Constitutive Model for Geotechnical Materials

Chapter 1 Introduction

1.1 Idealization of stress – strain relation

(1) Elastic

(2) Case (II) Rigid – perfectly plastic

(3) Case (III) Rigid – strain hardening

(4) Linear elastic – perfectly plastic

(5) Linear elastic – strain hardening

1.1.1 Elasticity

(1) Equilibrium Condition

(2) Compatibility Condition (Deformation of geometry)

(3) Stress – Strain relation (Constitutive Law)

1.1.2 Plasticity

(1) Elastic Stress – Strain Curve

(2) Yield Criterion

(3) Flow Rule

(4) Loading、Unloading、Reloading

1.2 Simplified Uniaxial Stress – Strain Curve

1.3 Experimental Results

1.4 Compressive test & Bauschinger Effects

(1) Hardening

(2) Effects on Stress – Strain Curve

(3) Tangent Modulus Et and Plastic Modulus Ep

(4) Hardening Rules

1.5 Vector and Tensor

1.5.1 Introduction

1.5.2 Vector and Index Notation

1.5.3 Summation Convention

1.5.4 Kronecker delta δ_{ij} (Substitution symbol)

1.5.5 Permutation Symbol e_{ijk}

1.5.6 Vector Manipulation

1.5.7 Matric & Determinate

1.5.8 Transformation of Coordination

Ex： The $\sigma-\varepsilon$ response in simple tension for a material is a material is approximated by：

$$\boldsymbol{\sigma}=\boldsymbol{\sigma}_0+\mathrm{m}\times\boldsymbol{\varepsilon}^{\mathrm{p}} \qquad \text{for} \quad \boldsymbol{\sigma}>\boldsymbol{\sigma}_0$$

$$\boldsymbol{\varepsilon}^{\mathrm{e}}=\boldsymbol{\sigma}/\mathrm{E}$$

$\boldsymbol{\sigma}_0=207\mathrm{MPa}$ 、 $\mathrm{E}=207\mathrm{GPa}$ 、 $\mathrm{m}=25.9\mathrm{GPa}$. material simple is first stretched to a total strain $\varepsilon=0.007$, is subsequently returned to its initial strain–free state ($\boldsymbol{\varepsilon}=0$) , and then is reloaded in tension gain to $\varepsilon=0.007$ .sketch the stress – strain Curve for following hardening rules：(1)Isotropic hardening (2)Kinematic hardening (3)Independent hardening acting tensile and Compressive hardening.

Sol：

$$\mathrm{E_p}=\frac{\mathrm{d}\boldsymbol{\sigma}}{\mathrm{d}\boldsymbol{\varepsilon}^{\mathrm{p}}}=\mathrm{m}=25.9\ \mathrm{Gpa}=25900\ \mathrm{Mpa}$$

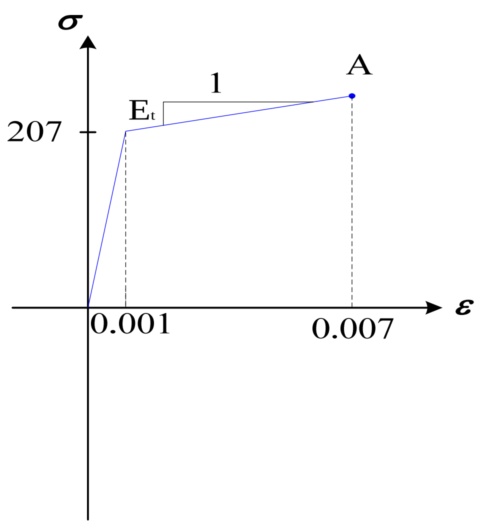
$$E_t = \frac{1}{\frac{1}{E}+\frac{1}{E_p}} = \frac{1}{\frac{1}{207\times10^3}+\frac{1}{25900}} = 23020\ \text{Mpa}$$

