Modern Wireless Communication

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Chapter 6

Diversity, Capacity and Space-Division Multiple Access

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6.2 “Space Diversity on Receive” Techniques

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6.2 “Space Diversity on Receive” Techniques

6.2.1 Selection Combining

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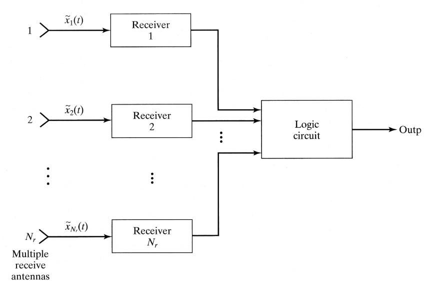

FIGURE 6.1 Block diagram of selection combiner, using N_r receive antennas.

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- Given N_r receiver outputs produced by common transmitted signal, logic circuit selects particular receiver output with largest signal-to-noise ratio as received signal.
- Assumption
  - Frequency-flat: all frequency components constituting the transmitted signal are characterized by same random attenuation and phase shift.
  - Slow-fading: fading remains unchanged during transmission
  - Fading phenomenon is described by Rayleigh distribution.

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- Complex envelope of received signal of kth diversity branch is

$$\tilde{x}_k(t) = \alpha_k e^{j\theta_k}\tilde{s}(t) + \tilde{w}_k(t) \quad 0 \le t \le T \text{ and } k = 1,2,\ldots,N_r \tag{6.1}$$

- With fading assumed to be slowly varying relative to symbol duration T, we simply the above Eq. to

$$\tilde{x}_k(t) \approx \alpha_k\tilde{s}(t) + \tilde{w}_k(t) \quad 0 \le t \le T \quad \text{and} \quad k = 1,2,\ldots,N_r \tag{6.2}$$

- The average signal-to-noise ratio at kth receiver output

$$(SNR)_k = \left(\frac{E\left[\left|\alpha_k\tilde{s}(t)\right|^2\right]}{E\left[\left|w_k(t)\right|^2\right]}\right) = \frac{E\left|\tilde{s}(t)\right|^2}{E\left|\tilde{w}_k(t)\right|^2}E\left[\alpha_k^2\right] \quad k = 1,2,\ldots,N_r \tag{6.3}$$

- As mean-square value of $\tilde{w}_k(t)$ is the same for all k , we have

$$(SNR)_k = \frac{E}{N_0}E\left[\alpha_k^2\right] \quad k = 1,2,\ldots,N_r \tag{6.4}$$

- Instantaneous signal-to-noise ratio

$$\gamma_k = \frac{E}{N_0}\alpha_k^2 \quad k = 1,2,\ldots,N_r \qquad (6.5)$$

- Assume average signal-to-noise ratio over short-term fading is same, the probability density functions of random variables Γ_k pertaining to individual branches as

$$f_{\Gamma_k}(\gamma_k) = \frac{1}{\gamma_{av}}\exp\left(-\frac{\gamma_k}{\gamma_{av}}\right) \quad \gamma_k \ge 0 \quad k = 1,2,\ldots,N_r \qquad (6.6)$$

- For some signal-to-noise ratio, the associated cumulative distributions of individual branches are

$$prob(\gamma_k \le \gamma) = \int_{-\infty}^{\gamma} f_{\Gamma_j}(\gamma_k)d\gamma_k = 1-\exp\left(-\frac{\gamma}{\gamma_{av}}\right) \quad \gamma \ge 0 \qquad (6.7)$$

- Probability that all the diversity branches have a signal-to-noise ratio less than threshold γ is

$$prob(\gamma_k < \gamma \text{ for } k = 1,2,\ldots,N_r) = \prod_{k=1}^{N_r} prob(\gamma_k < \gamma) = \prod_{k=1}^{N_r}\left[1-\exp\left(-\frac{\gamma}{\gamma_{av}}\right)\right] = \left[1-\exp\left(-\frac{\gamma}{\gamma_{av}}\right)\right]^{N_r} \quad \gamma \ge 0 \qquad (6.8)$$

- Cumulative distribution function of random variable Γ_{SC}

$$\gamma_{sc} = \max\{\gamma_1, \gamma_1, \ldots, \gamma_1\} \qquad (6.9)$$

- Cumulative distribution function of selection combiner

$$F_\Gamma(\gamma_{sc}) = \left[1-\exp\left(-\frac{\gamma_{sc}}{\gamma_{av}}\right)\right]^{N_r} \quad \gamma_{sc} \ge 0 \qquad (6.10)$$

- Probability density function of $f_\Gamma(\gamma_{sc})$

$$f_\Gamma(\gamma_{sc}) = \frac{d}{d\gamma_{sc}}F_\Gamma(\gamma_{sc}) = \frac{N_r}{\gamma_{av}}\exp\left(-\frac{\gamma_{sc}}{\gamma_{av}}\right)\left[1-\exp\left(-\frac{\gamma_{sc}}{\gamma_{av}}\right)\right]^{N_r-1} \quad \gamma_{sc} \ge 0 \qquad (6.11)$$

- For convenience of graphical presentation, we use the scaled probability density function

$f_X(x) = \gamma_{av} f_{\Gamma_{sc}}(\gamma_{sc})$ *where the normalized variable x is* $x = \gamma_{sc}/\gamma_{av}$

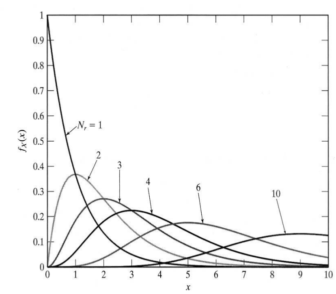

FIGURE 6.5 Normalized probability density function $f_X(x) = \frac{1}{(N_r-1)!}x^{N_r-1}\exp(-x)$ for a varying number N_r of receive antennas.

- From figure 6.2
  - As the number of branches increase, probability density function of normalized random variable moves progressively to right.
  - Probability density function becomes more symmetrical and Gaussian as N_r increase.
- Procedures of scanning version of selection-combining:
  - Selecting receiver with strongest output signal.
  - Maintain the procedure by using the output of this particular receiver as combiner's output.
  - When the instantaneous signal-to-noise ratio of combiner falls below the threshold, select a new receiver with strongest output signal.

6.2 “Space Diversity on Receive” Techniques

6.2.2 Maximal-Ratio Combining

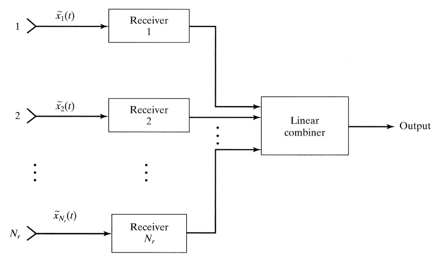

FIGURE 6.4 Block diagram of maximal-ratio combiner using N_r receive antennas.

- Limitation of selection combiner can be mitigated by maximal-ratio combiner
- Complex envelope of linear combiner output

$$\begin{aligned}\tilde{y}(t) &= \sum_{k=1}^{N_r} a_k \tilde{x}_k(t) \\ &= \sum_{k=1}^{N_r} a_k \left[\alpha_k e^{j\theta_k} \tilde{s}(t) + \tilde{w}_k(t)\right] \\ &= \tilde{s}(t)\sum_{k=1}^{N_r} a_k \alpha_k e^{j\theta_k} + \sum_{k=1}^{N_r} a_k \tilde{w}_k(t)\end{aligned} \tag{6.12}$$

- From Eq. (6.12), we notices that
  - Complex envelope of output signal equals $\tilde{s}(t)\sum_{k=1}^{N_r} a_k \alpha_k e^{j\theta_k}$
  - Complex envelope of output noise equals $\sum_{k=1}^{N_r} a_k \tilde{w}_k(t)$

