Chapter 4

Coding and Time-Division Multiple Access

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4.1 Introduction

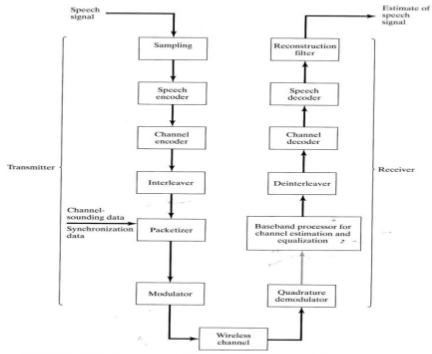

FIGURE 4.1 Block diagram of a basic TDMA link; the shaded arrow indicates a complex signal with real and imaginary parts.

4.2 Sampling

- Convert an analog information-bearing signal $m(t)$ to a sequence that spaced uniformly in time without significant loss of information.
- Sampling Theorem
  - A band-limited signal of finite energy that has no frequency components greater than W hertz is completely
    - *described by specifying the values of the signal at instants of time separated by $1/2W$ seconds.*
    - *recovered from a knowledge of its samples taken at the rate of $1/2W$ samples per seconds.*

- Aliasing
  - phenomenon of high frequency in the spectrum of information-bearing signal take on the identity of lower frequency in the spectrum of sampled version of signal.
- The sampling rate of 2W samples per second for a signal bandwidth of W hertz is called *Nyquist rate*.
- Corrective measures of aliasing
  - A low-pass antialiasing filter is used to attenuate high-frequency components of signal $m(t)$ before sampling.
  - The output of the low-pass filter is sampled at a rate slightly higher than the Nyquist rate.

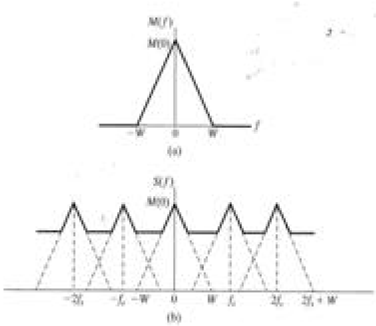

FIGURE 4.2 (a) Spectrum of a message signal band limited to $-W \le f \le W$.
(b) Spectrum of the corresponding sampled version of the signal for a sampling rate $f_s < 2W$.

4.3 Why Follow Sampling with Coding

- *Encoding*: translate the discrete set of sample values to a form best suited for transmission.
- *Source coding* is a way to remove the *redundant information*.
  - It has the net effect of reducing the channel bandwidth required to transmit the speech signal.
- Benefits of encoded version of speech signal
  - Offers potential for mitigating the effects of channel noise.
  - Allow system to have a capability to correct transmission errors.

4.4 Shannon's Information theory

- Issues of Shannon's Information Theory
  - The efficient encoding of a source signal
  - Reliable transmission over noisy channel

4.4 Shannon's Information theory

4.4.1 Source-Coding Theorem

- Entropy
  - a measure of average information content per symbol emitted by the source
- *Given discrete memoryless source characterized by a certain amount of entropy, the average code-word length for a distortionless source-encoding scheme is upper bounded by the entropy*
- Source encoder
- Entropy

$$h = \frac{H(S)}{\overline{L}} \qquad (4.1)$$

$$H(S) = \sum_{k=0}^{K-1} p_k \log_2\left(\frac{1}{p_k}\right) \qquad (4.2)$$

4.4 Shannon's Information theory

4.4.2 Channel-Coding Theorem

- Reliable transmission of sequence over noisy channel
- *If a discrete memoryless channel has capacity C and a source generates information at a rate less than C, then there exists a coding technique such that the output of the source may be transmitted over the channel with an arbitrarily low probability of symbol error.*
- Code rate

$$r = \frac{k}{n} \qquad (4.3)$$

- Channel-coding Theorem:
  – non-constructive nature

4.4 Shannon's Information theory

4.4.3 Information Capacity Theorem

- Trade-off between channel bandwidth and signal-to-noise ratio at channel output.
- Information Capacity

$$C = B\log_2\left(1+\frac{P}{s^2}\right) bits/s \qquad (4.4)$$

- Three key system parameters: channel bandwidth, average transmitted power and channel noise variance.
- easier to increase the information capacity of a wireless channel by expanding its bandwidth than the other.

4.4 Shannon's Information theory

4.4.4 Rate Distortion Theory

- A natural extension of source and channel coding theorem.
- Applications of rate distortion theory
  – Source coding, wherein the permitted alphabet of the source code cannot represent the source output exactly
    - thereby forcing us to put with lossy data compression
  – Information transmission
    - required at a rate greater than the permissible channel capacity.

