Modern Wireless Communication

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Chapter 5

Spread Spectrum and Code-Division Multiple Access

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5.2 Direct-Sequence Modulation

5.2.1 The Spreading Equation

- BPSK signal

$$\tilde{s}(t) = b\sqrt{E_b g(t)} \quad 0 \le t \le T \tag{5.2}$$

- Symbol-shaping function

$$g(t) = \begin{cases} \sqrt{\dfrac{1}{T'}} & 0 \le t \le T \\ 0 & otherwise \end{cases} \tag{5.3}$$

- For rectangular pulse shape, transmit spectrum is given by

$$S_g(f) = T \sin c^2(fT) \tag{5.4}$$

- For a direct-sequence signal, the symbol-shaping function is

$$g(t) = \sum_{q=1}^{Q} c(q) g_c(t - qT_c) \tag{5.5}$$

which is called spreading factor

- For DS modulation, the pulse shape is a sequence of shorter rectangular pulses called chips, the function of chip shape is

$$g_c(t) = \begin{cases} \sqrt{\dfrac{1}{T'}} & 0 \le t \le T_c \\ 0 & otherwise \end{cases} \tag{5.6}$$

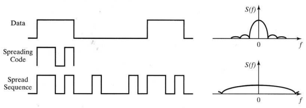


FIGURE 5.1 Spreading by a factor of four in the time and frequency domains.

5.2 Direct-Sequence Modulation

5.2.2 Matched-Filter Receiver

- Received signal

$$\tilde{x}(t)=\tilde{s}(t)+\tilde{w}(t) \qquad (5.7)$$

- Optimum receiver corresponds to the processing

$$y=\int_0^T \tilde{x}(t)\tilde{s}^*(t)dt \qquad (5.8)$$

- Optimum processing

$$\begin{aligned} y &= \int_0^T \tilde{x}(t)g^*(t)dt \\ &= b\sqrt{E_b}\int_0^T |g(t)|^2\,dt+\int_0^T \tilde{w}(t)g^*(t)dt \qquad (5.9) \\ &= b\sqrt{E_b}+\eta \end{aligned}$$

- The mean of the noise sample η is zero and the variance is given by

$$\begin{aligned} \sigma_\eta^2 &= E\left[\left|\int_0^T \tilde{w}(t)g^*(t)dt\right|^2\right]=\int_0^T\int_0^T E[\tilde{w}(t)\tilde{w}^*(s)]g(t)g^*(s)dtds \qquad (5.10) \\ &= \int_0^T\int_0^T N_0\delta(t-s)g(t)g^*(s)dtds=N_0\int_0^T |g(t)|^2\,dt=N_0 \end{aligned}$$

- In AWGN, the transmission performance of *DS spread spectrum* is identical to the nonspread system.
- The signal-to-noise ratio with optimum detection is

$$SNR=\frac{\left(E[y]^2\right)}{\sigma_\eta^2}=\frac{E_b}{N_0} \qquad (5.11)$$

5.2 Direct-Sequence Modulation

5.2.3 Performance with Interference

- DS modulation advantage: reduced receiver sensitivity to interference.
- To explain this, complex envelope is

$$\tilde{\xi}(t)=\sqrt{\frac{A_\xi}{T}}e^{-j2\pi f_\xi t} \qquad (5.13)$$

- Received signal, including interferer, is given by

$$\tilde{x}(t)=\tilde{s}(t)+\tilde{\xi}(t)+\tilde{w}(t) \qquad (5.14)$$

- Jamming term

$$y_\xi=\int_0^T \tilde{\xi}(t)g^*(t)dt \qquad (5.15)$$

- After multiplication by de-spreading sequence, the jamming tone becomes

$$\begin{aligned} \tilde{\xi}(t)g^*(t) &= \sqrt{\frac{A_\xi}{T}}e^{-j2\pi f_\xi t}g^*(t) \\ &= \sqrt{\frac{A_\xi}{T}}e^{-j2\pi f_\xi t}\sum_{q=1}^{Q}c^*(q)g_c(t-qT_c) \\ &= \sqrt{\frac{A_\xi}{T}}\sum_{q=1}^{Q}c^*(q)g_c(t-qT_c)e^{-j2\pi f_\xi t} \quad 0\le t\le T \qquad (5.16) \end{aligned}$$

- The corresponding spectrum is give by

$$\begin{aligned} S_{\xi g}(f) &= \frac{A_\xi}{T}\times Q\times\frac{T}{Q^2}\sin c^2((f+f_\xi)T_c) \\ &= \frac{A_\xi}{Q}\sin c^2((f+f_\xi)T_c) \qquad (5.17) \end{aligned}$$

- To estimate the contribution of jammer to the noise after demodulation, we assume:
  - The frequency offset is small
  - The noise bandwidth of the integrate and dump circuit is $1/T$

- Equivalent noise contribution of jamming is given by the frequency-domain equation

$$\begin{aligned} \sigma_\xi^2 &= \int_{-\infty}^{\infty} S_{\xi g}(f)S_p(f)df \qquad (5.18) \\ &\approx \frac{A_\xi}{Q}\times\frac{1}{T} \\ &\approx \frac{A_\xi}{P_G T} \end{aligned}$$