Assuming $\tilde{w}_k(t)$ are mutually independent for $k = 1,2,\ldots,N_r$, the output signal-to-noise ratio of linear combiner is

$$\begin{aligned}(SNR)_c &= \frac{\mathbf{E}\left[\left|\tilde{s}(t)\sum_{k=1}^{N_r} a_k \alpha_k e^{j\theta_k}\right|^2\right]}{\mathbf{E}\left[\left|\sum_{k=1}^{N_r} a_k \tilde{w}_k(t)\right|^2\right]} \\ &= \frac{\mathbf{E}\left[\left|\tilde{s}(t)\right|^2\right]}{\mathbf{E}\left[\left|\tilde{w}_k(t)\right|^2\right]} \cdot \frac{\mathbf{E}\left[\left|\sum_{k=1}^{N_r} a_k \alpha_k e^{j\theta_k}\right|^2\right]}{\mathbf{E}\left[\left|\sum_{k=1}^{N_r} \alpha_k\right|^2\right]} \\ &= \left(\frac{E}{N_0}\right) \frac{\mathbf{E}\left[\left|\sum_{k=1}^{N_r} a_k \alpha_k e^{j\theta_k}\right|^2\right]}{\mathbf{E}\left[\left|\sum_{k=1}^{N_r} \alpha_k\right|^2\right]}\end{aligned} \tag{6.13}$$

where E/N_0 is the symbol energy-to-noise spectral density ratio

- Let γ_c denote the instantaneous output signal-to-noise ratio of linear combiner, then using

$$\left|\sum_{k=1}^{N_r} a_k \alpha_k e^{j\theta_k}\right|^2 \quad and \quad \sum_{k=1}^{N_r} \left|a_k\right|^2$$

as the instantaneous values of expectations in numerator and denominator of eq.(6.13), we can write

$$\gamma_c = \left(\frac{E}{N_0}\right) \frac{\left|\sum_{k=1}^{N_r} a_k \alpha_k e^{j\theta_k}\right|^2}{\sum_{k=1}^{N_r} \left|a_k\right|^2} \tag{6.14}$$

- According to *Cauchy-Schwarz inequality* for complex number, we have

$$\left|\sum_{k=1}^{N_r} a_k b_k\right|^2 \le \sum_{k=1}^{N_r} \left|a_k\right|^2 \sum_{k=1}^{N_r} \left|b_k\right|^2 \tag{6.15}$$

- Applying Cauchy-Schwarz inequality to instantaneous output signal-to-noise ratio

$$\gamma_c \le \left(\frac{E}{N_0}\right) \frac{\sum_{k=1}^{N_r} \left|a_k\right|^2 \sum_{k=1}^{N_r} \left|\alpha_k e^{j\theta_k}\right|^2}{\sum_{k=1}^{N_r} \left|a_k\right|^2} \tag{6.16}$$

- Canceling common terms in Eq.(6.16)

$$\gamma_c \le \left(\frac{E}{N_0}\right) \sum_{k=1}^{N_r} \alpha_k^2 \tag{6.17}$$

- Therefore, the equality in Eq.(6.17) holds for

$$\begin{aligned}a_k &= c\left(\alpha_k e^{j\theta_k}\right)^* \\ &= c\alpha_k e^{-j\theta_k} \qquad k = 1,2,\ldots,N_r\end{aligned} \tag{6.18}$$

- Instantaneous output signal-to-noise ratio of maximal-ratio combiner

$$\gamma_{mrc} = \left(\frac{E}{N_0}\right) \sum_{k=1}^{N_r} \alpha_k^2 \tag{6.19}$$

- According to Eq.(6.5), the maximal-ratio combiner produces an γ_{mrc} that is the sum of instantaneous signal-to-noise ratios of individual branches

$$\gamma_{mrc} = \sum_{k=1}^{N_r} \gamma_k \tag{6.20}$$

- The term "maximal-ratio combiner" has been coined to describe the combiner of Fig. 6.4 that produces the produces optimum result.

- Chi-Square with $2N_r$ degrees of freedom

$$f_{\Gamma_{mrc}}(\gamma_{mrc}) = \frac{1}{(N_r - r)!}\frac{\gamma_{mrc}^{N_r-1}}{\gamma_{av}^{N_r}}\exp\left(-\frac{\gamma_{mrc}}{\gamma_{av}}\right) \quad (6.1)$$

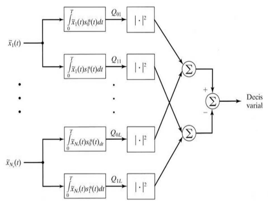

FIGURE 6.8 Block diagram of diversity receiver based on square-law combining.

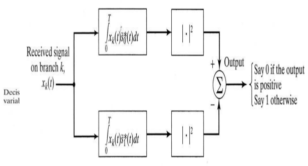

FIGURE 6.7 Illustration of receiver for orthogonal waveforms.

6.2 "Space Diversity on Receive" Techniques

6.2.3 Equal-Gain Combining

- Three issues for maximal-ratio combiner
  1. Significant instrumentation is needed to adjust the complex weighting parameters of maximal-ratio combiner to their exact values.
  2. Additional improvement in :
     1. output signal-to-noise ration gained by mrc over sc is not large
     2. Receiver performance is lost in inability to achieve the exact setting of mrc
  3. other details of the combiner may result in a minor improvement in overall receiver performance.
- Equal-gain combiner: all the complex weighting parameters have their phase angles set opposite to those of their respective multipath branches.

6.2 "Space Diversity on Receive" Techniques

6.2.4 Square-Law Combining

- No need phase estimation.
- Applicable to orthogonal modulation.
- With binary orthogonal signaling, receiver generates

$$Q_{0k} = \frac{1}{\sqrt{N_0}}\int_0^T \tilde{x}_k(t)\tilde{s}_0^*(t)dt \quad and \quad Q_{1k} = \frac{1}{\sqrt{N_0}}\int_0^T \tilde{x}_k(t)\tilde{s}_1^*(t)dt \quad (6.27,28)$$

- In *orthogonal modulation*, two signaling waveforms approximately satisfy the condition

$$\int_0^T \tilde{s}_i(t)\tilde{s}_j^*(t)dt = \begin{cases} E_b & i = j \\ 0 & i \neq j \end{cases} \quad (6.29)$$

- If binary symbol 0 is transmitted, the two decision variables are

$$Q_{0k} = \frac{\sqrt{E_b}\alpha_k e^{j\theta_k}}{\sqrt{N_0}} + \frac{w_{0k}}{\sqrt{N_0}} \quad and \quad Q_{1k} = \frac{w_{1k}}{\sqrt{N_0}} \quad (6.30,31)$$

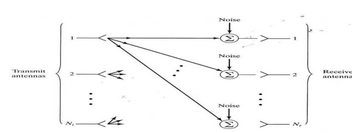

FIGURE 6.10 Block diagram of MIMO wireless link with N_t transmit antennas and N_r receive antennas.

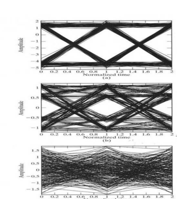

FIGURE 6.11 Effect of coantenna interference on the eye diagram for one receive antenna and different numbers of transmit antennas. (a) $N_t = 1$, (b) $N_t = 2$, (c) $N_t = 8$.

- Square-law receiver makes a decision between symbols 0 and 1 as

$$\text{if} \quad |Q_{0k}|^2 > |Q_{1k}|^2 \quad \text{say} \;\; 0 \quad \text{otherwise} \quad \text{say} \;\; 1 \qquad (6.32)$$

- With square-lave combining, we form the decision variable

$$Q_0 = \sum_{k=1}^{N_r} |Q_{0k}|^2 \quad \text{and} \quad Q_I = \sum_{k=1}^{N_r} |Q_{1k}|^2 \qquad (6.33,34)$$

- If $s_0(t)$ was transmitted, the variances are given by

$$Var(Q_{0k}) = \frac{E_b}{N_0} Var\left(\alpha_k e^{j\theta_k}\right) + \frac{Var(w_{0k})}{N_0} \quad \text{and} \quad Var(Q_{Ik}) = \frac{1}{N_0} Var(w_{1k})$$

$$= \frac{E_b}{N_0}\left(\mathbf{E}[\alpha_k^2]\right) + \frac{Var[w_{0k}]}{N_0} \qquad = 1$$

$$= \gamma_{av} + 1 \qquad (6.35,36)$$

- Probability density function for correct symbol is

$$f_{Q_0}(q) = \frac{1}{(N_r-1)!} \frac{q^{N_r-1}}{(\gamma_{av}+1)} \exp\left(-\frac{q}{\gamma_{av}+1}\right) \qquad (6.37)$$

- For incorrect symbol, it is

$$f_{Q_1}(q) = \frac{1}{(N_r-1)!} q^{N_r-1} \exp(-q) \qquad (6.38)$$

- As Q_1 and Q_0 are independent random variables, the probability of error is

$$prob(Q_0 < Q_1) = \int_0^\infty f_{Q_0}(q_0)\left(\int_{q_0}^\infty f_{Q_1}(q_1)dq_1\right)dq_0 \qquad (6.39)$$

6.3 Multiple-Input, Multiple-Output Antenna Systems

- MIMO wireless communications include space diversity
- Three important points:
  - Fading phenomenon is an environmental source of possible enrichment.
  - Space diversity provides the basis for a significant increase in channel capacity or spectral efficiency.
  - Increasing channel capacity with MIMO is achieved by increasing computational complexity with maintaining primary communication resources fixed.