在受張力時之初始降伏點(即$\varepsilon_p = 0$)之降伏應力$\sigma_0 = 207\ \text{MPa}$

即降伏時之應變$\varepsilon_0 = 207/207\times10^3 = 0.001$

當 Strain=0.007 之應力σ_A

$$\sigma_A = \sigma_0 + E_t\cdot\Delta\varepsilon = 207 + 23020\times(0.007-0.001) = 345.12\ \text{Mpa}$$



(1) Isotropic hardening

(a)當解拉，進入壓力時之初始降伏應力為$\sigma_B = -345.12\ \text{Mpa}$

$$\varepsilon_B = 0.007 - \frac{345.12+345.12}{207\times10^3} = 3.67\times10^{-3}$$

(b)當$\varepsilon = 0$時，$\sigma = \sigma_c$

$$\sigma_c = -345.12 + 23020\cdot(-3.67\times10^{-3}) = -429.50\ \text{Mpa}$$

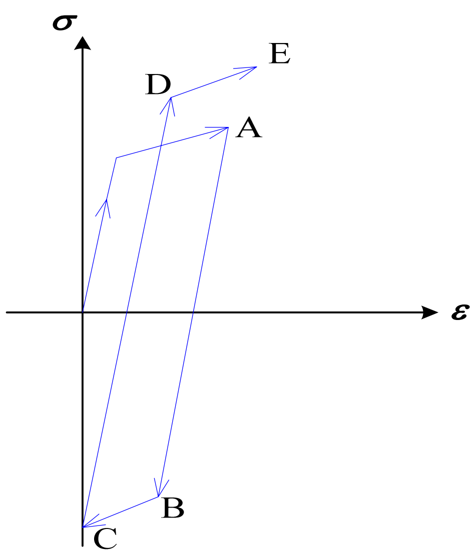
(c)再施加拉力至 D 點時再度降伏，$\sigma_D = 429.50\ \text{Mpa}$

$$\varepsilon_D = 0.007 - \frac{2\times429.50}{207\times10^3} = 4.15\times10^{-3}$$

(d)當拉力繼續施加至 E 點時，$\varepsilon_E = 0.007$

$$\sigma_E = 429.50 + E_t \cdot \Delta\varepsilon$$

$$= 429.50 + 23020 \times (0.007 - 4.15 \times 10^{-3}) = 495.11 \text{ Mpa}$$



(2) Kinematic hardening

(a)當拉力解壓至 B 點：

$$\sigma_D - \sigma_C = 2\sigma_0$$

$$\sigma_B = 2 \times 207 - 345.12 \quad \Rightarrow \quad \sigma_B = -68.88 \text{ Mpa}$$