- In nonspread system, the interference power after the integrate-and-dump circuit is

$$\begin{aligned} \sigma_\xi^2 &= \int_{-\infty}^{\infty} A_x\sin c^2((f+f_\xi)t)S_p(f)df \qquad (5.19) \\ &\approx \frac{A_\xi}{T} \end{aligned}$$

- *In a DS-SS system, the interference is reduced by the processing gain P_G relative to its effect in a nonspread system.*

5.3 Spreading Codes

- *With TDMA, the users are time orthogonal, with FDMA, the users are approximately frequency orthogonal, with CDMA, the users are approximately code orthogonal*
- To achieve this orthogonality, a different symbol-shaping function is assigned to each user k

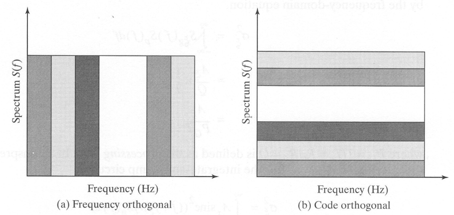
$$g_k(t)=\sum_{q=1}^{Q}c_k(q)g_c(t-qT_c) \quad 0\le t\le T \tag{5.20}$$

- The approximate orthogonality of $g_j(t)$ and $g_k(t)$ for different time offsets τ can be expressed as

$$R_{jk}(\tau)=\int_{-\infty}^{\infty}g_j(t+\tau)g_k^*(t)dt\approx 0 \quad for\ j\ne k \tag{5.21}$$

- Additional self-orthogonality requirement that minimizes a number of practical channel and receiver effects is

$$R_{kk}(\tau)\approx 0 \quad for\ \tau>0 \tag{5.22}$$



(a) Frequency orthogonal (b) Code orthogonal

FIGURE 5.3 Comparison of spectrum use of CDMA and FDMA.

5.3 Spreading Codes

5.3.1 Walsh-Hadamard sequences

- To construct a Walsh-Hadamard sequence, we begin with sequences of length 2

$$\mathbf{H}_1=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix} \tag{5.25}$$

- A four orthogonal sequences of length 4 from the sequences of length 2

$$\mathbf{H}_2=\begin{bmatrix}\mathbf{H}_1 & \mathbf{H}_1\\ \mathbf{H}_1 & -\mathbf{H}_1\end{bmatrix}=\begin{bmatrix}1 & 1 & 1 & 1\\ 1 & -1 & 1 & -1\\ 1 & 1 & -1 & -1\\ 1 & -1 & -1 & 1\end{bmatrix} \tag{5.26}$$

- In general, we may construct 2^n orthogonal sequences of length 2^n from sequences of length 2^{n-1} by operation

$$\mathbf{H}_n=\begin{bmatrix}\mathbf{H}_{n-1} & \mathbf{H}_{n-1}\\ \mathbf{H}_{n-1} & -\mathbf{H}_{n-1}\end{bmatrix} \tag{5.27}$$

5.3 Spreading Codes

5.3.2 Orthogonal Variable Spreading Factors (OVSF)

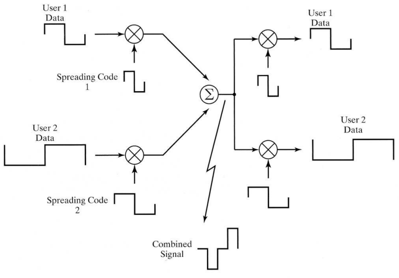


FIGURE 5.6 Illustration of orthogonal variable spreading factors.

- Suppose we need OVSF codes of length n_1 and n_2, with $n_1 < n_2$. The algorithm for constructing OVSF codes is
  1. Construct H_{n1} by the usual Walsh-Hadamard algorithm.
  2. Choose one row of the matrix H_{n1} as the code of length 2^{n_1}. Let H'_{n_1} represent the Hadamard matrix with the selected row removed.
  3. Continue the Walsh-Hadamard algorithm with H'_{n_1}; that is,

$$\mathbf{H}'_{n_1+1} = \begin{bmatrix} \mathbf{H}'_{n_1} & \mathbf{H}'_{n_1} \\ \mathbf{H}'_{n_1} & -\mathbf{H}'_{n_1} \end{bmatrix} \qquad (5.29)$$

  4. Then continue until the desired H'_{n_2} is constructed
  5. Choose any row of H'_{n_2} as the spreading code of length 2^{n_2}
  6. If a third code is needed, continue in a similar manner

5.3 Spreading Codes

5.3.3 Maximal-Length Sequences

- Sequences having properties similar to random sequences but can be generated simply at both the transmitter and receiver.
- Can be generated by Shift register
- Maximal-length sequences have five important properties:
  1. Length property: Each maximal length sequence is of length 2^m-1.
  2. Balance property: Each maximal length sequence has 2^m-1 ones and 2^{m-1}-1 zeros.
  3. Shift property:
     1. The modulo-2 sum of an m-sequence and any circular-shifted version of itself produces another circular-shifted version of itself.
  4. Subsequence property: Each maximal sequence contains a subsequence of $1,2,3,\ldots,m$-1 zeros and ones.
  5. Autocorrelation property.