6.3 Multiple-Input, Multiple-Output Antenna Systems

6.3.1 Co-antenna Interference

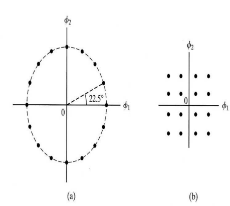

FIGURE 6.17 (a) Signal constellation of 16-PSK. (b) Signal constellation of 16-QAM.

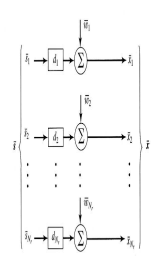

FIGURE 6.15 Set of N_r virtual decoupled channels resulting from the singular-value decomposition of the channel matrix **H**, assuming that $N_t \le N_r$.

6.3 Multiple-Input, Multiple-Output Antenna Systems

6.3.2 Basic Baseband Channel Model

- Spatial parameter, defines as *a new degree of freedom.*

$$n = \min\{N_t, N_r\} \qquad (6.40)$$

- N_t-by-1 vector, denotes the complex signal vector transmitted by the N_t antennas at discrete time n.

$$\mathbf{s}(n) = [\tilde{s}_1(n), \tilde{s}_2(n), \ldots, \tilde{s}_t(n)]^T \qquad (6.41)$$

- Total transmit power is fixed at the value

$$P = N_t \sigma_s^2 \qquad (6.42)$$

- For flat-fading, we can use $\tilde{h}_{ik}(n)$ denote sampled complex gain of channel, thus express the N_r-by-N_t complex channel matrix as

$$\mathbf{H}(n) = \underbrace{\begin{bmatrix} \tilde{h}_{11}(n) & \tilde{h}_{21}(n) & \cdots & \tilde{h}_{N_t1}(n) \\ \tilde{h}_{21}(n) & \tilde{h}_{22}(n) & \cdots & \tilde{h}_{N_t2}(n) \\ \vdots & \vdots & & \vdots \\ \tilde{h}_{N_r1}(n) & \tilde{h}_{N_r2}(n) & \cdots & \tilde{h}_{N_rN_t}(n) \end{bmatrix}}_{N_t \text{ transmit antennas}} \Bigg\} N_r \text{ receive antennas} \qquad (6.43)$$

- The system of equations, defines the complex signal received at ith antenna due to transmitted symbol $\tilde{s}_k(n)$ radiated by kth antenna

$$\tilde{x}_i(n) = \sum_{k=1}^{N_t} \tilde{h}_{ik}(n)\tilde{s}_k(n) + \tilde{w}_i(n) \quad \begin{matrix} i = 1,2,\ldots,N_r \\ k = 1,2,\ldots,N_t \end{matrix} \qquad (6.44)$$

- Complex received signal vector

$$\mathbf{x}(n) = [\tilde{x}_1(n), \tilde{x}_1(n), \ldots, \tilde{x}_1(n)] \qquad (6.45)$$

- Complex channel noise vector

$$\mathbf{w}(n) = [\tilde{w}_1(n), \tilde{w}_2(n), \ldots, \tilde{w}_{N_r}(n)]^T \qquad (6.46)$$

- Therefore, the compact matrix form of the system equation which is the *basic complex channel model for MIMO wireless communications*

$$\mathbf{x}(n) = \mathbf{H}(n)\mathbf{s}(n) + w(n) \qquad (6.47)$$

- To simply the exposition, we get $\mathbf{x} = \mathbf{H}\mathbf{s} + w$ (6.48)

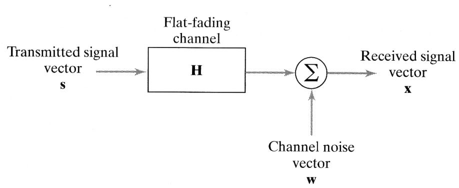

FIGURE 6.12 Depiction of the basic channel model of Eq. (6.48).

- For mathematical tractability, we assume *Gaussian model* made up of three elements
  1. Transmitter
  2. Channel
  3. Receiver
- 1. Correlation matrix of transmitted signal vector s is

$$\mathbf{R}_s = \mathbf{E}[\mathbf{s}\mathbf{s}^\dagger]$$
$$= \sigma_s^2 \mathbf{I}_{Nt}$$

where $\mathbf{I}_{N_t}$ is the N_t-by-N_t identity matrix

- 2. The N_t x N_r elements of channel matrix $\mathbf{H}$ is

$$h_{ik}: \; N\left(0,1/\sqrt{2}\right)+jN\left(0,1/\sqrt{2}\right) \quad \begin{matrix} i=1,2,\ldots,N_r \\ k=1,2,\ldots,N_r \end{matrix} \qquad (6.50)$$

- On this basis, the amplitude component h_{ik} is rayleigh distributes, this is the reason why MIMO channel is a rich Rayleigh scattering environment.
- Mean of squared amplitude component is a chi-square random variable

$$\mathbf{E}\left[|h_{ik}|^2\right]=1 \quad \text{for all } i \text{ and } k \qquad (6.51)$$

- 3. Correlation matrix of noise vector $\mathbf{w}$ is

$$\mathbf{R}_w = \mathbf{E}[\mathbf{w}\mathbf{w}^{\dagger}]$$
$$= \sigma^2_w \mathbf{I}_{Nr}$$

where $\mathbf{I}_{N_r}$ is the N_r-by-N_r identity matrix

- The average signal-to-noise ratio (SNR) at each receiver is

$$\rho = \frac{P}{\sigma_w^2} = \frac{N_t \sigma_s^2}{\sigma_w^2} \qquad (6.53)$$

6.4 MIMO Capacity for Channel Known at the Receiver

6.4 MIMO Capacity for Channel Known at the Receiver

6.4.1 Ergodic Capacity

- Information capacity of AWGN channel with fixed transmit power is defined as

$$C = B\log_2\left(1+\frac{P}{\sigma_w^2}\right) bit/s \qquad (6.54)$$

- With the sampling theorem, we can rewrite to

$$C = \frac{1}{2}\log_2\left(1+\frac{P}{\sigma_w^2}\right) bit/s/Hz \qquad (6.55)$$

- Capacity of complex, flat-fading channel is

$$C = \mathbf{E}\left[\log_2\left(1+\frac{|h|^2 P}{\sigma_w^2}\right)\right] bits/s/Hz \qquad (6.56)$$

- Assume the channel is stationary and ergodic, C is commonly referred to ergodic capacity of *single-input, single-output* (SISO) flat fading channel.

- Ergodic capacity of MIMO channel

$$C = \mathbf{E}\left[\log_2\left\{\frac{\det\left(\mathbf{R_W}+\mathbf{H}\mathbf{R_S}\mathbf{H}^{+}\right)}{\det\left(\mathbf{R_W}\right)}\right\}\right] bit/s/Hz \quad \text{which is subject to constraint} \quad \max_{\mathbf{R}_s} tr(\mathbf{R_S}) \le P \qquad (6.57)$$

- Substituting Eqs. (6.49) and (6.52) into Eq.(6.57)

$$C = \mathbf{E}\left[\log_2\left\{\det\left(\mathbf{I}_{N_r}+\frac{\sigma_s^2}{\sigma_w^2}\mathbf{H}\mathbf{H}^{+}\right)\right\}\right] bit/s/Hz \qquad (6.58)$$

- Invoking definition of average signal-to-noise ratio, we get

$$C = \mathbf{E}\left[\log_2\left\{\det\left(\mathbf{I}_{N_r}+\frac{\rho}{N_t}\mathbf{H}\mathbf{H}^{+}\right)\right\}\right] bit/s/Hz \qquad (6.59)$$

- This is the *log-det capacity formula* for a Gaussian MIMO channel.

- Asymptotic formula

$$\lim_{N\to\infty} \frac{C}{N} \geq constant \qquad (6.61)$$

- *The ergodic capacity of a MIMO flat-fading wireless link with an equal number N of transmit and receive antennas grows roughly proportionately with N*

6.4 MIMO Capacity for Channel Known at the Receiver

6.4.2 Two other special case of log-det formula: Capacities of Receiver and Transmit Diversity Links

- 1. *Diversity-on-receive channel*
  - Applying log-det capacity formula to this case, Eq. (6.60) reduces to

$$C = E\left[\log_2\left\{\left(1+\rho\sum_{i=1}^{N_r}|h_i|^2\right)\right\}\right] bit/s/Hz \qquad (6.62)$$

  - Eq. (6.62) expresses the ergodic capacity due to linear combination of receive-antenna outputs.
  - It designed to maximize the information contained in N_r received signals about the transmitted signal.