$$\varepsilon_B = 0.007 - \frac{2 \times 207}{207 \times 10^3} = 0.005$$

(b)加壓至 $\varepsilon = 0$ 時

$$\sigma_C = -68.88 + E_t \cdot \Delta\varepsilon$$

$$= -68.88 + 23020 \times (-0.005) = -183.98 \text{ Mpa}$$

(c)解壓，並施加拉力至 D 點時

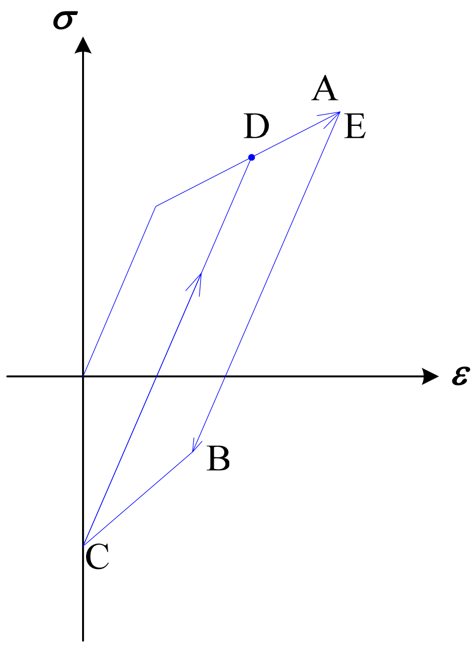
$$\sigma_D - \sigma_C = 2\sigma_0$$

$$\sigma_D = 2\times 207 - 183.98 = 230 \text{ Mpa}$$

$$\varepsilon_D = \frac{2\times 207}{207\times 10^3} = 2\times 10^{-3}$$

(d)達到時 $\varepsilon = 0.007$ ， $\sigma = \sigma_E$

$$\sigma_E = 230 + 23020\times(0.007 - 0.002) = 345.12 \text{ Mpa}$$



(3) Independent hardening

(a)當拉力解壓並加壓至 B 點， $\sigma_B = -207 \text{ Mpa}$

$$\varepsilon_B = 0.007 - \frac{345.12 + 207}{207\times 10^3} = 4.33\times 10^{-3}$$

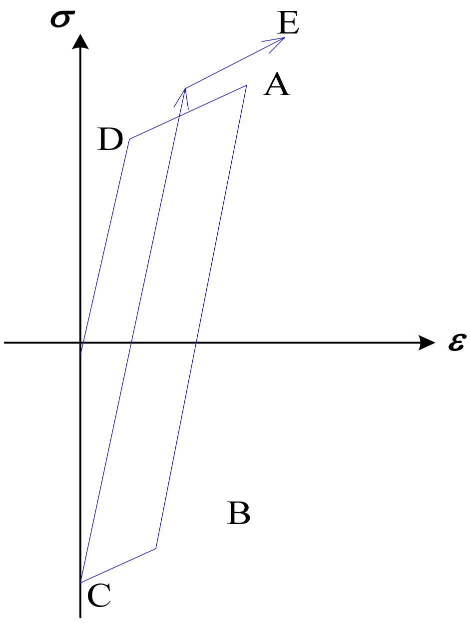
(b)當 $\varepsilon = \varepsilon_C = 0$ 時

$$\sigma_C = -207 + 23020\times(-0.00433) = -306.74 \text{ Mpa}$$

(c) $\sigma_D = \sigma_A = 345.12 \text{ Mpa}$

$$\varepsilon_B = \frac{306.74 + 345.12}{207\times 10^3} = 3.15\times 10^{-3}$$

(d) $\sigma_E = 345.12 + 23020 \times (0.007 - 3.15 \times 10^{-3}) = 433.77$ Mpa



Chapter 2 stress and Yielding Criterion

2.1 stress

2.2 Normal &Shear Stress

2.3 Principal Stress and Stress invariant

2.4 Principal Shear Stress

2.5 Deviatoric Stress Tensor

2.6 Octahedral Stresses

2.7 Geometric Repressentation of Strees

2.8 Yield Criteria

2.8.1 Elasticity

(1) Tresca Yield Criteruion

(2) von Mises Yield Criteruion

(3) Mohr – Coulomb Criteruion

(4) Drucker – Prager Criteruion

Ex： A Concrete has a compressive strength $fc' = 3000$ psi and a tensile strength $ft' = 300$ psi consider this concrete subjected a combination of compressive stress 1500 psi and shear stress $\tau = 600$ psi using (1) Mohr – Coulomb (2) Drucker – Prager Criteria to check if this material is yield ?