4.5 Speech Coding

- A recurrent theme in the design of digital wireless communication system is
  - *efficient utilization of the allotted spectrum.*
- Speech coding:
  - remove nearly all of the natural redundancy inherent in speech signal.
- *Linear Predictive coding (LPC)*

4.5 Speech Coding

4.5.1 Linear Prediction

- *Predictive mode*: predicting the present or future value of a discrete-time signal in a given set of past samples of signal.
- *Prediction error*: difference between the actual future value of the signal and the predicted value produced by the model.
- Set of samples $x(t), X(t-T_s), \ldots, x(t-NT_s)$
- Output of the predictive model

$$\hat{x}(t) = \sum_{n=1}^{N} a_n x(t - nT_s) = \mathbf{a}^T \mathbf{x} \qquad (4.5,7,6)$$

$$\mathbf{x} = [x(t-T_s), x(t-2T_s), \ldots, x(t-NT_s)]^T$$

$$\mathbf{a} = [a_1, a_2, \ldots, a_N]^T$$

- Mean-square-error criterion $MSE = E[x(t) - \hat{x}(t)^2]$ (4.8)
- Optimum value of parameter $\mathbf{a}$ $\mathbf{a} = \mathbf{R}^{-1}\mathbf{r}$ (4.9)

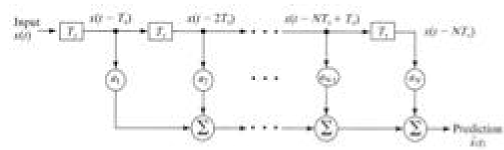

FIGURE 4.3 Structure of the FIR (i.e., tapped-delay-line) predictor.

4.5 Speech Coding

4.5.2 Multipulse Excited LPC

- Exploits the *principle of analysis by synthesis*
- Three parts of encoder
  - Synthesis Filter
    - Produce synthesis version of original speech with high quality.
  - Excitation generator
    - Producing the excitation applied to the synthesis filter.
  - Error minimization
    - Optimizing the perceptually weighted error between the original speech and the synthesized speech

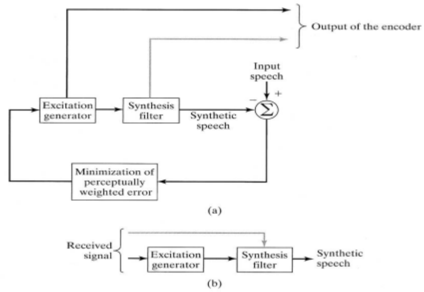

FIGURE 4.4 Multipulse excited linear predictive codec. (a) Encoder. (b) Decoder whose input (the received signal) consists of quantized filter parameters (signified by lighter arrows) and quantized excitation, as produced by the generator.

- Encoding Steps:
  1. Computation of free parameters of synthesis filter with the use of actual speech samples as input.
  2. Optimum excitation for synthesis filter is computed by minimizing the perceptually weighted error with the loop closed.
- Decoder which
  - located in the receiver
  - consists of excitation generator and synthesis filter,
  - using the received signal to produce a synthetic version of original speech signal.

4.5 Speech Coding

4.5.3 Code-Excited LPC

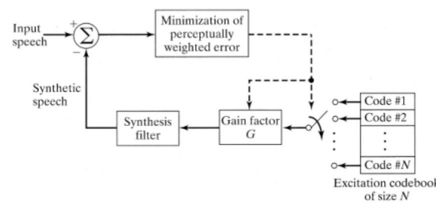

FIGURE 4.5 Encoder of the code-excited linear predictive codec (CELP). The transmitted signal consists of the address of the code selected from the codebook, the quantized gain factor G, and quantized filter parameters.

- Use of predetermined codebook of stochastic (zero-mean white Gaussian) vectors as the source of excitation for the synthesis filter.
- CELP us capable of producing good quality speech at bit rates below 8 kb/s
- Intensive computational complexity

4.6 Error-Control Coding

1. Forward error-correction (FEC) code
   - Classified into
     - *block codes*
     - *convolutional codes*.
   - Rely on the controlled use of redundancy in the transmitted code word for
     - *detection* and *correction* of errors.
2. Automatic-repeat request (ARQ) schemes
   - Use redundancy merely for the purpose of *error detection*.

4.6 Error-Control Coding

4.6.1 Cyclic Redundancy Check Codes

- Provide a powerful method of error detection for use in ARQ strategies.
- Cyclic codes: any cyclic shift of a code word in the code is also a code word.
- Cyclic codes are suited for error detection
  - Can be designed to detect many combinations of errors.
  - Implementation of encoding and error-detecting circuits is very simple.