- Define the circular autocorrelation function

$$R_{jj}(k) = \frac{1}{Q}\sum_{q=0}^{Q-1} c(q)c\left(\left(q+k\right)\mathrm{mod}(Q)\right) \qquad (5.30)$$

- Normalized autocorrelation

$$R_{jj}(k) = \begin{cases} 1 & k = 0 \\ \dfrac{-1}{Q} & k \neq 0 \end{cases} \qquad (5.31)$$

- *Each maximal-length sequence provides 2^m-1 approximately orthogonal sequences obtained by different (circular) time shifts of the original sequence.*
- Given the irreducible polynomial

$$x^m + c_{m-1}x^{m-1} + c_{m-2}x^{m-2} + \ldots + c_1 x + 1 = 0 \qquad (5.32)$$

A shift register for m-sequence can be constructed

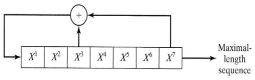


FIGURE 5.7 Shift register generator of maximal-length sequence.

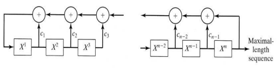


FIGURE 5.8 Maximal-length shift register corresponding to Eq. (5.32).

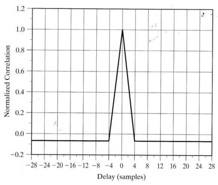


FIGURE 5.9 Circular autocorrelation function of maximal-length sequence of length 2^4 - with four times oversampling.

5.3 Spreading Codes

5.3.4 Scramblers

- Prevent transmission of long strings of zeros and ones that may appear in raw data.
- Long strings of 0 and 1 cause
  - difficulties for circuits tracking
  - Peaks in transmit spectrum →excessive interference.

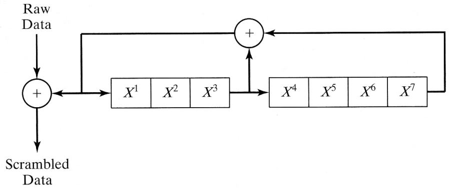


FIGURE 5.10 Scrambler implementation for $f(x) = x^7 + x^3 + 1$.

5.3 Spreading Codes

5.3.5 Gold Codes

- *Gold codes are created by summing the output of the two m-sequence generators to produce a new 0-1 sequence.*
- The cross-correlation of the Gold codes is approximately bounded by

$$\left|R_{jk}(\tau)\right| \le \frac{1}{Q} + \frac{2}{\sqrt{Q}} \qquad j \ne k \qquad (5.33)$$

- *For select pairs of maximal-length sequences, we can generate 2^m-1 distinct gold codes*

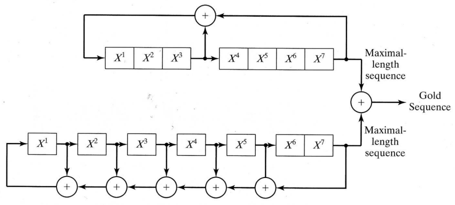


FIGURE 5.11 Generation of a Gold sequence.

5.3 Spreading Codes

5.3.6 Random Sequences

- For random sequences, we define cross-correlation of two sequence as

$$R_{xy}(k) = E\left[\frac{1}{Q}\sum_{q=1}^{Q} x_q y_{q+k}\right] \tag{5.34}$$

- The autocorrelation function of random sequence is given by

$$R_{xx}(k) = E\left[\frac{1}{Q}\sum_{q=1}^{Q} x_q y_{q+k}\right]$$

$$= \begin{cases} \frac{1}{Q}\sum_{q=1}^{Q} E[x_q]E[x_{q+k}] = 0\times 0 = 0 & k \neq 0 \\ \frac{1}{Q}\sum_{q=1}^{Q} E[x_q^2] = 1 & k = 0 \end{cases} \tag{5.35}$$

- If we compute the cross-correlation properties, we can get

$$R_{xy}(k) = 0 \quad \text{for all } k \tag{5.36}$$

- The average cross-correlation energy of 2 random sequences is give by

$$\begin{aligned} E\left[|R_{xy}|^2\right] &= E\left[\left(\frac{1}{Q}\sum_{q=1}^{Q} x_q y_q\right)^2\right] \\ &= E\left[\frac{1}{Q^2}\sum_{q=1}^{Q}\sum_{m=1}^{Q} x_q y_q x_m y_m\right] \\ &= \frac{1}{Q^2}\sum_{m=1}^{Q} x_m^2 y_m^2 \\ &= \frac{1}{Q^2}\sum_{m=1}^{Q} 1\times 1 \\ &= \frac{1}{Q} \end{aligned} \tag{5.37}$$

- *The interference power between two random sequences is inversely proportional to the length of the sequence.*

5.4 The advantages of CDMA for Wireless

- Effect on multiple-access performance when spreading codes are not perfectly orthogonal.
- 5.4.1 Multiple-Access Interference
- 5.4.2 Multipath Channels
- 5.4.3 RAKE Receiver
- 5.5.5 Fading Channels
- 5.4.5 Summary of Benefits of DS-SS

5.4 The advantages of CDMA for Wireless

5.4.1 Multiple-Access Interference

- Received complex baseband signal of synchronous CDMA

$$\tilde{x}(t) = \sum_{k=1}^{K} \alpha_k \tilde{s}_k(t) + \tilde{w}(t) \quad 0 \le t \le T \qquad (5.38)$$