Sol：

(1) Mohr – Coulomb

$ft' = 300 = \sigma_1 \quad \sigma_3 = 0$

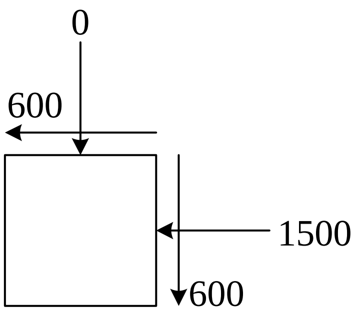
$$ft' = \frac{2c \cdot \cos\phi}{1+\sin\phi} = 300 \qquad \ldots① \qquad 2c \cdot \cos\phi = 300(1+\sin\phi)$$

$$fc' = 3000 = \frac{2c \cdot \cos\phi}{1-\sin\phi} \qquad \ldots② \qquad 2c \cdot \cos\phi = 3000(1-\sin\phi)$$

$$\therefore \quad 3000 - 3000\sin\phi - 300 - 3000\sin\phi = 0$$

$$\sin\phi = \frac{2700}{3300}$$

$\phi = 54.9°$ ，代入①得 $c = 474.34$ psi



$$\sigma_1 = -\frac{1500}{2} + \sqrt{(\frac{1500}{2})^2 + 600^2} = 210.47$$

$$\sigma_3 = -\frac{1500}{2} - \sqrt{(\frac{1500}{2})^2 + 600^2} = -1710.47$$

求在下 $\sigma_3 = -1710.47$ 下最大之 $\sigma_1 = ?$ (即材料降伏時提供之最大

主應力 σ_1)

$$\sigma_1 = \frac{1+\sin 54.9}{2\times 474.34\cos 54.9} + 1710.47 \times \frac{1-\sin 54.9}{2\times 474.34\cos 54.9} = 1$$

得 $\sigma_1 = 128.94$ psi < 210.47 $\qquad\qquad$ ∴材料 yield

(2) Drucker – Prager

$$\alpha I_1 + \sqrt{J_2} - K_d = 0$$

①Tension $\sigma_1 = 300$psi $\quad \sigma_2 = \sigma_3 = 0$

$$\therefore I_1 = 300$$

$$J_2 = \frac{1}{6}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\right]$$

$$= \frac{1}{6}\left[(300)^2 + 300^2\right] = 30000$$

$$\therefore \quad 300\alpha + \sqrt{30000} - K_d = 0 \qquad \ldots(1)$$

② Compressio $\sigma_1 = \sigma_2 = 0 \quad \sigma_3 = -3000$

$$\therefore I_1 = -3000$$

$$J_2 = \frac{1}{6}\left[2\times(3000)^2\right] = 3000000$$

$$-3000\alpha + \sqrt{3000000} - K_d = 0 \qquad \ldots(2)$$

$(1)-(2)$

$$3300\alpha = 1558.85 \quad \therefore \alpha = 0.4724$$

$$K_d = 300\times 0.4724 + \sqrt{30000} \quad \therefore K_d = 314.92$$

$$I_1 = -1500$$

$$J_2 = \frac{1}{6}\left[(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2\right] + \sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2$$

$$= \frac{1}{6}\left[2\times(-1500)^2\right] + 600^2 = 1110000$$

$$f(I_1, J_2) = 0.4724\times(-1500) + \sqrt{1110000} - 314.92 = 30.14 > 0$$

$\therefore$ yield

Chapter 3 Strain tensor & Stress – Strain Relation

3.1 Displacement & deformation

3.2 Rigid Body Motion & Deformation

3.3 Strain vectior $\vec{\delta}$ and Normal Strain $\vec{\varepsilon}_n$ 、Shear Strain $\vec{\varepsilon}_{ns}$

3.4 Engineering Strain

3.5 Principl Strain

3.6 Octahedral Strains

3.7 Deviatoric Strain tensor

3.8 Invariant of deviatoric Strain

3.9 $\sigma - \varepsilon$ relation for isotropic linear elastic material

3.10 Isotropic Linear Elastic Stress – Strain Relations in Matrix Form

3.11 Non – linear Elastic Isotropic Stress – Strain Relation

Ex： Assume the Constitutive law for a nonlinear elastic material is given by Complementary Energy density

$$\Omega(\sigma_{ij}) = aJ_2 + bI_1J_2$$

The $\sigma - \varepsilon$ relation of the material in simple tension is

$$10^3\varepsilon = \sigma/10 + (\sigma/10)^2$$

(a) Find the Constant a、b

(b) Find all Strain Components when loading path from $(\sigma,\tau) =$ (0,0) to (30,10) ksi