- Binary (n,k) CRC codes are capable of detecting the following patterns
  1. All error bursts of length n-k or less.
  2. A fraction of error bursts of length equal or greater then n-k+1; the fraction equals $1\text{-}2^{-(n\text{-}k\text{-}1)}$
  3. All combinations of $d_{\min}$-1 (or fewer) errors.
  4. All error patterns with an odd number of errors if the generator polynominal for the code has an even number of nonzero coefficients.

4.7 Convolutional codes

- Convolutional coder generates redundant bits by using modulo-2 convolutions.
- The encoder of a binary convolutional code with rate $1/n$, measured in bits per symbol, is called *finite-state machine (FSM)*.
- Code rate of convolutional code

$$r = \frac{L}{n(L+M)} \text{bits/symbol for } L >> M,\ r \approx \frac{1}{n} \text{bit/symbol} \quad (4.10)$$

- The *constraint length* defines as the number of shifts over which a single message bit can influence the encoder output.

- *Impulse response*
  - the response of the path connecting output to input of convolutional encoder to a symbol 1 applied to its input.
- *Generator polynomial*
  - the *unit-delay transform* of the impulse response.

$$g^{(i)}(D) = g_0^{(i)} + g_1^{(i)} D + g_2^{(i)} D^2 + \ldots + g_M^{(i)} D^M \quad \text{for} \quad i = 1, 2, \ldots, n \tag{4.12}$$

4.7 Convolutional codes

4.7.1 Trellis and State Diagrams of Convolutional Codes

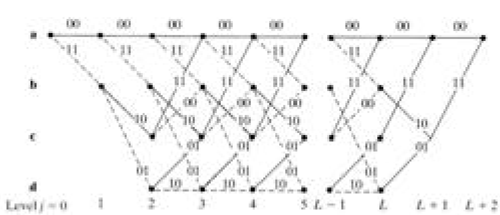

FIGURE 4.7 Trellis for the convolutional encoder of Figure 4.6.

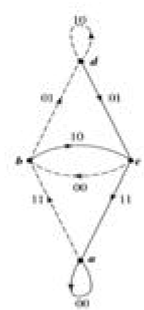

FIGURE 4.8 State diagram of the convolutional encoder of Figure 4.6.

4.8 Maximum-Likelihood Decoding of Convolutional Codes

- Maximum-likelihood decoder

$$\textit{Choose the estimate } \hat{\mathbf{c}} \textit{ for which the } \log\textit{-likelihood function } \log p(\mathbf{r} \mid \mathbf{c}) \textit{ is } \max\textit{imum}.$$

- Consider the special case of memoryless binary symmetric channel, the conditional probability

$$p(\mathbf{r} \mid \hat{\mathbf{c}}) = \prod_{i=1}^{N} p(r_i \mid \hat{c}_i) \tag{4.14}$$

- Log-likelihood function for convolutional decoder

$$\log p(\mathbf{r} \mid \hat{\mathbf{c}}) = \sum_{i=1}^{N} \log p(r_i \mid \hat{c}_i) \tag{4.15}$$

- Let the transition probability be

$$p(r_i \mid \hat{c}_i) = \begin{Bmatrix} p & \text{if } r_i \neq \hat{c}_i \\ 1-p & \text{if } r_i = \hat{c}_i \end{Bmatrix} \quad (4.16)$$

- Then we may rewrite the log-likelihood function as

$$\begin{aligned} \log p(r|\hat{c}) &= d \log p + (N-d)\log(1-p) \\ &= d \log\left(\frac{p}{1-p}\right) + N \log(1-p) \end{aligned} \quad (4.17)$$

- Restate the maximum-likelihood decoding rule for binary symmetric channel as

Choose the estimate $\hat{c}$ that miminizes the Hamming distance d between the candidate code vector $\hat{c}$ and the received vector r

4.9 The Viterbi Algorithm

TABLE 4.4 Summary of the Viterbi Algorithm.

Initialization

Label the states of the trellis from top to bottom as shown in Fig. T.1. Label the leftmost column of the trellis as time $j = 0$. Initialize the *cumulative path metric* to state $s = 0, 1, \ldots, S-1$, at time step 0 to

$$J_0(s) = \begin{Bmatrix} 0 & \text{if } s = 0 \\ \infty & \text{if } s \neq 0 \end{Bmatrix}$$

If the encoder produces L bits per transition, then we index the received vector $\mathbf{r}$ in groups of L bits; that is, $\mathbf{r} = (r_{1,1}, \ldots, r_{1,L}, r_{2,1}, \ldots, r_{2,L}, r_{3,1}, \ldots)$. We also label the L bits that are output from the encoder on a transition

TABLE 4.4 Summary of the Viterbi Algorithm.