- 2. Diversity-on-transmit channel
  - The log-det capacity formula reduced to

$$C = \mathbf{E}\left[\log_2\left(1+\frac{\rho}{N_t}\sum_{k=1}^{N_t}|h_k|^2\right)\right] bits/s/Hz \qquad (6.63)$$

6.4 MIMO Capacity for Channel Known at the Receiver

6.4.3 Outage Capacity

- *Outage probability* of MIMO link is defined as *the probability for which the link is in a state of outage for data transmitted across the link at a certain rate, R.*
- To proceed on this probabilistic basis, we invoke a *quasi-static model*:
  - The burst is long enough to accommodate the transmission of large number of symbols.
  - Yet the burst is short enough that can be treated as quasi static.
  - Channel matrix is permitted to change.
  - The different realizations of transmitted signal vector are drawn from a white gaussian codebook.

- In light of the log-det capacity formula, we may view the random variable

$$C_k = \mathbf{E}\left[\log_2\left\{\det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{H}_k\mathbf{H}_k^+\right)\right\}\right] bit/s/Hz \quad (6.64)$$

as the expression for a sample of the wireless link.

- Outage probability at rate R is

$$P_{outage}(R) = prob\{C_k < R \text{ for some burst } k\}$$

or equivalently,

$$P_{outage}(R) = prob\left\{\log_2\left\{\det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{H}_k\mathbf{H}_k^+\right)\right\} < R \text{ for some burst } k\right\} \quad (6.65)$$

- *Outage capacity* of the MIMO link is the *maximum bit rate that can be maintained across the link for all bursts of data transmissions for a prescribed outage probability.*

6.4 MIMO Capacity for Channel Known at the Receiver

6.4.4 Channel Known at the Transmitter

- Log-det capacity formula is based on the premise that the transmitter has no knowledge of channel state.
- Knowledge of channel state can be gathered by
  - first estimating the channel matrix at receiver
  - then sending to the transmitter via feedback channel.
  - Capacity is optimized.

6.5 Singular-Value Decompostion of the Channel Matrix

- Diagonalize $\mathbf{HH}^+$ by invoking *eigendecomposition* of Hermitian Matrix

$$\mathbf{U}^+\mathbf{HH}^+\mathbf{U} = \Lambda \quad (6.67)$$

- The matrix Λ is a *diagonal matrix* whose N_r elements are eigenvalues of matrix product $\mathbf{HH}^+$
- The matrix $\mathbf{U}$ is a *unitary matrix* whose N_r columns are the eigenvectors associated with eigenvalues of $\mathbf{HH}^+$.
- Inverse of unitary matrix is equal to Hermitian transpose of matrix, as shown by

$$\mathbf{U}^{-1} = \mathbf{U}^+ \quad \text{or, equivalently,} \quad \mathbf{UU}^+ = \mathbf{U}^+\mathbf{U} = \mathbf{I}_{N_r} \quad (6.68,69)$$

- Let N_t-by-N_t matrix $\mathbf{V}$ be another unitary matrix, that is

$$\mathbf{VV}^+ = \mathbf{V}^+\mathbf{V} = \mathbf{I}_{N_t} \quad (6.70)$$

- Inject the matrix product $\mathbf{VV}^+$ into the center of left-hand side of Eq. (6.67)

$$\mathbf{U}^+\mathbf{H}(\mathbf{VV}^+)\mathbf{H}^+\mathbf{U} = \Lambda \quad (6.71)$$

- Let the N_r-by-N_t matrix $\mathbf{D}$ denate a new diagonal matrix related to N_r-by-N_r diagonal matrix Λ with $N_r \le N_t$ by

$$\Lambda = [D \quad 0][D \quad 0]^+ \quad (6.72)$$

- Examining Eqs.(6.71) and (6.72) and comparing terms, we deduce the new decomposition, which is *singular-value decomposition (SDV)*

$$\mathbf{U}^+\mathbf{HV} = [\mathbf{D} \quad 0] \quad (6.73)$$

- According to SVD theorem, we have
  - Elements of diagonal matrix

$$\mathbf{D} = diag(d_1, d_2, \ldots, d_{N_t}) \tag{6.74}$$

are the *singular values* of channel matrix $\mathbf{H}$.

  - Columns of unitary matrix

$$\mathbf{U} = [\mathbf{u}_1, \mathbf{u}_1, \ldots, \mathbf{u}_{N_r}] \tag{6.75}$$

are the *left singular vectors* of matrix $\mathbf{H}$

  - Columns of second unitary matrix

$$\mathbf{V} = [\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_{N_t}] \tag{6.76}$$

are the *right singular vectors* of matrix $\mathbf{H}$

- Using the definitions of Eqs. (6.74) through (6.76), the decomposed channel model can changed to scalar form

$$\tilde{x}_i = d_i \tilde{s}_i + \tilde{w}_i \qquad i = 1, 2, \ldots, N_r$$

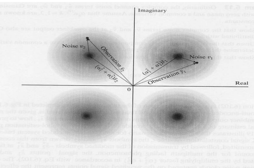

FIGURE 6.19 Signal-space diagram for Alamouti code, using the QPSK signal constellation. The signal point $\tilde{s}_1$ and $\tilde{s}_2$ are weighted by $(\alpha_1^2 + \alpha_2^2)$. The corresponding linear combiner outputs $\tilde{y}_1$ and $\tilde{y}_2$ are noisy versions of these weighted signals.

6.5 Singular-Value Decompostion of the Channel Matrix

6.5.1 Eigendecomposition of the Log-det Capacity Formula

- Substituting Eq. (6.59)into Eq. (6.67) leads to spectral decomposition of $\mathbf{HH}^+$ in terms of N_r *eigenmodes*, we may write

$$\begin{aligned}\mathbf{HH}^+ &= \mathbf{U\Lambda U}^+ \\ &= \sum_{i=1}^{N_r} \lambda_i \mathbf{u}_i \mathbf{u}_i^+\end{aligned} \tag{6.82}$$

- Substituting the first line of this decomposition into determinant part of Eq. (6.59) yields

$$\det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{HH}^+\right) = \det\left(I_{N_r} + \frac{\rho}{N_t}\mathbf{U\Lambda U}^+\right) \tag{6.83}$$

- Invoking the *determinant identity*.

$$\det(\mathbf{I} + \mathbf{AB}) = \det(\mathbf{I} + \mathbf{BA}) \tag{6.84}$$

- And then using the defining Eq. (6.69), we can rewrite Eq. (6.83)

$$\begin{aligned}\det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\mathbf{HH}^+\right) &= \det\left(I_{N_r} + \frac{\rho}{N_t}\mathbf{U}^+\mathbf{U\Lambda}\right) \\ &= \det\left(\mathbf{I}_{N_r} + \frac{\rho}{N_t}\Lambda\right) \\ &= \prod_{i=1}^{N_r}\left(1 + \frac{\rho}{N_t}\lambda_i\right)\end{aligned} \tag{6.85}$$

- Finally, subsitute Eq. (6.85) into Eq. (6.59)

$$C = E\left[\sum_{i=1}^{N_r}\log_2\left(1 + \frac{\rho}{N_t}\lambda_i\right)\right] bits/s/Hz \tag{6.86}$$

- *Ergodic capacity of a MIMO wireless communication system is the sum of capacity of N_r virtual single-input, single-output channels defined by the spatial eigenmodes of the matrix product $\mathbf{HH}^+$.*
- Using the log-det capacity formula of Eq.(6.60), for $N_t \le N_r$, we can show that

$$C = E\left[\sum_{i=1}^{N_t}\log_2\left(1 + \frac{\rho}{N_r}\lambda_i\right)\right] bits/s/Hz \tag{6.87}$$

6.6 Space-Time Codes for MIMO Wireless Communications

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- Space-Time codes
  - Joint channel encoding of multiple transmit antennas
  - Employ redundancy for purpose of providing protection against channel fading, noise, interference.
  - Maximize outage capacity
  - Classified into
    - Space-time trellis codes
    - Space-time block codes

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- Space-time trellis code
  - Permits serial transmission of symbols.
  - Designed for two to four transmit antennas
  - Well for slow-fading environment but not indoor data transmission.
  - For decoding, multidimensional version of Viterbi algorithm is required
  - *Decoding complexity of space-time trellis codes increases exponentially* as the function of spectral efficiency.

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- Space-time block code
  - Transmission of signal takes place in *blocks*. Code is defined by transmission matrix with 3 parameters
    - Number of transmitted symbols l
    - Number of transmit antennas, N_t, defines the size of transmission matrix.
    - Number of time slots in data block m
  - Ratio l/m defines of *rate* of code.
  - For efficient transmission, transmitted symbols are expressed in *complex* form.