- The individual transmitted signals reflect their respective data and spreading waveforms, given by

$$\tilde{s}_k(t) = b_k \sqrt{E_b}\, g_k(t) \qquad (5.39)$$

- The output is given by

$$\begin{aligned} y &= \int_0^T \tilde{x}(t) g_1^*(t) dt \\ &= \alpha_1 b_1 \sqrt{E_b} + \eta_1 + \sum_{k=2}^{K} \alpha_k b_k \sqrt{E_b} \int_0^T g_k(s) g_1^*(s) dt \qquad (5.40) \\ &= \alpha_1 b_1 \sqrt{E_b} + \eta_1 + \sqrt{E_b} \sum_{k=2}^{K} \alpha_k b_k R_{1k} \quad 0 \le t \le T \end{aligned}$$

- Average amplitude at correlator output is given by

$$\begin{aligned} E[y] &= \alpha_1 b_1 \sqrt{E_b} + E[\eta_1] + E\left[\sqrt{E_b} \sum_{k=0}^{K} \alpha_k b_k R_{1k}\right] \qquad (5.41) \\ &= \alpha_1 b_1 \sqrt{E_b} + 0 + 0 \\ &= \alpha_1 b_1 \sqrt{E_b} \end{aligned}$$

- The variance of output is

$$\begin{aligned} \sigma_\gamma^2 &= E\left[Y - E[y]^2\right] \\ &= E\left[|\eta_1|^2 + 2\,\mathrm{Re}\left[\eta_1^*\left(\sqrt{E_b} \sum_{k=2}^{K} \alpha_k b_k R_{1k}\right)\right] + \left|\sqrt{E_b} \sum_{k=2}^{K} \alpha_k b_k R_{1k}\right|^2\right] \\ &= N_0 + 0 + E_b \sum_{k=2}^{K} \sum_{l=2}^{K} \alpha_k \alpha_l^* E[b_k b_l^*] E[R_{1k} R_{1l}^*] \qquad (5.42) \\ &= N_0 + E_b \sum_{k=2}^{K} \sum_{l=2}^{K} \alpha_k \alpha_l^* \delta(k-1) E[R_{1k} R_{1l}^*] \\ &= N_0 + E_b \sum_{k=2}^{K} |\alpha_k|^2 E|R_{1k}|^2 \end{aligned}$$

- From Eq.(5.37), we can have

$$E\left[|R_{jk}|^2\right] \approx \frac{1}{Q} \qquad (5.43)$$

- Contribution of multiple-access term to the noise variance given by

$$\sigma_{MAI}^2 \approx E_b \sum_{k=2}^{K} |\alpha_k|^2 \frac{1}{Q} \qquad (5.44)$$

- The transmitters are power controlled, which means

$$\alpha_k = 1 \quad for\ all\ k \qquad (5.45)$$

- 1st and 2nd order statistics of MAI contribution to noise

$$\mu_{MAI} = E\left[y - \alpha_1 b_1 \sqrt{E_b}\right] = 0 \quad and \quad \sigma_{MAI}^2 = \frac{K-1}{Q} E_b \qquad (5.46)$$

- Signal-to-interference-pulse-noise ratio

$$\begin{aligned} SINR &= \frac{\left(E[y]^2\right)}{\sigma_Y^2} \\ &= \frac{E_b}{\left(N_0 + \frac{K-1}{Q} E_b\right)} \qquad (5.47) \\ &= \frac{E_b}{N_0} \left(\frac{1}{1 + \frac{K-1}{Q} \frac{E_b}{N_0}}\right) \\ &= \frac{E_b}{N_0} D_g \end{aligned}$$

- Degradation

$$D_g = \left(1 + \frac{K-1}{Q} \frac{E_b}{N_0}\right)^{-1} \qquad (5.48)$$

5.4 The advantages of CDMA for Wireless

5.4.2 Mutlipath

- Complex envelope of channel impulse response is given by

$$\tilde{h}(t) = \sum_{l=1}^{L} \alpha_l \delta(t - \tau_l) \qquad (5.49)$$

- Define windowed channel spectrum by

$$H_R(f) = \begin{cases} H(f) & |f| < R_c / 2 \\ 0 & otherwise \end{cases} \qquad (5.50)$$

- By Nyquist smapling theorem, the time domain equivalent of $H_R(f)$ can be represented by $\{h_n\}$

$$h_w(t) = \sum_{n=-\infty}^{\infty} h_n \sin c\left(R_c\left(t - n/R_c\right)\right) \qquad (5.51)$$

- With $R_c=1/T_c$, the chip rate, by the convolutional sampling theorem, the received signal is

$$\begin{aligned}\tilde{x}_{mp}(t) &= \tilde{h}_w(t)\otimes\tilde{s}(t)+\tilde{w}(t)\\ &= \sum_{l=-\infty}^{\infty} h_l s(t-lT_c)+\tilde{w}(t) \qquad (5.52)\\ &= \sum_{l=0}^{L'} h_l s(t-lT_c)+\tilde{w}(t)\end{aligned}$$