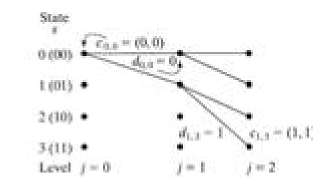

FIGURE T.1 Labeling of trellis states and transitions.

Computation step $j+1$

Final Step

4.9 The Viterbi Algorithm

4.9.1 Modifications of the Viterbi Algorithm

- When the received sequence is very long, the storage requirement of Viterbi algorithm becomes too high.
- *Decoding window* of acceptable length *l* is specified and the Viterbi algorithm operates on a frame of received sequence, always stopping after *l* steps.
- Decision is made on the best path and symbol associated with 1st branch on that path is released to user.
- Decoding window is moved forward one time interval and the decision on next code frame is made.
- No longer truly maximum likelihool.

4.10 Interleaving

- Minimization of information to be transmitted
  - *Reducing the amount of data to be transmitted means less pwer has to be transmitted.*
  - *Reducing the spectral (or radio frequency) resources that are required for satisfactory performance.*
- Interleaving
  - Obtain the maximum benefit from FEC coding.
  - Resolving the two conflicting phenomena
    - *Wireless channel that produces bursts of correlated bit errors.*
    - *Convolutional decoder that cannot handle error burst.*
  - No need exact statistical characterization of wireless channel but only the coherence time.

- Coherence time for fast fading

$$T_{coherence} \approx \frac{0.3}{2 f_D} \tag{4.19}$$

- Interleaver randomizes the order of encoded bits after the channel encoder in transmitter.
- Deinterleaver undoes the randomization before the data reach the channel decoder in the receiver.

4.10 Interleaving

4.10.1 Block Interleaving

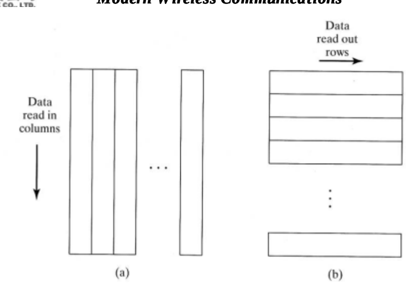

FIGURE 4.10 Block interleaver structure.
(a) Data "read in."
(b) Data "read out."

4.10 Interleaving

4.10.2 Convolutional Interleaving

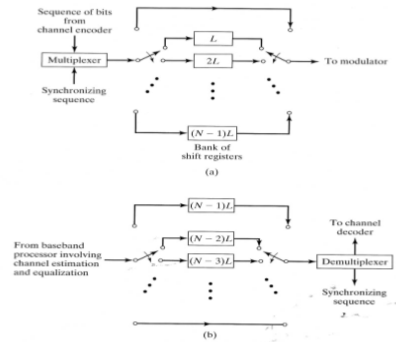

FIGURE 4.12 (a) Convolutional interleaver. (b) Convolutional deinterleaver.

4.10 Interleaving

4.10.3 Random Interleaving

Two steps algorithm:

1. Choose an integer i_1 from the uniformly distributed set $A = \{1, 2, \ldots, N\}$, with the probability of choosing i_1 being $P(i_1) = 1/N$. The chosen integer i_1 is set to $p(1)$.

2. For $k > 1$, choose an integer i_k from the uniformly distributed set $A_k = \{i \in A, i \neq i_1, i_2, \ldots, i_{k-1}\}$, with the probability of choosing i_k being $P(i_k) = 1/(N-k+1)$. The chosen integer i_k is set to $p(k)$. Note that the size of the set A_k is progressively reduced for $k > 1$. When $k = N$, we are left with a single integer i_N that is set to $p(N)$.

4.11 Noise Performance of Convolutional Codes

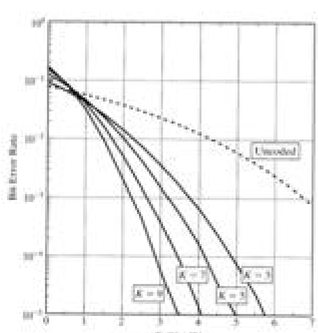

FIGURE 4.13 Comparison of uncoded and coded performance in AWGN channel.