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- Linear processing
  - Estimate transmitted symbols at receiver and simplify the receiver design.
- Different receiver design procedures:
  - *Complex orthogonal design*
  - *Generalized complex orthogonal design*
- *Complex orthogonality of transmission matrix in temporal sense is a sufficient condition for linear processing at receiver.*
- *Alamouti code*: code with code rate of unity.

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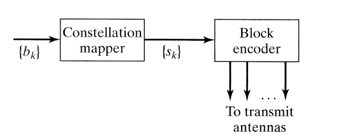

FIGURE 6.16 Block diagram of orthogonal space–time block encoder.

6.6 Space-Time Codes for MIMO Wireless Communications

6.6.1 Preliminaries

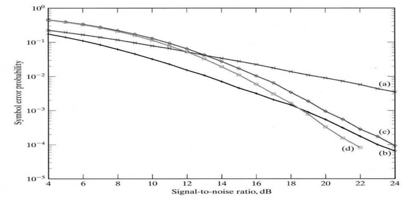

FIGURE 6.22 Comparison of bit error rate performance of different space–time block codes:
(a) No space-time coding.
(b) Alamouti code (QPSK).
(c) Code $\mathbf{G}_3$ (16-QAM).
(d) Code $\mathbf{G}_4$ (16-QAM).

- Two functional units:
  - Mapper
    - Take incoming binary data stream and generate a new *sequence of blocks*.
  - Block encoder
    - Converts each block of complex symbols produced by mapper into an l-by- N_t transmission matrix $\mathbf{S}$, where l and N_t are *temporal* dimension and *spatial* dimension

6.6 Space-Time Codes for MIMO Wireless Communications

6.6.2 Alamouti Code

- Two-by-one orthogonal space-time block code.

$$\mathbf{S} = \begin{bmatrix} \tilde{s}_1^* & -\tilde{s}_2^* \\ -\tilde{s}_2^* & \tilde{s}_1^* \end{bmatrix} \rightarrow Space \quad \text{(6.88)}$$

$\downarrow$ *Time*

- Hermitian transpose of $\mathbf{S}$

$$\mathbf{S}^+ = \begin{bmatrix} \tilde{s}_1^* & -\tilde{s}_2^* \\ \tilde{s}_2^* & \tilde{s}_1^* \end{bmatrix} \rightarrow Time \quad \text{(6.89)}$$

$\downarrow$ *Space*

- Multiply code matrix $\mathbf{S}$ by its Hermitian transpose, obtaining

$$\begin{aligned} \mathbf{SS}^+ &= \begin{bmatrix} \tilde{s}_1^* & \tilde{s}_2^* \\ -\tilde{s}_2^* & \tilde{s}_1^* \end{bmatrix} \begin{bmatrix} \tilde{s}_1^* & -\tilde{s}_2^* \\ \tilde{s}_2^* & \tilde{s}_1^* \end{bmatrix} \\ &= \begin{bmatrix} |\tilde{s}_1|^2 + |\tilde{s}_1|^2 & -\tilde{s}_1\,\tilde{s}_2 + \tilde{s}_2\,\tilde{s}_1 \\ -\tilde{s}_2^*\tilde{s}_1^* + \tilde{s}_1^*\tilde{s}_2^* & |\tilde{s}_1|^2 + |\tilde{s}_1|^2 \end{bmatrix} \\ &= \left(|\tilde{s}_1|^2 + |\tilde{s}_2|^2\right) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \end{aligned} \quad \text{(6.90)}$$

- This result also holds for alternative matrix product $\mathbf{S}^+\mathbf{S}$, which is proof of orthogonality in the temporal sense

- The transmission matrix of Alamouti code satisfies the unique condition

$$\mathbf{S}\mathbf{S}^{+} = \mathbf{S}^{+}\mathbf{S} = \left(|\tilde{s}_1|^2 + |\tilde{s}_2|^2\right)\mathbf{I} \qquad (6.91)$$

- And Note that

$$\mathbf{S}^{-1} = \frac{1}{|\tilde{s}_1|^2 + |\tilde{s}_2|^2}\mathbf{S}^{+} \qquad (6.92)$$

- Four Properties of Alamouti code
- Property 1. Unitarity (Complex Orthogonality)
  - *Alamouti code is an orthogonal space-time block code*
  - *Product of transmission matrix with its Hermitian transpose is equal to the two-by-two identity matrix scaled by the sum of squared amplitudes of transmitted symbols*.
- Property 2: Full-Rate Complex Code
  - *Only complex space-time block code with a code rate unity*.

- Property 3. Linearity
  - *Alamouti code is linear in transmitted symbols*.
- Property 4. Optimality of Capacity
  - *For two transmit antennas and a single receive antenna, the Alamouti code is the only optimal space-time block code that satisfies the log-det capacity formula of Eq. (6.63)*.

With $\tilde{s}_1$ and $\tilde{s}_2$ transmitted simultaneously at time t, the complex received signal at time $t' > t$, is

$$\tilde{x}_1 = \alpha_1 e^{j\theta_1}\tilde{s}_1 + \alpha_2 e^{j\theta_2}\tilde{s}_2 + w_1 \qquad (6.95)$$

The complex signal received at time $t' + T$ is

$$\tilde{x}_2 = -\alpha_1 e^{j\theta_1}\tilde{s}_2^* + \alpha_2 e^{j\theta_2}\tilde{s}_1^* + w_2 \qquad (6.96)$$

Reformulate the variable $\tilde{x}_1$ and complex conjugate of the second variable $\tilde{x}_2$ in matrix form

$$\begin{bmatrix}\tilde{x}_1\\ \tilde{x}_2^*\end{bmatrix} = \begin{bmatrix}\alpha_1 e^{j\theta_1} & \alpha_1 e^{j\theta_2}\\ \alpha_2 e^{-j\theta_2} & -\alpha_1 e^{-j\theta_1}\end{bmatrix}\begin{bmatrix}\tilde{s}_1\\ \tilde{s}_2\end{bmatrix} + \begin{bmatrix}\tilde{w}_1\\ \tilde{w}_2^*\end{bmatrix} \qquad (6.97)$$

- Multiply Eq.(6.97) by hermitian transpose of two-by-two channel matrix

$$\begin{bmatrix}\alpha_1 e^{-j\theta_1} & \alpha_2 e^{-j\theta_2}\\ \alpha_2 e^{-j\theta_2} & -\alpha_1 e^{-j\theta_1}\end{bmatrix}\begin{bmatrix}\tilde{x}_1\\ \tilde{x}_2^*\end{bmatrix} = \left(\alpha_1^2 + \alpha_2^2\right)\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\tilde{s}_1\\ \tilde{s}_2\end{bmatrix} + \begin{bmatrix}\alpha_1 e^{-j\theta_1} & \alpha_2 e^{-j\theta_2}\\ \alpha_2 e^{-j\theta_2} & -\alpha_1 e^{-j\theta_1}\end{bmatrix}\begin{bmatrix}\tilde{w}_1\\ \tilde{w}_2\end{bmatrix}$$

$$= \left(\alpha_1^2 + \alpha_2^2\right)\begin{bmatrix}\tilde{s}_1\\ \tilde{s}_2\end{bmatrix} + \begin{bmatrix}\alpha_1 e^{-j\theta_1}\tilde{w}_1 & \alpha_2 e^{-j\theta_2}\tilde{w}_2^*\\ \alpha_2 e^{-j\theta_2}\tilde{w}_1 & -\alpha_1 e^{-j\theta_1}\tilde{w}_2^*\end{bmatrix} \qquad (6.98)$$

- And let

$$\begin{bmatrix}\tilde{y}_1\\ \tilde{y}_2\end{bmatrix} = \begin{bmatrix}\alpha_1 e^{-j\theta_1} & \alpha_2 e^{-j\theta_2}\\ \alpha_2 e^{-j\theta_2} & -\alpha_1 e^{-j\theta_1}\end{bmatrix}\begin{bmatrix}\tilde{x}_1\\ \tilde{x}_2^*\end{bmatrix} \quad \text{and} \quad \begin{bmatrix}\tilde{v}_1\\ \tilde{v}_2\end{bmatrix} = \begin{bmatrix}\alpha_1 e^{-j\theta_1}\tilde{w}_1 & +\alpha_2 e^{-j\theta_2}\tilde{w}_2^*\\ \alpha_2 e^{-j\theta_2}\tilde{w}_1 & -\alpha_1 e^{-j\theta_1}\tilde{w}_2^*\end{bmatrix}$$

$$= \begin{bmatrix}\alpha_1 e^{-j\theta_1}\tilde{x}_1 & +\alpha_2 e^{-j\theta_2}\tilde{x}_2^*\\ \alpha_2 e^{-j\theta_2}\tilde{x}_1 & -\alpha_1 e^{-j\theta_1}\tilde{x}_2^*\end{bmatrix} \qquad (6.99)$$