5.4 The advantages of CDMA for Wireless

5.4.3 RAKE Receiver

- Operating in multipath environments.
- The output of each delay element is processed by a single-user receiver.
- This receiver has (L+1) fingers and gets the name “RAKE” from its resemblance to common garden rake.
- The output of ith finger

$$\begin{aligned}y_i &= \int_{-iT_c}^{T-iT_c}\tilde{x}(t+iT_c)g^*(t)dt\\ &= \int_{-iT_c}^{T-iT_c}\sum_{l=0}^{L} h_l\tilde{s}(t-lT_c+iT_c)g^*(t)dt+\int_{-iT_c}^{T-iT_c}\tilde{w}(t+iT_c)g^*(t)dt \qquad (5.53)\\ &= \sum_{l=o}^{L} h_l\int_{-iT_c}^{T-iT_c}\tilde{s}(t-lT_c+iT_c)g^*(t)dt+\eta_i\\ &\approx h_i b\sqrt{E_b}+b\sqrt{E_b}\sum_{l\neq i} h_l R_{gg}(l-i)+\eta_i\end{aligned}$$

- If the delay spread of channel is much less than the symbol period T, then

$$\int_0^T g(t-(l-i)T_c)g^*(t)dt \approx R_{gg}(l-i) \qquad (5.54)$$

- Contribution of self-interference to total noise *for the first finger* is

$$\sigma_Y^2 = N_0 + E_b\sum_{l=1}^{L}|h_l|^2|R_{gg}(lT_c)|^2 \qquad (5.55)$$

- If spread code has cross-correlation properties similar to random codes, then we can approximate result

$$\sum_{l\neq j}|h_l|^2|R_{gg}(lT_c)|^2 \approx \frac{1}{Q}\sum_{l\neq j}|h_l|^2 \qquad (5.56)$$

- The final estimate of data is given by weighted sum of output of each finger of RAKE receiver

$$\begin{aligned}y &= \sum_{i=1}^{L} h_i^* y_i \qquad (5.57)\\ &\approx \sqrt{E_b}\sum_{l=0}^{L}|h_l|^2 b+\sum_{l=0}^{L} h_l^*\eta_l\end{aligned}$$

which is called *maximal-ratio combining (MRC)*

- The resulting output signal-to-noise ratio is

$$\begin{aligned}SNR &= \frac{(E[y]^2)}{\sigma_Y^2} = \frac{E_b\left(\sum_{l=0}^{L}|h_l|^2\right)^2}{N_0\sum_{l=0}^{L}|h_l|^2} \qquad (5.58)\\ &= \frac{E_b\sum_{l=0}^{L}|h_l|^2}{N_0}\end{aligned}$$

5.4 The advantages of CDMA for Wireless

5.4.4 Fading Channels

- Assumption: Transmission systems are usually designed such that channel is approximately constant, at least over the duration of a symbol interval.
- Therefore, the fading performance of single-user spread spectrum BPSK or QPSK is same as non-spread case.
- For the transmission of coherent BPSK over a single-ray Rayleigh-fading transmission path, BER performance is

$$P_e \approx \frac{1}{4E_b/N_0} \qquad (5.59)$$

- BPSK performance is given by

$$P_e \approx \binom{2L-1}{L}\left(\frac{1}{4E_b/N_0}\right)^L \qquad (5.60)$$

- *With second-order diversity, there is an approximate squaring of the bit error rate*

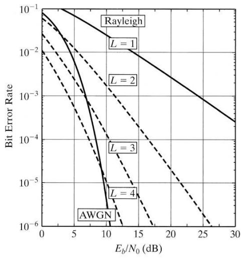


FIGURE 5.19 Comparison of performance in AWGN with that of Rayleigh fading with different diversity orders.

5.4 The advantages of CDMA for Wireless

5.4.5 Summary of the Benefits of DS-SS

Important features in a multiple-access system

1. Spectral density of transmitted signal is reduced by factor equal to processing gain.
2. Effect of interference by processing gain.
3. Under ideal conditions, there is no difference in BER performance between spread and non-spread forms of BPSK or QPSK.
4. In multipath channels, if it is treated as interference, its effect can be reduced by processing gain.
5. Multipath can improve receiver performance by capturing the energy in paths having different transmission delays.
6. In fading channels, spread-spectrum receiver can obtain important advantage in diversity by using RAKE receiver.
7. Choice of spreading codes is critical to reduce multiple-access interference and multipath self-interference.

5.5 Code Synchronization

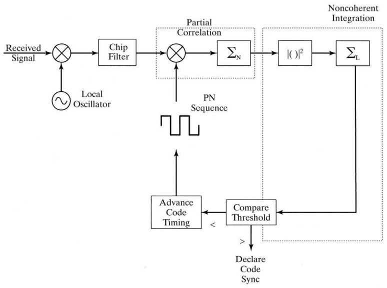


FIGURE 5.20 Illustration of the code synchronization circuit for BPSK.