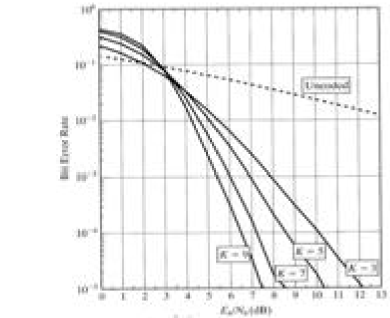

FIGURE 4.14 Comparison of uncoded and coded performance in Rayleigh-fading channel (with infinite interleaving).

- For low values of E_b/N_0,
  - the uncoded performance is better than the coded performance.
- For a prescribed E_b/N_0 ,
  - the noise performance improves with increasing constraint length K for both AWGN and fading channels.

- For a prescribed constraint length K
  - the E_b/N_0 must be increased for the fading channel to exhibit a noise performance comparable to that attainable with corresponding AWGN channel
- For constraint length K=9 in the Rayleigh-fading channel
  - we can realize a bit error rate of $2x10^{-4}$ by using an E_b/N_0 =6, by using the forward error-correction coding.

4.12 Turbo Codes

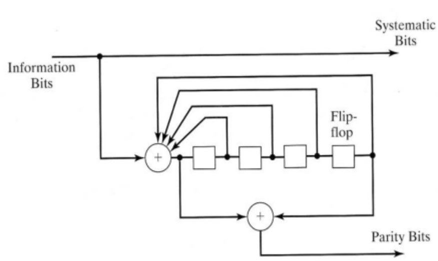

FIGURE 4.16 A systematic recursive convolutional code structure; delay T is equal to the symbol duration.

4.12 Turbo Codes

4.12.1 Turbo Encoding

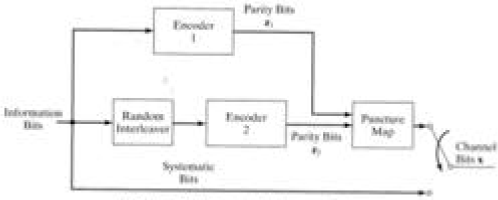

FIGURE 4.15 Illustration of turbo encoding strategy.

4.12 Turbo Codes

4.12.2 Turbo Decoding

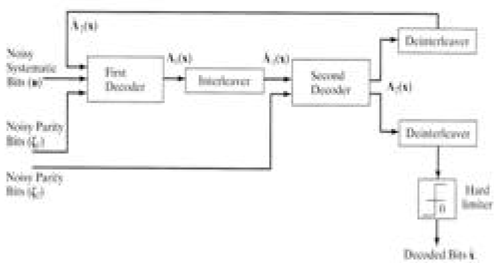

FIGURE 4.17 Block diagram of Turbo decoder.

- Soft-input, soft-output (SISO) decoding algorithm.
- 1st decoding stage uses MAP algorithm to produce a soft estimate, which is expressed as the equivalent log-likelihood ratio

$$\Lambda_1(x(j)) = \log\left(\frac{\mathrm{Prob}(x(j)=1|u,z_1,\tilde{\Lambda}_2(x))}{\mathrm{Prob}(x(j)=0|u,z_1,\tilde{\Lambda}_2(x))}\right) \qquad (4.20)$$

- The second decoding stage uses MAP algorithm and the second set of parity bits to produce a further refined estimate

$$\mathrm{Prob}(x(j)|\tilde{\Lambda}_1((x),z_2))$$

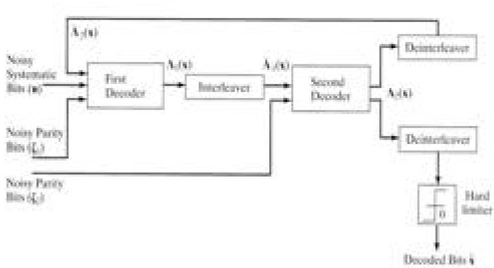

FIGURE 4.17 Block diagram of Turbo decoder.

- Single-loop feedback system
- To increase the independence of inputs from one processing stage to the next
  – turbo algorithm use the concepts of intrinsic and extrinsic information.
- Intrinsic information
  – information inherent in a sample prior to a decoding operation.
- Extrinsic information
  – incremental information obtained through decoding.