- Recast Eq.(6.98) in matrix form of input-output relations describing the overall behavior of Alamouti code

$$\begin{bmatrix}\tilde{y}_1\\ \tilde{y}_2\end{bmatrix} = \left(\alpha_1^2 + \alpha_2^2\right)\begin{bmatrix}\tilde{s}_1\\ \tilde{s}_2\end{bmatrix} + \begin{bmatrix}\tilde{v}_1\\ \tilde{v}_2\end{bmatrix} \quad \text{in expanded form,} \quad \tilde{y}_k = \left(\alpha_1^2 + \alpha_2^2\right)\tilde{s}_k + \tilde{v}_k \quad k = 1,2 \qquad (6.102)$$

- Due to complex orthogonality of Alamouti code, the unwanted symbols are cancelled out in the equations. This cancellations are responsible for the simplification of receiver.
- The detrimental effect of fading arises when diversity paths suffer from it. This means a wireless communication system based on the Alamouti code enjoys a two-level diversirty gain.

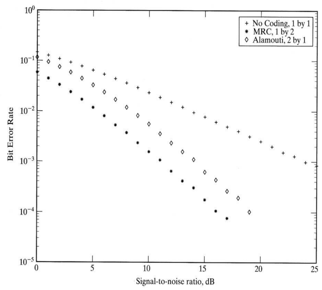

FIGURE 6.20 Comparison of the bit error rate performance of coherent BPSK over flat-fading Rayleigh channel for three configurations:
(a) No diversity.
(b) Maximal-ratio combiner ($N_t = 1, N_r = 2$).
(c) Alamouti code ($N_t = 2, N_r = 1$).

- Maximum-likelihood decoding rule:
  - Given that receiver has knowledge of
    - Channel fading coefficients α_1 and α_2.
    - Set of all possible transmitted symbols in the mapper's constellation denoted by S, the maximum-likelihood estimates of transmitted symbols defined by

$$\hat{s}_1 = \arg\min_{\varphi \in S} \left\{ d^2\left(\tilde{y}_1, \left(\alpha_1^2 + \alpha_2^2\right)\varphi\right)\right\} \text{ and } \hat{s}_2 = \arg\min_{\varphi \in S} \left\{ d^2\left(\tilde{y}_2, \left(\alpha_1^2 + \alpha_2^2\right)\varphi\right)\right\} \quad (6.1)$$

where the φ denote the different hypotheses for the linear combiner output.

6.6 Space-Time Codes for MIMO Wireless Communications

6.6.3 Performance Comparison of Diversity-on-Receive and Diversity-on-Transmit Schemes

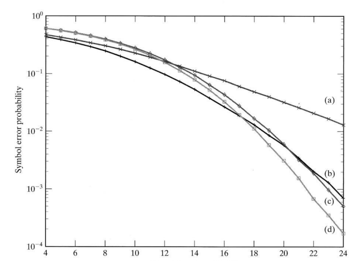

FIGURE 6.21 Comparison of the bit error rate performance of different space–time block codes:
(a) No space-time coding.
(b) Alamouti code (8-PSK).
(c) Sporadic code $\mathbf{H}_3$ (16-QAM).
(d) Sporadic code $\mathbf{H}_4$ (16-QAM).

6.6 Space-Time Codes for MIMO Wireless Communications

6.6.4 Generalized Complex Orthogonal Space-Time Block Codes

- Generalized complex orthogonal designs of space–time block codes distinguish themselves from the Alamouti code in three respects:
  - Nonsquare transmission matrix
  - Fractional code rate
  - Orthogonality of transmission matrix only in temporal sense.

- Let $\mathbf{G}$ be a an m-by- N_t matrix, N_t is the number of transmit antennas and m is the number of time slots, the entries of the matrix

$$0, \pm s_1, \pm s_1^*, \pm s_2, \pm s_2^*, \ldots, \pm s_l, \pm s_l^*$$

- $\mathbf{G}$ is said to be generalized complex orthogonalized design of size N_t and code rate is l/m if

$$\mathbf{G}^+\mathbf{G} = \left(\sum_{j=1}^{l} |s_j|^2\right)\mathbf{I} \qquad (6.106)$$

- Construction of space-time block codes using generalized complex orthogonal design is exemplified by rate-1/2 codes.
- Case 1: For three transmit antennas (l=4, m=8)

$$\mathbf{G}_3 = \begin{bmatrix} \tilde{s}_1 & \tilde{s}_2 & \tilde{s}_3 \\ -\tilde{s}_2 & \tilde{s}_1 & -\tilde{s}_4 \\ -\tilde{s}_3 & \tilde{s}_4 & \tilde{s}_1 \\ -\tilde{s}_4 & -\tilde{s}_3 & \tilde{s}_2 \\ \tilde{s}_1^* & \tilde{s}_2^* & \tilde{s}_3^* \\ -\tilde{s}_2^* & \tilde{s}_1^* & -\tilde{s}_4^* \\ -\tilde{s}_3^* & \tilde{s}_4^* & \tilde{s}_1^* \\ -\tilde{s}_4^* & -\tilde{s}_3^* & \tilde{s}_2^* \end{bmatrix} \rightarrow Space \qquad (6.107)$$

$\downarrow$ *Time*

- Case 2: For four transmit antennas (l=4,m=8)

$$\mathbf{G}_4 = \begin{bmatrix} \tilde{s}_1 & \tilde{s}_2 & \tilde{s}_3 & \tilde{s}_4 \\ -\tilde{s}_2 & \tilde{s}_1 & -\tilde{s}_4 & \tilde{s}_3 \\ -\tilde{s}_3 & \tilde{s}_4 & \tilde{s}_1 & -\tilde{s}_2 \\ -\tilde{s}_4 & -\tilde{s}_3 & \tilde{s}_2 & \tilde{s}_1 \\ \tilde{s}_1^* & \tilde{s}_2^* & \tilde{s}_3^* & \tilde{s}_4^* \\ -\tilde{s}_2^* & \tilde{s}_1^* & -\tilde{s}_4^* & \tilde{s}_3^* \\ -\tilde{s}_3^* & \tilde{s}_4^* & \tilde{s}_1^* & -\tilde{s}_2^* \\ -\tilde{s}_4^* & -\tilde{s}_3^* & \tilde{s}_2^* & \tilde{s}_1^* \end{bmatrix} \rightarrow Space \qquad (6.108)$$

$\downarrow$ *Time*

- Compare with Alamouti code, space-time codes G_3 and G_4 are at a disadvantage in two respects:
  1. The bandwidth efficiency is reduced by a factor of two.
  2. The number of time slots across which channel is required to have a constant fading envelope is increased by a factor of four.

- To improve the bandwidth efficiency, we may use rate-3/4 generalized complex linear processing orthogonal designs referred to as *sporadic codes*:
- Case 1: For three transmit antennas (l=3, m=4)

$$\mathbf{H}_3 = \begin{bmatrix} \tilde{s}_1 & \tilde{s}_2 & \tilde{s}_3/\sqrt{2} \\ -\tilde{s}_2^* & \tilde{s}_1^* & \tilde{s}_3/\sqrt{2} \\ \tilde{s}_3^*/\sqrt{2} & \tilde{s}_3^*/\sqrt{2} & \left(-\tilde{s}_1 - \tilde{s}_1^* + \tilde{s}_2 - \tilde{s}_2^*\right)/\sqrt{2} \\ \tilde{s}_3^*/\sqrt{2} & -\tilde{s}_3^*/\sqrt{2} & \left(\tilde{s}_2 + \tilde{s}_2^* + \tilde{s}_1 - \tilde{s}_1^*\right)/\sqrt{2} \end{bmatrix} \rightarrow Space \qquad (6.109)$$