(c) Find the γ_{xy} & I'_1 when loading path from (0,0) to (0,10)

Sol： $\Omega = (\sigma_{ij}) = aJ_2 + bI_1J_2$

(a) $\varepsilon_{ij} = \partial\Omega/\partial\sigma_{ij} = a\partial J_2/\partial\sigma_{ij} + bI_1\cdot\partial J_2/\partial\sigma_{ij} + bJ_2\cdot\partial I_1/\partial\sigma_{ij}$

$$= \partial\Omega / \partial\boldsymbol{\sigma}_{ij} = (a + bI_1)\partial J_2 / \partial\boldsymbol{\sigma}_{ij} + bJ_2 \cdot \partial I_1 / \partial\boldsymbol{\sigma}_{ij} \qquad \ldots①$$

$$\because \quad \partial J_2 / \partial\boldsymbol{\sigma}_{ij} = S_{ij} \quad ; \quad \partial I_1 / \partial\boldsymbol{\sigma}_{ij} = \boldsymbol{\delta}_{ij}$$

$$\therefore \quad \boldsymbol{\varepsilon}_{ij} = (a + bI_1) \cdot S_{ij} + bJ_2 \cdot \boldsymbol{\delta}_{ij} \qquad \ldots②$$

For simple tension $\boldsymbol{\sigma}_{11} = \boldsymbol{\sigma}$, $\boldsymbol{\sigma}_{ij}$ others=0

$$\therefore \quad I_1 = \boldsymbol{\sigma}$$

$$J_2 = \frac{1}{3}\boldsymbol{\sigma}^2$$

$$S_{11} = \boldsymbol{\sigma} - \frac{1}{3}\boldsymbol{\sigma} = \frac{2}{3}\boldsymbol{\sigma}$$

$$\therefore \quad \boldsymbol{\varepsilon}_{11} = \boldsymbol{\varepsilon} = (a + b\boldsymbol{\sigma}) \cdot \frac{2}{3}\boldsymbol{\sigma} + b\frac{1}{3}\boldsymbol{\sigma}^2$$

$$= \frac{2a}{3}\boldsymbol{\sigma} + b\boldsymbol{\sigma}^2 \qquad \ldots③$$

$$\because \quad \boldsymbol{\varepsilon} = 10^{-4}\boldsymbol{\sigma} + 10^{-5}\boldsymbol{\sigma}^2$$

$$\therefore \quad a = \frac{3}{2} \times 10^{-4}\boldsymbol{\sigma} \quad , \quad b = 10^{-5}$$

$$\therefore \quad \boldsymbol{\varepsilon}_{ij} = (\frac{3}{2} \times 10^{-4} + 10^{-5} I_1) \cdot S_{ij} + 10^{-5} J_2 \cdot \boldsymbol{\delta}_{ij}$$

(b) $\boldsymbol{\varepsilon}_{ij} = (1.5 \times 10^{-4} + 10^{-5} I_1) S_{ij} + 10^{-5} J_2 \cdot \boldsymbol{\delta}_{ij}$

Stress path from (0,0) to (30,10)

$$\therefore \quad I_1 = 30 \quad \boldsymbol{\sigma}_{11} = 30\text{ksi} \quad \boldsymbol{\tau}_{12} = \boldsymbol{\sigma}_{12} = 10\text{ksi} \quad \text{others} = 0$$

$$J_2 = \frac{1}{6}(2 \times 30^2 + 6 \times 10^2) = 400$$

$$\therefore \quad \boldsymbol{\varepsilon}_{ij} = (4.5 \times 10^{-4}) \cdot S_{ij} + 4 \times 10^{-3} J_2 \cdot \boldsymbol{\delta}_{ij}$$