- Extrinsic information at the output of 2^{nd} stage

$$\tilde{\Lambda}_2(x)=\Lambda_2(x)-\tilde{\Lambda}_1(x) \qquad (4.21)$$

- The extrinsic information supplied to the second stage by 1^{st} stage

$$\tilde{\Lambda}_1(x)=\Lambda_1(x)-\tilde{\Lambda}_2(x) \qquad (4.22)$$

- On the last iteration of the decoding process, a hard decision is applied to the output of the 2^{nd} decoder to produce an estimate of the jth information bit

$$\tilde{x}(j)=sign(\Lambda_2(x(j))) \qquad (4.23)$$

4.12 Turbo Codes

4.12.3 Noise Performance

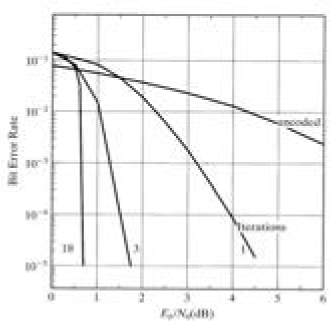

FIGURE 4.18 Performance of turbo codes as a function of number of iterations. (Reproduced from Berrou et al., 1993, with permission of the IEEE.)

4.12 Turbo Codes

4.12.4 Maximum a Posteriori Probability Decoding

- MAP algorithm includes a backward an forward recursion.
- Derive the *a posteriori* probabilities of the states and transitions of the trellis
- S-vector of state probabilities at time j based on set of observations

$$?(j)=\Pr ob(\mathbf{s}(j|\mathbf{r})) \quad ?(j)\in R^S$$
$$where\ jth\ elementis\quad l_{S(j)}=\Pr ob(s(j)=s|\mathbf{r}) \qquad (4.24,25)$$

- Assume the transmitted information bit is least significant bit (LSB) of the state, the probability that a 1 was the information bit is given by

$$\Pr ob(x(j)=1|r)=\sum_{s:LSB(s)=1} l_s(j) \qquad (4.26)$$

- Forward estimator of state probabilities

$$\mathbf{a}(j)=\Pr ob[s(j)|\mathbf{r}_{[1,j]}] \quad \mathbf{a}(j)\in R^M \qquad (4.27)$$

- Backward estimator of state probabilities

$$\mathbf{ß}(j)=\Pr ob[s(j)|\mathbf{v}_{[j,1]}] \quad b(j)\in R^M \qquad (4.28)$$

- Define the L_1 norm for probability vectors as

$$\|\mathbf{a}\|=\sum_s a(s) \quad therefore \quad l(j)=\frac{a(j)\cdot b(j)}{\|a(j)\cdot b(j)\|_1} \qquad (4.29,30)$$

- Transition probability at time j

$$g_{s',s}(j) = \Pr ob(s(j)=s, w(j) \mid s(j-1)=s') \qquad (4.31)$$

- Matrix of the above probabilities

$$\Gamma(j) = [g_{s',s}(j)] \quad \Gamma(j) \in R^{S\times S} \qquad (4.32)$$

- Recursion equations for calculating the forward and backward state estimates:

$$a^T(j) = \frac{a(j-1)^T \Gamma(j)}{\left\| a(j-1)^T \Gamma(j) \right\|_1} \quad and \quad b(j) = \frac{\Gamma(j+1) b(j+1)}{\left\| \Gamma(j+1) b(j+1) \right\|_1} \qquad (4.33,34)$$

- MAP algorithm:
  - $\boldsymbol{\alpha}(0)$ and $\boldsymbol{\beta}(T)$ are initialized according to the trellis structure.
    - For a trellis beginning and ending in the all-zero state,
    - we have $\boldsymbol{\alpha}_0(0)=1$ and $\boldsymbol{\alpha}_S(0)=0$ for all s not equal to zero and similarly for $\boldsymbol{\beta}(T)$.
  - When $r(j)$ is received, the decoder computes $\boldsymbol{\Gamma}(j)$ and then $\boldsymbol{\alpha}(j)$ using Eq.(4.33).
    - The computed values of $\boldsymbol{\alpha}(j)$ are stored for all j and s .
    - Note that $r(j)$ is a vector of length L, defined by

- MAP algorithm:
  - After the complete sequence $r[1,T]$ has been received, the decoder recursively computes $\boldsymbol{\beta}(j)$, using Eq.(4.34).
    - Then, when $\boldsymbol{\beta}(j)$ have been computed, they are multiplied by the appropriate $\boldsymbol{\alpha}(j)$ to obtain $\boldsymbol{\lambda}(j)$, using Eq.(4.30)

4.13 Comparison of Channel-Coding Strategies for Wireless Communication

- 4.13.1 Encoding
- 4.13.2 Decoding
- 4.13.3 AWGN Channel
- 4.13.4 Fading Wireless Channels
- 4.13.5 Latency
- 4.13.6 Joint Equalization and Decoding

4.13 Comparison of Channel-Coding Strategies for Wireless Communication

4.13.1 Encoding

- Convolutional encoder can assume of one of two forms:
  - Nonrecursive nonsystematic
  - Recursive systematic
- Comparing turbo codes with convolutional (RSC) encoder:
  - Unlike Shannon's random codes, turbo codes are decodable .
  - Turbo codes work better than classical convolutional codes when code rates are high or signal to noise ratios are low.
  - Both types of codes require use of flush bits. With parallel encoding structure of turbo codes, it is not straightforward to flush the second encoder, so flushing is often not done.
  - Unlike convolutional codes, turbo codes have an error floor.
  - Turbo codes rely on soft inputs to work. Convolutional codes can work with either soft-decision or hard-decision inputs.