$\downarrow$ *Time*

- Case 2: For four transmit antennas (l=3,m=4)

$$\mathbf{H}_3 = \begin{bmatrix} \tilde{s}_1 & \tilde{s}_2 & \tilde{s}_3/\sqrt{2} & \tilde{s}_3/\sqrt{2} \\ -\tilde{s}_2^* & \tilde{s}_1^* & \tilde{s}_3/\sqrt{2} & -\tilde{s}_3/\sqrt{2} \\ \tilde{s}_3^*/\sqrt{2} & \tilde{s}_3^*/\sqrt{2} & \left(-\tilde{s}_1 - \tilde{s}_1^* + \tilde{s}_2 - \tilde{s}_2^*\right)/\sqrt{2} & \left(-\tilde{s}_2 - \tilde{s}_2^* + \tilde{s}_1 - \tilde{s}_1^*\right)/\sqrt{2} \\ \tilde{s}_3^*/\sqrt{2} & -\tilde{s}_3^*/\sqrt{2} & \left(\tilde{s}_2 + \tilde{s}_2^* + \tilde{s}_1 - \tilde{s}_1^*\right)/\sqrt{2} & -\left(\tilde{s}_2 + \tilde{s}_2^* + \tilde{s}_1 - \tilde{s}_1^*\right)/\sqrt{2} \end{bmatrix} \rightarrow Space$$

$\downarrow$ *Time* (6.110)

6.6 Space-Time Codes for MIMO Wireless Communications

6.6.5 Performance Comparisons of Different Space-Time Block Codes Using a Single Receiver

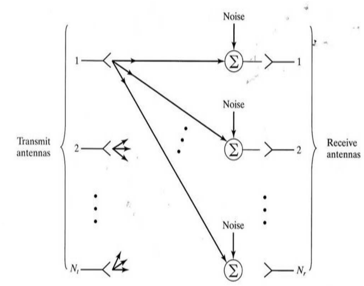

FIGURE 6.10 Block diagram of MIMO wireless link with N_t transmit antennas and N_r receive antennas.

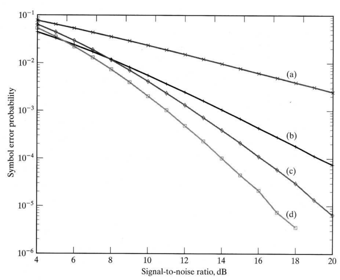

FIGURE 6.23 Comparison of bit error rate performance of different space–time block codes:
(a) No space-time coding.
(b) Alamouti code (binary PSK).
(c) Generalized code $\mathbf{G}_3$ (QPSK).
(d) Generalized code $\mathbf{G}_4$ (QPSK).

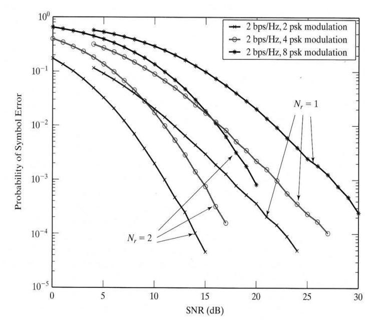

FIGURE 6.27 Noise–performance comparison of differential space–time block coding for specific M-ary PSK modulation strategies in a Rayleigh-fading environment.

6.7 Differential Space-Time Block Codes

6.7 Differential Space-Time Block Codes

6.7.1 Differential Space-Time Block Coding

- Given the new pair of complex signals to be transmitted at time $t+2$, the row vector can be expressed as

$$[\tilde{s}_{3,t+2}, \tilde{s}_{4,t+2}] = a_{1,t+2}[\tilde{s}_{1,t}, \tilde{s}_{2,t}] + a_{2,t+2}[-\tilde{s}_2^*, \tilde{s}_{1,t}^*] \tag{6.111}$$

- $a_{1,t+2}$ and $a_{2,t+2}$ are coefficients of linear combination and defined as inner products of the row vector, therefore

$$\begin{aligned}[a_{1,t+2}, a_{2,t+2}] &= [\tilde{s}_{3,t+2}, \tilde{s}_{4,t+2}]\left[[\tilde{s}_{1,t}, \tilde{s}_{2,t}]^+, [-\tilde{s}_{2,t+1}^*, \tilde{s}_{1,t+1}^*]^+\right] \\ &= [\tilde{s}_{3,t+2}, \tilde{s}_{4,t+2}]\begin{bmatrix} \tilde{s}_{1,t}^* & -\tilde{s}_{2,t+1} \\ \tilde{s}_{2,t}^* & \tilde{s}_{1,t+1} \end{bmatrix} \\ &= [\tilde{s}_{3,t+2}, \tilde{s}_{4,t+2}]\mathbf{S}_{t,t+1}^+ \end{aligned} \tag{6.112}$$

- Similarly, we can write

$$[-a_{2,t+3}^*, a_{1,t+3}^*] = [-\tilde{s}_{4,t+3}^*, \tilde{s}_{3,t+3}^*]\mathbf{S}_{t,t+1}^+ \tag{6.113}$$

- Coefficients matrix is a *product of two orthogonal Alamouti (quaternionic) matrices.*

$$\begin{aligned} A_{t+2,t+3} &= \begin{bmatrix} a_{1,t+2} & a_{2,t+2} \\ -a_{2,t+3}^* & a_{1,t+3}^* \end{bmatrix} \\ &= \mathbf{S}_{t+2,t+3}\mathbf{S}_{t,t+1}^+ \end{aligned} \tag{6.114}$$

- Since $\mathbf{S}_{t,t+1}$ is a unitary matrix by virtue of orthonormality of its tow constituent row vectors, it follow that

$$\mathbf{S}_{t,t+1}^{-+} = \mathbf{S}_{t,t+1}$$

- Basis for differential space-time block encoding at the transmitter

$$\begin{aligned} \mathbf{S}_{t+2,t+3} &= \mathbf{A}_{t+2,t+3}\mathbf{S}_{t,t+1}^{-+} \\ &= \mathbf{A}_{t+2,t+3}\mathbf{S}_{t,t+1} \end{aligned} \tag{6.115}$$

- In the absence of channel noise, the received signal matrix in response to transmitted signal matrix $\mathbf{S}_{t,t+1}$ is given by

$$\mathbf{X}_{t,t+1} = \mathbf{S}_{t,t+1}\mathbf{H} \tag{6.116}$$

- Similarly, the received signal matrix in response to the next transmitted signal matrix $\mathbf{S}_{t+2,t+3}$ is

$$\mathbf{X}_{t+2,t+3} = \mathbf{S}_{t,t+1}\mathbf{H} \tag{6.117}$$

- New two-by-two matrix

$$\begin{aligned} \mathbf{Y}_{t+2,t+3} &= \mathbf{X}_{t+2,t+3}\mathbf{X}_{t,t+1}^+ \\ &= \mathbf{S}_{t+2,t+3}\mathbf{H}(\mathbf{S}_{t,t+1}\mathbf{H})^+ \\ &= \mathbf{S}_{t+2,t+3}\mathbf{H}\mathbf{H}^+\mathbf{S}_{t,t+1}^+ \end{aligned} \tag{6.118}$$

- From the solution to problem6.12, we note that

$$\mathbf{H} = \begin{bmatrix} \alpha_1 e^{-j\theta_1} & \alpha_2 e^{-j\theta_2} \\ \alpha_2 e^{-j\theta_2} & -\alpha_1 e^{-j\theta_1} \end{bmatrix} \quad \text{and} \quad \mathrm{H}^+\mathrm{H} = \mathrm{H}\mathrm{H}^+ = (\alpha_1^2 + \alpha_2^2)\mathbf{I} \qquad (6.119, 6.120)$$

- Accordingly. Eq.(6.118) reduces to

$$\begin{aligned} \mathbf{Y}_{t+2,t+3} &= (\alpha_1^2 + \alpha_2^2)\mathbf{S}_{t+2,t+3}\mathbf{S}_{t,t+1}^+ \\ &= (\alpha_1^2 + \alpha_2^2)\mathbf{A}_{t+2,t+3} \end{aligned} \tag{6.121}$$

- This is the *basis for differential space-time block decoding at the receiver.*

- With two sets of signal points involved in formulation of Eqs (6.115) and (6.121), we identify two signal spaces
  - $\mathcal{A}$, is spanned by pair of complex coefficients (a_1, a_2) constituting matrix $\mathbf{A}$
  - $\mathcal{S}$, is spanned by the complex signals $\tilde{s}_1, \tilde{s}_2$ constituting matrix $\mathbf{S}$
- Properties of these signals:
- 1. With M-ary PSK as the method of modulation transmitting Alamouticode,
  - points representing signal space $\mathcal{S}$ are uniformly distributed on circle of unit radius
  - Points representing signal space $\mathcal{A}$ constitute a quadrature amplitude modulation (QAM) constellation

- 2. Minimum distance between the points in signal space $\mathcal{S}$ is equal to the minimum distance between the points in signal space $\mathcal{A}$.
- 3. To construct matrix $\mathbf{A}$
  - *Bijective* mapping of 2b bits onto the signal space $\mathcal{A}$.
  - The mapping bijective in the sense that it is *one-to-one* and *onto*.

