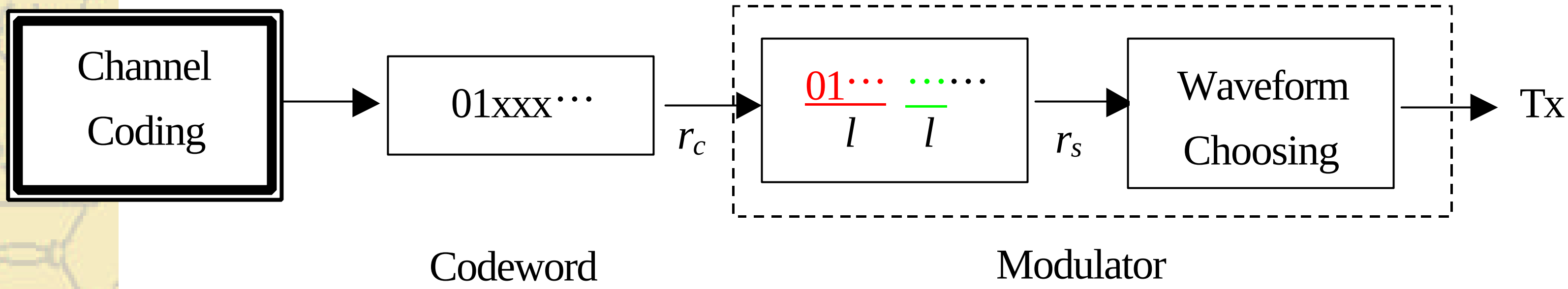


Fig. 3 Signal Flow in Modulator

The M-ary modulator maps a block of *l* binary digits (symbol) from the channel encoder into one of the M possible waveforms, where $M=2^l$.

The duration of the modulator output waveform is ***T*** sec. So, the symbol rate is $r_s=1/T$.

The minimum signal bandwidth is equal to $r_s$ Hz.  Hence,

$$r_s = \frac{r_c}{l} = \frac{r_b}{Rl} \quad (1)$$

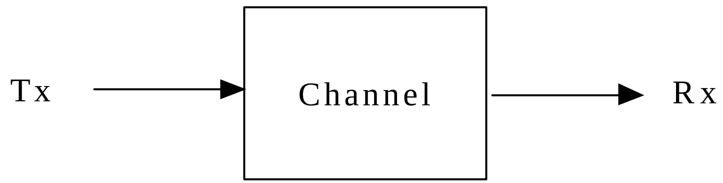


Fig. 4 Signal Flow through Channel

The term of channel is referred to the frequency range located to a particular service.

For example: IS-95C System in Taiwan (CDMA-2000-1X)

(1)AMPS system with the bandwidth 5M：

(Down) 875~880MHz (869~894)

(Up) 830~835MHz (824~849)

(2)5MHz = (4 Channels) ×(1.25MHz/Channel)

= 4×(CDMA Carriers)

The goal of the decoder is to minimize the effects of channel impairment. In the receiver, the demodulator generates a binary or analog sequences as the best estimation of the transmitted sequences.

In order to increase the bandwidth efficiency, the more effective signal design is obtained by combining coding and modulation scheme. The combined coding and modulation is called the coded modulation, e.g. TCM.

Every communication system has a limited bandwidth due to the capacitances and inductances which prevent instantaneous change of signals. The system bandwidth ***B*** limits the speed of signal variations.

The character of bandwidth limitation is quantified as *spectral efficiency* and denoted as $h$, defined as

$$h = \frac{r_b}{B} \qquad (2)$$

It can be expressed as

$$h = \frac{r_s l R}{B} \qquad (3)$$

where $r_s$ is the symbol rate and $\boldsymbol{R}$ is the coding rate as defined previously.

As the minimum required bandwidth to transmit the modulated signal is $r_s$ Hz, i.e. $B_{min} = r_s$.