- Assume spreading code is transmitted repeatedly
- Complex baseband receieved signal prior to chip filters

$$\begin{aligned}\tilde{x}(t) &= \sqrt{E_b}\,g(t)e^{j(2\pi\Delta ft+\phi)} & 0 \le t \le T \\ &= \sqrt{E_b}\,g(t-T)e^{j(2\pi\Delta fkT_c+\phi)} & T \le t < 2T\end{aligned} \quad (5.61)$$

- Output of sampling device

$$\begin{aligned}\tilde{x}(kT_c) &= \sqrt{E_b}\,g(kT_c \bmod T)e^{j(2\pi\Delta fkT_c+\phi)} \\ &= \sqrt{E_b}\,c(k \bmod Q)e^{j(2\pi\Delta fkT_c+\phi)}\end{aligned} \quad (5.62)$$

- Partial correlation

$$\begin{aligned}C_x(k_1) &= \sum_{k=k_1}^{k_1+\Delta k} x(kT_c)c^*((m_0+(k-k_1))\bmod Q) \\ &= E_b\sum_{k=k_1}^{k_1+\Delta k} c(k \bmod Q)c^*((m_0+(k-k_1))\bmod Q)e^{j(2\pi\Delta fkT_c+\phi)} \quad (5.63) \\ &\approx E_b e^{j\phi'}\sum_{k=k_1}^{k_1+\Delta k} c(k \bmod Q)c^*((m_0+(k-k_1))\bmod Q)\end{aligned}$$

- Ideally the vale if $C_x(k_1)$ should be

$$C_x(k_i) = \begin{cases} E_b\Delta k e^{j\phi'} & \textit{if the sequencesalign} \\ 0 & \textit{otherwise} \end{cases} \quad (5.64)$$

- To increase the confidenceof synchronization, we form a decision variable

$$D = \sum_{l=1}^{L_1} \left|C_x(k_l)\right|^2 \quad (5.65)$$

5.6 Channel Estimation

- In flat-fading channel, a frequency shift will be happened there the
- amplitude and phase of the received signal can be changed.
- The mobile channel effects on phase, frequency and amplitude are slowly time-varying.
- It is critical at receiver performance to have an accurate channel estimation and tracking strategy to tack these variations.

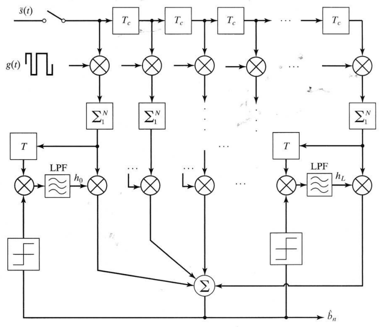


FIGURE 5.21 Channel estimation with a RAKE receiver.

- Channel-tracking algorithm
  - Reliable data estimates, which involves transmitting training sequence to adapt the channel estimates initially.
  - Bandwidth of low-pass filter must be chosen appropriately.
  - If data stream is channel encoded to operate at a lower SNR, reliability of the uncoded data at the output of RAKE receiver is liable to be poor at all times.

5.7 Power Control: The Near-Far Problem

- The complex baseband received signal is

$$\tilde{x}_c(t) = \sum_{k=1}^{K} \alpha_k \tilde{s}_k(t) + \tilde{w}(t) \qquad (5.67)$$

- SINR of 1st user is

$$\begin{aligned} SINR &= \frac{(E[y])^2}{\sigma_Y^2} \\ &= \frac{\alpha_1^2 E_b}{N_0 + \frac{1}{Q} E_b \sum_{k=2}^{K} \alpha_k^2} \qquad (5.68) \\ &= \frac{\alpha_1^2 E_b}{N_0} \frac{1}{1 + \frac{K-1}{Q} \frac{\alpha_1^2 E_b}{N_0} \left(\frac{1}{(K-1)} \sum_{k=2}^{K} \left(\frac{\alpha_k}{\alpha_1} \right)^2 \right)} \\ &= \alpha_1^2 \frac{E_b}{N_0} D_g' \qquad \textit{where} \qquad D_g' = \frac{1}{1 + \frac{K-1}{Q} \frac{\alpha_1^2 E_b}{N_0} \left(\frac{1}{K-1} \sum_{k=2}^{K} \left(\frac{\alpha_k}{\alpha_1} \right)^2 \right)} \end{aligned}$$

- Comparing the definition of D'_g with definition of D_g , we can see
  - The received energy for the desired signal is $\alpha_1^2 E_b$ instead of E_b.
  - Difference is that the denominator of D'_g included the factor, multiplier applied to the multiple-access interference.

$$\frac{1}{(K-1)} \sum_{k=2}^{K} \left(\frac{\alpha_k}{\alpha_1} \right)^2 \qquad (5.70)$$

- Near-far problem: If any of the signal strengths is a greater than the desired signal($\alpha_k >> \alpha_1$), the multiple-access interference will be increased.
- Solution for near-far problem: Power control.
- Implementation issues of power control:
  - Latency
    - Power must be measured at the receiver and then relayed to the transmitter to adjust the transmit power.
  - Accuracy
    - Minimal averaging of potentially very noisy signal.

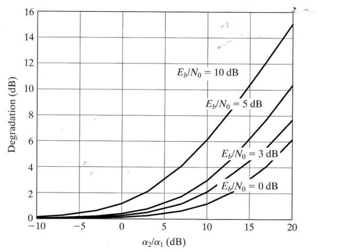


FIGURE 5.22 Performance degradation of first of two users as a function of relative power and processing gain.