6.7 Differential Space-Time Block Codes

6.7.2 Transmitter and Receiver Structures

Differential space-time block encoder

1. Mapper
   - Generate the entries that make up matrix $\mathbf{A}_{t+2,t+3}$
2. Differential encoder
   - Transforms the matric $\mathbf{A}_{t+2,t+3}$ into matrix $\mathbf{S}_{t+2,t+3}$
     - Delay unit feeds back the matrix $\mathbf{S}_{t,t+1}$ to input of differential encoder
     - Multiplier multiplies matrix inputs $\mathbf{A}_{t+2,t+3}$ and $\mathbf{S}_{t,t+1}$ to provide the transmitted signal matrix $\mathbf{S}_{t,t+3}$ in accordance with Eq.(6.115)

Differential space-time block decoder

1. Differential decoder
   - Delay unit feeds forward the matrix $\mathbf{X}_{t,t+1}$
   - Multiplier multiplies the matrices $\mathbf{X}_{t+2,t+3}$ and $\mathbf{X}_{t,t+1}$ to produce new matrix $\mathbf{Y}_{t+2,t+3}$
2. Signal estimator
   - Computes the matrix that $\hat{\mathrm{A}}_{t+2,t+3}$ is closest to $\mathbf{Y}_{t+2,t+3}$
3. Inverse mapper
   - Operates on estimate $\hat{\mathrm{A}}_{t+2,t+3}$ to produce corresponding estimates of original pair of data bits transmitted at time $t+2$ and $t+3$.

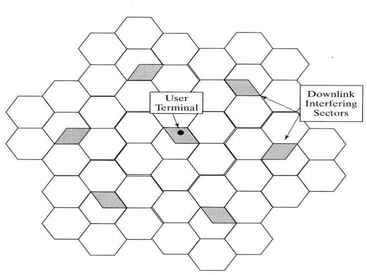

FIGURE 6.28 Cellular system with 120° sector antennas.

6.7 Differential Space-Time Block Codes

6.7.3 Noise Performance

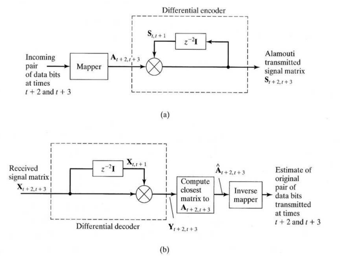

FIGURE 6.26 Block diagrams of (a) differential space–time block encoder. (b) differential space–time block decoder, where z^{-1} = unit-delay operator and $\mathbf{I}$ = two-by-two identity matrix.

6.8 Space-Division Multiple Access and Smart Antennas

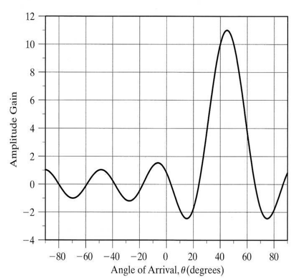

FIGURE 6.31 Gain for maximum signal strength with $\theta_0 = 45°$.

- Advantages of 120° sector antennas at base station
  - Can be applied with FDMA, TDMA or CDMA
  - Allows multiple users to operate on same frequency and/or time slot in same cell
  - More users in same spectrum and improved capacity
  - Can be applied at base station without affecting mobile terminals

- SDMA
  - user terminals can spatially separated by virtue of their angular directions.
- SDMA relies on smart antennas
- Examples of smart antennas
  - Sector antenna
  - Switched-beam antennas
  - Adaptive antenna

- Advantages of smart antenna
  - Greater range
  - Fewer base stations
  - Better building penetration
  - Less sensitivity to power control errors
  - More responsive to traffic hot spots
- SDMA improves system capacity by
  - Minimization of effects of interference.
  - Increasing signal strength

6.8 Space-Division Multiple Access and Smart Antennas

6.8.1 Antenna Arrays

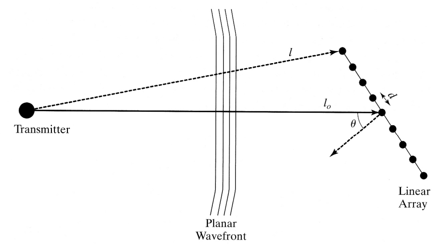

FIGURE 6.29 Plane wave incident on a linear antenna array.

- Complex envelope of transmitted signal

$$\tilde{s}(t) = m(t)e^{j2\pi ft} \quad (6.122)$$

- Received symbols

$$\tilde{r}(t,l) = A(l)m(t-l/c)e^{j2\pi f(l/c)} \quad (6.123)$$

- Key assumptions
  1. Incident field is a plane wave
  2. The attenuation $A(l) \approx A(l_0) \equiv A_0$
  3. $\cdot m(t-l/c) \approx (t-l_0/c) \equiv m_0(t)$
  4. There is no mutual coupling between the antenna elements.

- Under the above assumptions, analysis depends solely on phase relationship of different elements.
- If antenna element k is at distance l_k from the transmitting antenna, then the carrier phase is

$$\begin{aligned}\phi(t) &= 2\pi f(t - l_k/c)\\ &= 2\pi f(t-l_0/c) - 2\pi f(l_k - l_0/c)\\ &\equiv \phi_0(t) - 2\pi f \Delta l_k / c\\ &\equiv \phi_0(t) - \Delta\phi_k\end{aligned} \quad (6.124)$$

- The phase offset is

$$\begin{aligned}\Delta\phi_k &= (2\pi f/c)(kd \sin\theta)\\ &= 2\pi\left(\frac{kd}{\lambda}\right)\sin\theta\end{aligned} \quad (6.125)$$

- The received signal at element k is

$$\begin{aligned}s_k(t) &= s_0(t)e^{j2\pi\left(\frac{kd}{\lambda}\right)\sin\theta}\\ &= 2\pi\left(\frac{kd}{\lambda}\right)\sin\theta \qquad \text{where} \quad s_0(t) = A_0 m_0(t)e^{j\phi_0(t)}\end{aligned} \quad (6.126,127)$$

- Complex rotation

$$a_k(\theta) = e^{j2\pi\left(\frac{kd}{\lambda}\right)\sin\theta} \quad (6.128)$$

- A phased array computes a linear sum of the signals received at each element in Fig 6.29, yielding the received signal

$$\begin{aligned}r(t) &= \sum_{k=-N/2}^{N/2} w_k^* s_k(t)\\ &= (\mathbf{w}^+\mathbf{a})s_0(t) \quad \text{where } w = [w_{-N/2},\ldots,w_{N/2}]^T \text{ and } \mathrm{a}(\theta) = [a_{-N/2}(\theta),\ldots,a_{N/2}(\theta)]^T\end{aligned} \quad (6.129)$$

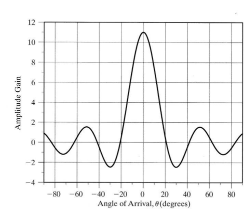

FIGURE 6.30 Antenna gain for constant weighting with N = 6 and d = λ/4.

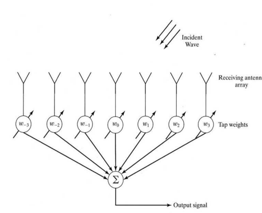

FIGURE 6.32 Antenna with adjustable elemental weights.

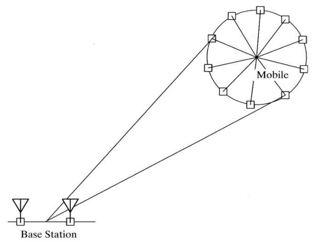

FIGURE 6.33 Lee's model for multipath propagation in macrocells.

6.8 Space-Division Multiple Access and Smart Antennas

6.8.2 Multipath with Direction Antennas

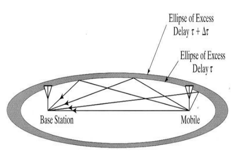

FIGURE 6.34 Single-bounce elliptical model for multipath propagation in microcells.

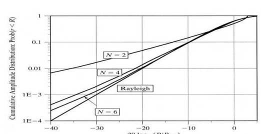

FIGURE 6.35 Amplitude distribution of multipath as a function of number of paths N.

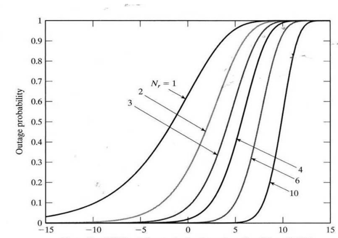

FIGURE 6.6 Outage probability of maximal-ratio combiner for a varying number N_r of receive antennas.