$$S_{ij} = \begin{pmatrix} 20 & 10 & 0 \\ 10 & -10 & 0 \\ 0 & 0 & -10 \end{pmatrix}$$

$$\therefore \quad \boldsymbol{\varepsilon}_{ij} = (4.5\times10^{-4})\cdot\begin{pmatrix} 20 & 10 & 0 \\ 10 & -10 & 0 \\ 0 & 0 & -10 \end{pmatrix} + 4\times10^{-3}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 130 & 45 & 0 \\ 45 & -5 & 0 \\ 0 & 0 & -5 \end{pmatrix}\times10^{-4}$$

(c) Stress path from (0,0) to (0,10)

$$I_1 = 0 \quad J_2 = 100 \qquad \Rightarrow \qquad \boldsymbol{\varepsilon}_{ij} = 1.5\times10^{-4}\cdot S_{ij} + 10^{-3}\cdot\boldsymbol{\delta}_{ij}$$

$$\gamma_{xy} = 2\boldsymbol{\varepsilon}_{12} = 2\times(1.5\times10^{-4}S_{12} + 0) = 2\times(1.5\times10^{-4}\times10) = 3\times10^{-3}$$

$$\boldsymbol{\varepsilon}_{KK} = I'_1 = 1.5\times10^{-4}\cdot S_{KK} + 10^{-3}\cdot\boldsymbol{\delta}_{KK} = 3\times10^{-3}$$

Chapter 4 $\sigma - \varepsilon$ Relations for perfectly plastic Materials

4.1 Introduction

4.1.1 Elastic Limit & Yield Function

4.1.2 Criterion for Loading & Unloding

4.1.3 Elastic & Plastic Strain Increment ensors

4.2 Plastic & Potential Function and Flow Rule

4.3 Flow rule Associated with von Mises Yield Function

4.4 Flow rule Associated with Tresca Yield Function

4.5 Flow rule Associated with Mohr－Coulomb Function

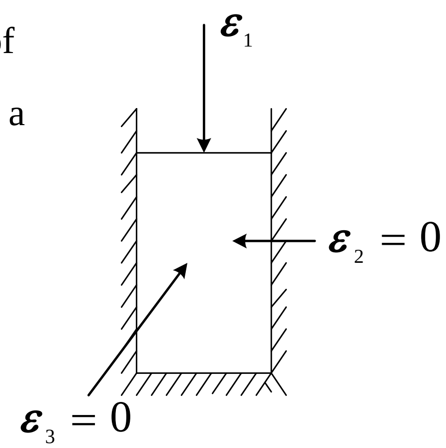
4.6 Flow rule Associated with Drucker－Prger Yield Function

4.7 Incremental Stress – Strain Relationshis

4.7.1 Constitutive Relation in Form

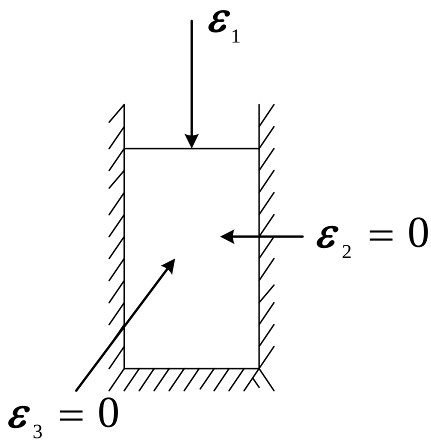
4.7.2 Constitutive Relation in General Form of Non – associated Material

Ex： Examine the $\sigma_1 - \varepsilon_1$ behavior of Drucker－Prger material under a uniaxial state of strain test.