Hence, the maximum spectral efficiency is

$$h_{\max} = l \cdot R \qquad (4)$$

Power efficiency is defined as the required bit energy to noise power spectral density ratio, $\boldsymbol{E_b/N_0}$, to achieve a specified bit error rate, $\boldsymbol{P_b}$, which is the important parameter used to measure the reliability of a digital communication system. The *signal-to-noise ratio* (SNR), $\boldsymbol{S/N}$, is related to $\boldsymbol{E_b/N_0}$ as

$$\frac{S}{N} = \frac{E_b \cdot h \cdot B}{N_0 \cdot B} \qquad (5)$$

When the spectral efficiency is maximum, the SNR becomes

$$\frac{S}{N} = \frac{E_b}{N_0} \cdot l \cdot R \qquad (6)$$

Shannon's main theorem introduces the concepts of channel capacity, $\boldsymbol{C}$ , as the maximum rate at which the information can be transmitted through a noisy channel.

For AWGN (additive white Gaussian noise) channel, it is given as

$$C = B \cdot \log_2(1 + \frac{S}{N}) \quad \text{bits/sec} \qquad (7)$$

It guarantees the existence of codes that can be achieve arbitrary small probability of error if the information transmission rate $r_b$ is smaller than the channel capacity.

In the Shannon theorem, it doesn't indicate how to design the specific codes to achieve the maximum information rate with an arbitrary small error probability. Hence, it is the motivation to develop an effective coding scheme.

Assume the information rate takes its maximum possible value for error-free transmission (equal to the channel capacity, ***C***). Then, the maximum spectral efficiency can be expressed as

$$h_{\max} = \frac{C}{B} = \log_2(1 + \frac{E_b}{N_0} \cdot Rl) \quad (8)$$

Substitute the maximum spectral efficiency, $h_{max}$=*Rl*, the equation becomes

$$h_{\max} = \log_2(1 + h_{\max} \cdot \frac{E_b}{N_0}) \quad (9)$$

Hence, the minimum required $\boldsymbol{E_b/N_0}$ for error-free transmission is

$$\frac{E_b}{N_0} = \frac{2^{h_{\max}} - 1}{\boldsymbol{h}_{\max}} \qquad (10)$$

If the bandwidth is unlimited, i.e. ***B*** approaches infinite ($\boldsymbol{B} \rightarrow \infty$ ), the spectral efficiency approaches to zero ($\eta_{max} \rightarrow \infty$ ), the minimum required $\boldsymbol{E_b/N_0}$ becomes

$$\lim_{B \to \infty} \frac{E_b}{N_0} = \ln 2 = 0.693$$

$$= -1.59 dB$$

Fig. 5 shows the bandwidth efficiency of various modulation and coding schemes at the bit error rate of $10^{-5}$

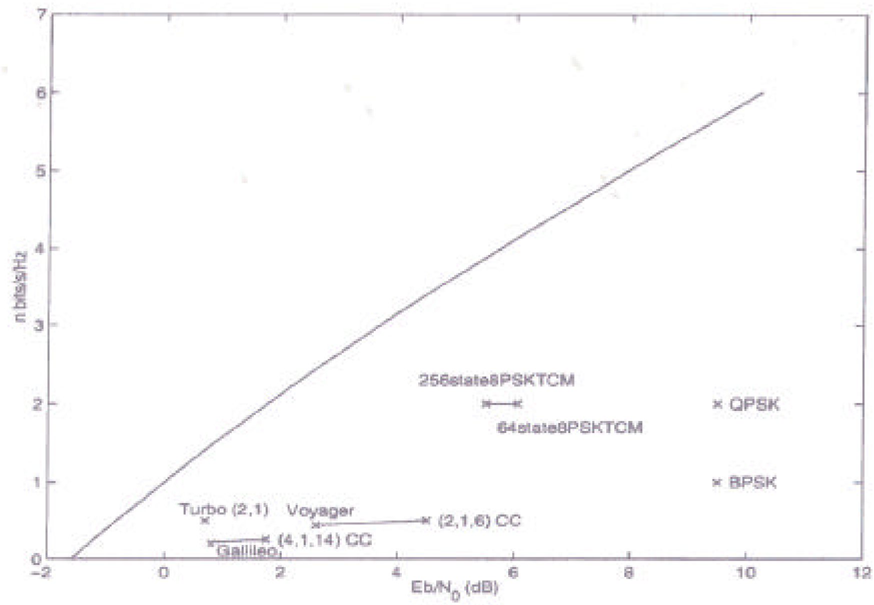


Fig. 5 Bandwidth efficiency of various modulation and coding schemes