5.8 FEC Coding and CDMA

- *Forward error correction (FEC)* coding increases the bandwidth by a factor of 2 to 3, but includes redundancy.
- Decompose the maximum spreading rate into two parts $Q = Q_s \times \frac{1}{r}$
- If each FEC-encoded bit is spread by factor Q_s, we can write Eq. (5.48) as

$$D_g = \left(1 + \left(\frac{K-1}{Q_s}\right)\frac{E_s}{N_0}\right)^{-1} = \left(1 + \left(\frac{K-1}{Q/r}\right)\frac{E_b}{rN_0}\right)^{-1} = \left(1 + \left(\frac{K-1}{Q}\right)\frac{E_b}{N_0}\right)^{-1} \quad (5.72)$$

- *This is the general expression for the degradation due to multiple-access interference when coding is present and unchanged from the uncoded expression.*
- *However, since the operating E_b/N_0 is much lower in coded situation, there is significant improvement in overall performance.*

- *The equation is the general expression for the degradation due to multiple-access interference when coding is present and unchanged from the uncoded expression.*
- *Since the operating E_b/N_0 is much lower in coded situation, there is significant improvement in overall performance*
- *FEC coding often provides a form of time tiversity*

5.9 Multiuser Detection

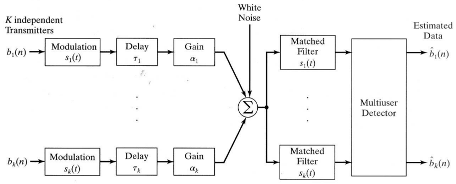


FIGURE 5.24 Multiuser system model.

- Matched-filter outputs can undergo further processing in the form of *multiuser detection* so that the multiple-access interference can be reduced.
- In synchronous CDMA, an equivalent baseband discrete-time model for the system over one symbol period is

$$\mathbf{y} = \mathbf{ARb} + \eta \quad 0 \le \mathrm{t} \le \mathrm{T} \quad \text{where} \quad \mathrm{b} = [b_1, b_2, \ldots b_K]^T \quad \text{and} \quad \mathbf{A} = \begin{bmatrix} \alpha_1 & 0 & \cdots & 0 \\ 0 & \alpha_2 & \cdots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & \alpha_K \end{bmatrix} \quad (5.73,74)$$

- For matrix R is the cross-correlation matrix of symbol-shaping waveforms of different users, *ij*th element of **R** and the vector of correlated Gaussian noise samples with element is

$$R_{ij} = \int_0^T g_i(t) g_j^*(t)\,dt \quad \text{and} \quad \eta_{\mathrm{i}} = \int_0^T g_i^*(t)\tilde{w}(t)\,dt \quad (5.75,76)$$

- Multiuser detection algorithm is to perform the equivalent of interference cancellation
- Method 1:
  - Conventional receiver: estimates each user's bits on basis of sign of individual elements.

$$\hat{\mathbf{b}} = sign\{y\} \quad (5.77)$$

- Method 2
  - Optimum (maximum-likelihood) detector

$$\hat{\mathbf{b}} = \arg\min_b |\mathbf{y} - \mathbf{ARb}|^2 \quad (5.78)$$

- Method 3
  - de-correlating detector

$$\hat{\mathbf{b}} = sign\{\mathbf{R}^{-1} y\} \quad (5.80)$$

5.10 CDMA in a Cellular Environment

- Interference caused by another CDMA signal appears to be approximately equivalent to additive Gaussian noise.
- Multiple-access interference is directly proportional to channel loading.
- Define intracellular interference as

$$I_{\text{intracell}} = \frac{K-1}{Q} E_b \approx \frac{K}{Q} E_b \qquad (5.81)$$

- Intercellular or other-cell interference is the second type of interference.
- Define relative other-cell interference factor as

$$f = \frac{I_{\text{other-cell}}}{I_{\text{intracell}}} \qquad (5.82)$$

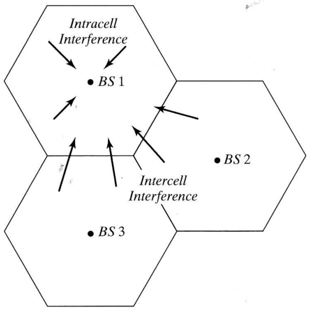


FIGURE 5.25 Intracell and intercell interference.

- Ping Pong effect:
  - With the mobile terminal switching back and forth between two base stations as the strength of the signal varies at the boundary between two cells.
- The size of relative other-cell interference factor depends on

1. *Propagation-loss exponent*:
   - the larger the propagation loss exponent is, the quicker adjacent-cell interference will be attenuated.
2. *Variations in signal strength due to shadowing*:
   - the larger the variation due to shadowing, the greater the potential is to cause problems.

- 3. *The handoff technique between cells*:
  - Hard handover:
    - the case where communications are terminated with one base station immediately upon acquisition of 2nd base station.
    - This will result a ping-pong effect.
  - Soft handover:
    - mobile terminal maintains communications with both base stations until one of them becomes significantly stronger than the other.
    - This can prevent ping-pong effect.