Sol：

$$\boldsymbol{\varepsilon}_{ij} = \begin{pmatrix} \boldsymbol{\varepsilon}_1 \\ 0 \\ 0 \end{pmatrix} \qquad \boldsymbol{\varepsilon}_{ij} = \begin{pmatrix} 2/3\boldsymbol{\varepsilon}_1 \\ -1/3\boldsymbol{\varepsilon}_1 \\ -1/3\boldsymbol{\varepsilon}_1 \end{pmatrix}$$



(1)在 elastic ranre

$$\boldsymbol{\sigma}_{ij} = 2Ge_{ij} + K \cdot \boldsymbol{\varepsilon}_{KK} \cdot \boldsymbol{\delta}_{ij}$$

$$\boldsymbol{\sigma}_1 = 2G \cdot 2\boldsymbol{\varepsilon}_1/3 + K \cdot \boldsymbol{\varepsilon}_1 \cdot 1$$

$$= (K + 4G/3) \cdot \boldsymbol{\varepsilon}_1 \qquad ...(1)$$

∴ 斜率 $\boldsymbol{\sigma}_1/\boldsymbol{\varepsilon}_1 = K + 4/3G$

$$\boldsymbol{\sigma}_2 = \boldsymbol{\sigma}_3 = 2G \cdot (-\boldsymbol{\varepsilon}_1/3) + K \cdot \boldsymbol{\varepsilon}_1 = (K - 2G/3)\boldsymbol{\varepsilon}_1 \qquad \ldots(2)$$

(2)Yield

$$I_1 = \boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2 + \boldsymbol{\sigma}_3 = 3K \cdot \boldsymbol{\varepsilon}_1 \qquad \ldots(3)$$

$$J_2 = \frac{1}{6}\left[(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)^2 + (\boldsymbol{\sigma}_2 - \boldsymbol{\sigma}_3)^2 + (\boldsymbol{\sigma}_3 - \boldsymbol{\sigma}_1)^2\right]$$

$$= \frac{1}{3}(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)^2 = \frac{1}{3}(2G \cdot \boldsymbol{\varepsilon}_1)^2 = \frac{4}{3}(G \cdot \boldsymbol{\varepsilon}_1)^2 \qquad \ldots(4)$$

$$f(\boldsymbol{\sigma}_{ij}, K)\boldsymbol{\alpha} I_1 + \sqrt{J_2} - K_d = 0 \qquad \ldots(5)$$

(3) & (4) sub into eq (5)

$$32 \cdot K \cdot \boldsymbol{\varepsilon}_1 + \frac{2}{\sqrt{3}} G \cdot \boldsymbol{\varepsilon}_1 - K_d = 0$$

$$\therefore \quad \boldsymbol{\varepsilon}_{1y} = \frac{K_d}{(3\boldsymbol{\alpha} K + 2G/\sqrt{3})} \qquad \ldots(6)$$

代入(1)得

$$\sigma_{1y} = (4G/3 + K)\frac{K_d}{(3\alpha K + 2G/\sqrt{3})} \qquad \ldots(7)$$

(3)Plastic range

$$d\varepsilon_{ij} = \begin{pmatrix} \varepsilon_1 \\ 0 \\ 0 \end{pmatrix} \quad ; \quad de_{ij} = \begin{pmatrix} 2/3\, d\varepsilon_1 \\ -1/3\, d\varepsilon_1 \\ -1/3\, d\varepsilon_1 \end{pmatrix}$$

$$d\sigma_{ij} = \begin{pmatrix} d\sigma_1 \\ d\sigma_2 \\ d\sigma_3 \end{pmatrix} \quad ; \quad dS_{ij} = \begin{pmatrix} 2(d\sigma_1 - d\sigma_2)/3 \\ (d\sigma_2 - d\sigma_1)/3 \\ (d\sigma_2 - d\sigma_1)/3 \end{pmatrix}$$

$$\sigma_{ij} = \begin{pmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{pmatrix} \quad ; \quad S_{ij} = \begin{pmatrix} 2(\sigma_1 - d\sigma_2)/3 \\ (\sigma_2 - d\sigma_1)/3 \\ (\sigma_2 - d\sigma_1)/3 \end{pmatrix}$$