4.13 Comparison of Channel-Coding Strategies for Wireless Communication

4.13.2 Decoding

- Decoding for convolutional code is to use the Viterbi algorithm
  - complexity depends on number of states and
  - there is a trade off between the performance and decoder complexity.
- Turbo decoding relies on
  - exchange extrinsic information between 2 SISP decoding stage on iterative basis.
  - Complexity is twice as Viterbi algorithm. But it can achieve large coding gains with simple component codes.
- EXIT chart is defined as the *function that maps the prior information to the extrinsic information applied to the decoder in question*, with the information capacity of its communication channel treated as a parameter.

4.13 Comparison of Channel-Coding Strategies for Wireless Communication

4.13.3 AWGN Channel

- Ability to achieve near-optimum performance.
- Optimality is defined in terms of theoretical limits imposed by Shannon's information capacity theorem.
- For finite size of information block, performance is measured in terms of increase in E_b/N_0.

4.13 Comparison of Channel-Coding Strategies for Wireless Communication

4.13.4 Fading Wireless Channels

- To combat error bursts, using of interleavers in transmitter and deinterleavers in receiver is needed.
- Performance curves of turbo codes, brick-wall in shape.
- Convolutional codes, the performance codes exhibit a slow roll-off characteristic.
- For short block lengths, which are most robust for communication over fading wireless channels. The improvement offered by turbo codes over convolutional codes is usually small.

4.13 Comparison of Channel-Coding Strategies for Wireless Communication

4.13.5 Latency

- Delay incurred by a channel decoder in processing the received signal in order to recover the original sequence of information bits.
- Proportional to interleaver size.
- Smaller block sizes mean smaller latency.

4.13 Comparison of Channel-Coding Strategies for Wireless Communication

4.13.6 Joint Equalization and Decoding

- Turbo codes
  - have a very small free distance.
  - Upon decoding, they have fewer error events at this free distance than convolutional codes have.
- Convolutional encoders
  - used in the turbo encoder are recursive
  - non-return-to zero encoders in that they return to initial state only with probability 2^{-v}

4.14 RF Modulation Revisited

- Partial-response modulation
  - ensure the phase response of the modulated signal is spread over several symbol periods.
- GMSK: example of partial-response modulation.

FIGURE 4.19 Block diagram of Gaussian minimum-shift keying (GMSK) signal generator.

4.15 Baseband Processing for Channel Estimation and Equalization

- Signal resulting from convolution operation plus AWGN at channel output constitutes the received RF signal:

$$x(t) = s(t) \otimes h(t) + w(t) \qquad (4.35)$$

- Complex baseband signal

$$\tilde{x}(t) = x_I(t) + x_Q(t) \qquad (4.36)$$

- Complex equivalent baseband form of real impulse response of channel

$$\tilde{h}(t) = h_I(t) + jh_Q(t) \qquad (4.37)$$

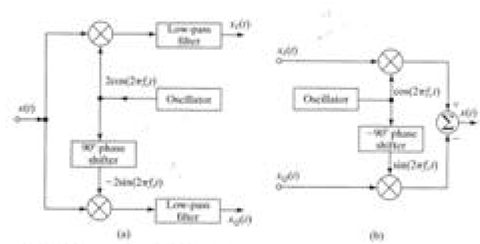

FIGURE 4.20 (a) Scheme for deriving the in-phase and quadrature components of RF-modulated signal x(t). (b) Scheme for reconstructing the RF-modulated signal from its in-phase and quadrature components.

4.15 Baseband Processing for Channel Estimation and Equalization

4.15.1 Channel Estimation

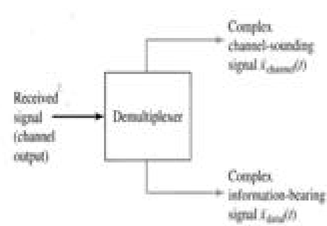

FIGURE 4.21 Block diagram of demultiplexer at the receiver input, following the quadrature demodulator.