- Total interference density

$$\begin{aligned} I_0 &= I_{intracell} + I_{other-cell} \\ &\approx (1+f)\frac{K}{Q}E_b \end{aligned} \qquad (5.83)$$

- Signal-to-interference-plus-noise at individual receiver

$$\begin{aligned} SINR &= \frac{E_b}{N_0 + I_0} \\ &= \frac{E_b}{I_0\left(1+\frac{N_0}{I_0}\right)} \end{aligned} \qquad (5.84)$$

- Cellular CDMA system are often *interference limited*.
- Substitute Eq.(5.83) into Eq. (5.84), we get

$$SINR = \frac{1}{(1+f)\frac{K}{Q}\left(1+\frac{N_0}{I_0}\right)} \qquad (5.85)$$

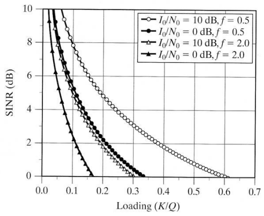


FIGURE 5.26 Maximum channel loading as a function of the other-cell interference factor f and the ratio of the interference noise density to that of the receiver.

- Refer to Figure 5.26
  - For a constant SINR moving from a noise-limited system to an interference-limited system increases the permissible channel loading.
  - For a constant SINR, increasing other-cell interference significantly reduces the permissible channel loading.
  - Reducing the SINR required by the receiver can significantly improve the permissible channel loading.
- *Cell sectorization*:
  - Installation of directional antennas as opposed to omnidirectional antennas to improve system capacity

5.11 Frequency-Hopped Spread Spectrum

- Advantages of FH-SS
  - High tolerance of narrowband interference.
  - Relatively straightforward interference avoidance.
  - Current technology-hopped bandwidths on the order of several gigahertz.
- Disadvantages of FH-SS
  - Noncoherent detection
  - Higher probability of detection

5.11 Frequency-Hopped Spread Spectrum

5.11.1 Complex Baseband Representation of FH-SS

- As DS-SS, complex baseband signal can be expressed as

$$\tilde{s}(t) = m(t)\exp\left(j2\pi\zeta_k t + \theta_k\right) \quad 0 \le t \le T \qquad (5.86)$$

- For linear modulation, the data modulation is

$$m(t) = b\sqrt{E_s}\, g(t) \quad 0 \le t \le T \qquad (5.87)$$

- For frequency-hopped systems, M-ary FSK will be used, in which case we have

$$m(t) = \sqrt{E_s}\exp\left(j2\pi f_i t\right) \quad 0 \le t \le T \qquad (5.88)$$

- Hop period is the period when the transmit frequency is constant.
- When hop period larger than the symbol time, multiple data symbols transmitted on each hop, this is called *slow-frequency hopping*.

- Chip refers to the tone of shortest duration.
- For slow-frequency-hopped system, chip rate = symbol rate.
- For fast-frequency-hopped system, chip rate = hopping rate.

$$R_c = \max\{R_s, R_h\} \qquad (5.89)$$

- The tone (signal) which produced by combination of pseudonoise (PN) code generator output and MFSK modulator output is

$$\begin{aligned} s(t) &= \sqrt{E_s}\,\mathrm{Re}\{m(t)\exp(j(2\pi(\zeta_k + f_c)t + \phi_k))\} \\ &= \sqrt{E_s}\,\mathrm{Re}\{\exp(j(2\pi(f_i + \zeta_k + f_c)t + \phi_k))\} \quad where \quad 0 \le t \le T \end{aligned}$$

(5.90)

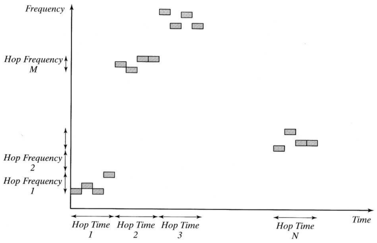


FIGURE 5.28 Frequency–time plot of output of FH/MFSK system.

5.11 Frequency-Hopped Spread Spectrum

5.11.2 Slow-Frequency Hopping

$$\min\{\zeta_k - \zeta_j\} \approx MR_s \qquad k \neq j$$

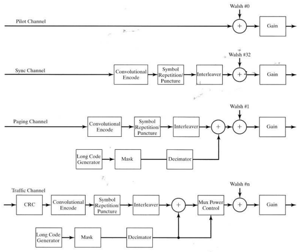


FIGURE 5.29 Bit processing of IS-95 downlink, showing various channels of the system.

5.11 Frequency-Hopped Spread Spectrum

5.11.3 Fast-Frequency Hopping

- Same symbol is transmitted on multiple hops.
- For combining the outputs from multiple hops, two algorithms are:
  - *Majority logic detector*
    - Hard decision on symbol is made every hope
    - Outs of different hops are combined by choosing the symbol that appears the most often.
  - *Noncoherent combining*
    - Soft measure related to likelihood is used for combining outputs.
    - It related to signal energy in a given frequency bin.

5.11 Frequency-Hopped Spread Spectrum

5.11.4 Processing Gain

- Jammer: intentional interferer
- If jammer spreads its power in interference over the whole spread bandwidth, the resulting interference density is $J_0=J/W_{ss}$
- Received signal-to-noise ratio

$$\begin{aligned} SNR &= \frac{E_S}{J_0} \\ &= \frac{C/R_s}{J/W_{ss}} \\ &= \frac{C}{J}\left(\frac{W_{ss}}{R_S}\right) \end{aligned} \quad (5.93)$$

- Processing gain

$$P_G = \frac{W_{ss}}{R_s} \quad (5.94)$$