$$d\sigma_{ij} = 2G \cdot de_{ij} + K \cdot d\varepsilon_{KK} \cdot \delta_{ij} - (3K\alpha\delta_{ij} + G \cdot S_{ij}/\sqrt{J_2}) \cdot d\lambda \qquad \ldots(8)$$

$$d\lambda = \frac{3\alpha K \cdot d\varepsilon_{KK} + G \cdot S_{mn} \cdot de_{mn}/\sqrt{J_2}}{9\alpha^2 \cdot K + G} \qquad \ldots(9)$$

$$d\varepsilon_{KK} = d\varepsilon_1 \qquad \ldots(10)$$

$$S_{mn} \cdot de_{mn} = \frac{4}{9}(\sigma_1 - \sigma_2) \cdot d\varepsilon_1 - \frac{1}{9}(\sigma_2 - \sigma_1) \cdot d\varepsilon_1 - \frac{1}{9}(\sigma_2 - \sigma_1) \cdot d\varepsilon_1$$

$$= 2(\sigma_1 - \sigma_2) \cdot d\varepsilon_1/3 \qquad \ldots(11)$$

$$\therefore \quad d\lambda = \frac{3\alpha K \cdot d\varepsilon_1 + \frac{G}{\sqrt{J_2}} \cdot \frac{2}{3}(\sigma_1 - \sigma_2) \cdot d\varepsilon_1}{9\alpha^2 \cdot K + G}$$

$$= \frac{3\alpha K \cdot d\varepsilon_1 + \frac{G}{1/\sqrt{3}(\sigma_1 - \sigma_2)} \cdot \frac{2}{3}(\sigma_1 - \sigma_2) \cdot d\varepsilon_1}{9\alpha^2 \cdot K + G}$$

$$= \frac{(3\alpha K + 2G/\sqrt{3}) \cdot d\varepsilon_1}{9\alpha^2 \cdot K + G} \qquad \ldots(12)$$

由 eq (8) 得

$$\therefore \quad d\boldsymbol{\sigma}_1 = 2\mathrm{G} \times \frac{2}{3} d\boldsymbol{\varepsilon}_1 + \mathrm{K} \cdot d\boldsymbol{\varepsilon}_1 - \left[3\boldsymbol{\alpha}\mathrm{K} + \frac{\mathrm{G}}{(\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) / \sqrt{3}}) \cdot \frac{2}{3} (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \right]$$

$$= \frac{(3\boldsymbol{\alpha}\mathrm{K} + 2\mathrm{G}/\sqrt{3}) \cdot d\boldsymbol{\varepsilon}_1}{9\boldsymbol{\alpha}^2 \cdot \mathrm{K} + \mathrm{G}}$$

$$= \left[\frac{3}{4}\mathrm{G} + \mathrm{B} - \frac{(3\boldsymbol{\alpha}\mathrm{K} + 2\mathrm{G}/\sqrt{3})^2}{9\boldsymbol{\alpha}^2 \cdot \mathrm{K} + \mathrm{G}} \right] \cdot d\boldsymbol{\varepsilon}_1$$

$$\therefore \quad d\boldsymbol{\sigma}_1 / d\boldsymbol{\varepsilon}_1 = \mathrm{K} + \frac{4}{3}\mathrm{G} - \frac{(3\boldsymbol{\alpha}\mathrm{K} + 2\mathrm{G}/\sqrt{3})^2}{9\boldsymbol{\alpha}^2 \cdot \mathrm{K} + \mathrm{G}}$$

Constitutive Model for Geotechnical Materials

Chapter 5 雙曲線模式之理論推導與參數之求取

5.1 前言

5.2 內容

5.2.1 理論推導

(1)雙曲線應力－應變的關係

(2)雙曲線之參數值的求法

(3)參數值之修正

5.2.2 土壤－構造物間剪應力－剪位移的關係