- Complex channel-sounding signal

$$\tilde{x}_{channel}(t) = \tilde{c}(t) \otimes \tilde{h}(t) \quad w(t) = 0 \tag{4.38}$$

- The complex channel-sounding signal is applied to matched filter. Appropriately delayed to satisfy causality, the impulse response is

$$\tilde{q}(t) = \tilde{c}^*(T_c - t) \tag{4.39}$$

- Match-filter output

$$\begin{aligned} \tilde{z}(t) &= \tilde{q}(t) \otimes \tilde{x}_{channel}(t) \\ &= \tilde{c}^*(T_c - t) \otimes \tilde{c}(t) \otimes \tilde{h}(t) \end{aligned} \tag{4.40}$$

- Property of match filter

  The output of a matched filter, in response to an input signal to which the filter is matched, is equal to the autocorrelation of the input signal

- Mathematical representation of matched filter property

$$\tilde{c}^*(T_c - t) \otimes \tilde{c}(t) = r_{\tilde{c}}(T_c - t) \tag{4.41}$$

- Hence, Eq(4.40) reduces to

$$\tilde{z}(t) = r_{\tilde{c}}(T_c - t) \otimes \tilde{h}(t) \tag{4.42}$$

- Real value of autocorrelation function

$$r_{\tilde{c}}(t) = r(t) \quad for\ all\ t \quad therefore \quad r(-t) = r(t) \tag{4.43,44}$$

- Rewrite Eq.(4.42)

$$\tilde{z}(t) = r(t - T_c) \otimes \tilde{h}(t) \tag{4.45}$$

- Suppose autocorrelation is real and in form of delta function, then

$$\tilde{z}(t) = d(t - T_c) \otimes \tilde{h}(t) = \tilde{h}(t - T_c) \tag{4.46}$$

- Eq.(4.46) is idealized when
  - Channel noise is zero
  - Probing signal is ling enough
- Estimate of complex impulse response

$$\tilde{h}_{est}(t) = r(t) \otimes \tilde{h}(t + T_c) \tag{4.47}$$

4.15 Baseband Processing for Channel Estimation and Equalization

4.15.2 Viterbi Equalization

- Metric for design the equalizer
  - Estimated received waveforms
  - Compensated received waveform
- Squared Euclidean distance

$$m_{k,v}^2(i) = \left(\tilde{x}_{data,I}(k,i) - \tilde{x}_{est,I}(v,i)\right)^2 + \left(\tilde{x}_{data,Q}(k,i) - \tilde{x}_{est,Q}(v,i)\right)^2$$

- Transition metric of equalizer

$$m_{k,v} = \sum_{i=0}^{h-1} m_{k,v}^2(i) \tag{4.49}$$

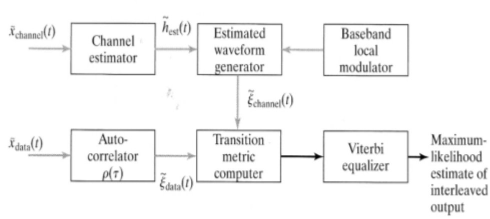

FIGURE 4.22 Block diagram of baseband processor for channel estimation and equalization. The lighter arrows indicate complex signals.

- Steps in Viterbi equalization
  1. Compute the transition metric
  2. Compute the accumulated transition metric for every possible path in the trellis representing the equalizer.
  3. Repeat the computation for every bit of received signal.
  4. Active path discovered by the algorithm defines the I-bit sequence applied to the local modulator.

4.16 Time-Division Multiple Access

- TDMA system permits a number of user to access a wireless communication channel of bandwidth on a time shared basis.
- Distinguish TDMA from FDMA
  - Each user has access to the full bandwidth of the channel.
  - Each user accesses the channel for only a fraction of time.
- TDMA frame are divided into two functional groups:
  - Traffic data bits
  - Overhead bits

4.16 Time-Division Multiple Access

4.16.1 Advantages of TDMA over FDMA

- With TDMA,
  - the use of a diplexer can be avoided at the mobile terminal.
  - only one RF carrier at a time is present in the channel.
  - same channel unit is shared between multiple sessions.
- With voice, a significant portion of the consists of quiet time, when neither party is speaking.

4.16 Time-Division Multiple Access

4.16.1 Advantages of TDMA over FDMA

- Three basic forms of TDMA
  - Wideband TDMA
  - Medium-band TDMA
  - Narrowband TDMA
- Appropriate choice of granularity for FDMA systems
  - In a cellular system, the granularity has to be sufficient to allow different frequency assignment and perform flexible interference management.
  - System complexity increases with the channel bandwidth and data-transmission rate.
  - Propagation conditions may favor higher bandwidth system.