Final exam on *Constitutive Models for Geotechnical Materials*

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1. An elastic-perfectly plastic non-associated material satisfies Drucker-Prager yield criterion ($f = \alpha I_1 + \sqrt{J_2} - \kappa_d = 0$) and has a von Mises plastic potential function ($g = J_2 - \kappa_v^2$).
   (a) Show that the stress-strain relation of this material can be expressed as
   $$d\varepsilon_{ij} = \frac{1}{2G} \cdot dS_{ij} + \frac{1}{9K} dI_1 \cdot \delta_{ij} + \left(\frac{3\alpha K \cdot d\varepsilon_{kk}}{2G\sqrt{J_2}} + \frac{1}{2J_2} S_{mn} \cdot de_{mn} \right) \cdot S_{ij} \quad (15\%)$$
   (b) Now suppose the material has a uniaxial compressive strength of $fc' = 210\ kg/cm^2$ and a shear strength of $\tau_y = 0.53\sqrt{fc'}\ kg/cm^2$, find the material constants α and κ_d? (5%)
   (c) The material has a Young's modulus of $E = 15100\sqrt{fc'}\ \ kg/cm^2$ and a Poisson's ratio of $\nu = 0.17$, now this element is subjected a strain state of $(\varepsilon_1, \varepsilon_2, \varepsilon_3) = (\varepsilon, 0, 0)$, find the values of σ_{1y} and ε_{1y}. (10%)

2. (a) Show that the relation of deviatoric stress $(\sigma_1 - \sigma_3)$ vs. axial strain ε_a and volumetric strain ε_v vs. axial strain ε_a response for the *Hyperbolic Model* can be express as:
   (1) $$\varepsilon_a = \frac{(\sigma_1 - \sigma_3)}{k * P_a * \left(\frac{\sigma_3}{P_a}\right)^n * \left(1 - \frac{R_f(\sigma_1 - \sigma_3)(1 - \sin\phi)}{2C\cos\phi + 2\sigma_3 * \sin\phi}\right)} \quad (10\%)$$
   (2) $$\varepsilon_v = \frac{k}{3k_b} * \left(\frac{\sigma_3}{P_a}\right)^{(n-m)} * \left(1 - \frac{R_f(\sigma_1 - \sigma_3)(1 - \sin\phi)}{2C\cos\phi + 2\sigma_3 * \sin\phi}\right) * \varepsilon_a \quad (5\%)$$
   (b) The $\sigma - \varepsilon$ response of a soil element follows the *Hyperbolic Model*. Parameters of this soil are shown as below:
   $C' = 0.5kg/cm^2, \phi' = 30^o, K = 350, n = 0.8, R_f = 0.95, K_{ur} = 450, K_b = 400,$
   $m = 0.4.$
   Now, suppose the soil specimen is consolidated under cell pressure of 4 kg/cm^2 and backed pressure of 2.0 kg/cm^2, respectively. The soil element was then applied deviatoric stress σ_d (drained condition) to the value of 5.0 kg/cm^2. Find the elastic axial strain ε_a^e, plastic axial strain ε_a^p and volumetric strain ε_v. (10%)
   (c) Why does the friction angle ϕ of cohesionless soil decreased, when the confining pressure σ_3 increased? (5%)

Mid-term exam on ***Constitutive Model for Geotechnical Materials***

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1. Explain following two nomenclatures: (a) Bauschinger effect (7%); (b) Yield criterion. (3%)

2. The σ–ε response in simple tension for a material, which is approximated by the form of a Ramberg-Osgood formula: $\varepsilon = \varepsilon_e + \varepsilon_p = 0.01\sigma + 0.001*(0.1\sigma)^2$. Where the unit of σ is MPa.
   (a) Find the E, E_t, E_p, σ, and W_p, when plastic strain ε^p =0.02. (15%)
   (b) When plastic strain ε^p =0.02 and is subsequently unloaded and reversely loaded to $\varepsilon^p = 0$, find the associated stress σ, if the hardening rule is isotropic hardening, and the hardening parameter is defined as $\kappa = \int (d\varepsilon_{ij}^p * d\varepsilon_{ij}^p)^{0.5}$. (10%)

3. Show that (a) $\partial J_2 / \partial \sigma_{ij} = s_{ij}$, and (5%); (b) $\tau_{oct} = \frac{\sqrt{2}}{3}(I_1^2 - I_2)^{\frac{1}{2}}$. (10%)

4. A soil element has a yield stress state: $\sigma_{ij} = \begin{bmatrix} 5 & 2 & 1 \\ 2 & -4 & 0.5 \\ 1 & 0.5 & 3 \end{bmatrix} MPa$

   (1) Find the new stress state for a new x_1', x_2', x_3' coordinate system defined by rotating x_1, x_2, x_3 axes through an angle of 30° clockwise about x_3 axis. (5%)

   (2) An acting plane has a normal vector, the angles between the normal vector and X_1, X_2 and X_3 axes are 73.677°, 35°, and 60°, respectively. Find the stress vector $\vec{T_i}$, magnitude and direction of the normal stress and shear stress. (15%)

   (3) Find the principal stresses, and the principal direction for $\sigma_{\max}$. (10%)

   (Note: Use Newton's method and guess σ=6.0 MPa to get principal stresses)

   (4) Find the invariants of σ_{oct}, τ_{oct}, ξ, ρ and θ. (10%)

   (5) Now suppose the material element is subjected a uniaxial compressive stress q_u, for what values of q_u when the soil element occurs yield? According to (a) Tresca yield criterion (3%); (b) von Mises yield criterion (4%); (c) Osgood yielding criterion: $J_2^3 - 2.25J_3^2 = \kappa_o^6$ (8%)