# C H A P T E R  0
## A Precalculus Review

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CHAPTER 0
A Precalculus Review

Section 0.1 The Real Number Line and Order

Solutions to Odd-Numbered Exercises

1. Since \(0.7 = \frac{7}{10}\), it is rational.

3. \(\frac{3\pi}{2}\) is irrational because \(\pi\) is irrational.

5. \(4.3\overline{5}\) is rational because it has a repeating decimal expansion.

7. Since \(\sqrt[3]{64} = 4\), it is rational.

9. \(\sqrt[3]{60}\) is irrational, since 60 is not the cube of a rational number.

11. (a) Yes, if \(x = 3\), then \(5(3) - 12 = 3 > 0\).

(b) No, if \(x = -3\), then \(5(-3) - 12 = -27 < 0\).

(c) Yes, if \(x = \frac{5}{2}\), then \(5 \left(\frac{5}{2}\right) - 12 = \frac{1}{2} > 0\).

(d) No, if \(x = \frac{3}{2}\), then \(5 \left(\frac{3}{2}\right) - 12 = -\frac{9}{2} < 0\).

13. \(0 < \frac{x - 2}{4} < 2\)

(a) Yes, if \(x = 4\), then \(2 < x < 10\).

(b) No, if \(x = 10\), then \(x\) is not less than 10.

(c) No, if \(x = 0\), then \(x\) is not greater than 2.

(d) Yes, if \(x = \frac{7}{2}\), then \(2 < x < 10\).

15. \(x - 5 \geq 7\)

\(x - 5 + 5 \geq 7 + 5\)

\(x \geq 12\)

17. \(4x + 1 < 2x\)

\(2x < -1\)

\(x < -\frac{1}{2}\)

19. \(4 - 2x < 3x - 1\)

\(4 - 5x < -1\)

\(-5x < -5\)

\(x > 1\)

21. \(-4 < 2x - 3 < 4\)

\(-4 + 3 < 2x - 3 + 3 < 4 + 3\)

\(-1 < 2x < 7\)

\(-\frac{1}{2} < x < \frac{7}{2}\)

23. \(\frac{3}{4} > x + 1 > -\frac{1}{4}\)

\(-\frac{1}{4} > x > -\frac{3}{4}\)

\(-\frac{3}{4} < x < -\frac{1}{4}\)

25. \(\frac{x}{2} + \frac{x}{3} > 5\)

\(3x + 2x > 30\)

\(5x > 30\)

\(x > 6\)
27. 
\[ 2x^2 - x < 6 \]
\[ 2x^2 - x - 6 < 0 \]
\[ (2x + 3)(x - 2) < 0 \]
Zeros of the polynomial (2x + 3)(x - 2) are \( x = -\frac{3}{2} \) and \( x = 2 \). Testing the intervals \((-\infty, -\frac{3}{2}), \left(-\frac{3}{2}, 2\right), \) and \((2, \infty)\), you see that the solution set is \(-\frac{3}{2} < x < 2\).

29. Hydrochloric acid 
Pure water 
Oven cleaner 
Lemon juice 
Black coffee 
Baking soda

31. \( R = 115.95x \) and \( C = 95x + 750 \) and we have:
\[ R > C \]
\[ 115.95x > 95x + 750 \]
\[ 20.95x > 750 \]
\[ x > \frac{750}{20.95} = 35.7995 \ldots \]
Therefore, \( x \geq 36 \) units.

33. Let \( x \) be the number of miles driven each week. Then the company reimburses according to the formula
\[ C = 100 + 0.35x \]
According to the allocations, you have
\[ 200 \leq C \leq 250 \]
\[ 200 \leq 100 + 0.35x \leq 250 \]
\[ 100 \leq 0.35x \leq 150 \]
\[ 100 \leq x \leq 150 \]
\[ 0.35 \leq x \leq 0.35 \]
\[ 285.71 < x < 428.57 \]
Thus, the minimum is approximately 285.7 miles per week, and the maximum is approximately 428.6.

Section 0.2 Absolute Value and Distance on the Real Number Line

1. (a) The directed distance from \( a \) to \( b \) is \( 75 - 126 = -51 \).
(b) The directed distance from \( b \) to \( a \) is \( 126 - 75 = 51 \).
(c) The distance between \( a \) and \( b \) is \( |75 - 126| = 51 \).

3. (a) The directed distance from \( a \) to \( b \) is \( -5.65 - 9.34 = -14.99 \).
(b) The directed distance from \( b \) to \( a \) is \( 9.34 - (-5.65) = 14.99 \).
(c) The distance between \( a \) and \( b \) is \( |-5.65 - 9.34| = 14.99 \).

5. (a) The directed distance from \( a \) to \( b \) is \( \frac{112}{73} - \frac{46}{73} = \frac{66}{73} \).
(b) The directed distance from \( b \) to \( a \) is \( \frac{46}{73} - \frac{112}{73} = \frac{-66}{73} \).
(c) The distance between \( a \) and \( b \) is \( \left| \frac{112}{73} - \frac{46}{73} \right| = \frac{66}{73} \).

7. \( |x| \leq 2 \) 
9. \( |x| > 2 \) 
11. \( |x - 4| \leq 2 \)
13. $|x - 2| > 2$
15. $|x - 4| < 2$
17. $|y - a| \leq 2$

19. $-5 < x < 5$
21. $\frac{x}{2} < -3$ or $\frac{x}{2} > 3$
23. $-5 < x + 2 < 5$

25. $\frac{x - 3}{2} \leq -5$ or $\frac{x - 3}{2} \geq 5$
27. $10 - x < -4$ or $10 - x > 4$

29. $-1 < 9 - 2x < 1$
31. $-b \leq x - a \leq b$
33. $-2b < \frac{3x - a}{4} < 2b$

35. Midpoint $= \frac{7 + 21}{2} = 14$
37. Midpoint $= \frac{-6.85 + 9.35}{2} = 1.25$
39. Midpoint $= \frac{-\frac{1}{2} + \frac{3}{2}}{2} = \frac{1}{2} = \frac{1}{8}$

41. $1083.4 - 0.2 \leq M \leq 1083.4 + 0.2$

$1083.2 \leq M \leq 1083.6$

$|M - 1083.4| \leq 0.2$

43. $\left| \frac{h - 68.5}{2.7} \right| \leq 1$

45. $|x - 200,000| \leq 25,000$

$-25,000 \leq x - 200,000 \leq 25,000$

$175,000 \leq x \leq 225,000$

47. (a) $|E - 4750| \leq 500 \Rightarrow 4250 \leq E \leq 5250$

$b = 237.50$

$|E - 4750| \leq 237.50 \Rightarrow 4512.50 \leq E \leq 4987.50$

(b) $\$5116.37$ is not within 5% of the specified budgeted amount; at variance.
49. (a) \(|E - 20,000| \leq 500 \Rightarrow 19,500 \leq E \leq 20,500
0.05(20,000) = 1000
|E - 20,000| \leq 1000 \Rightarrow 19,000 \leq E \leq 21,000
(b) $22,718.35 is at variance with both budget restrictions.

Section 0.3  Exponents and Radicals

1. \(-3(2)^3 = -3(8) = -24\)

5. \(\frac{1 + (2)^{-1}}{2^{-1}} = \frac{1 + \frac{1}{2}}{1/2} = \frac{3/2}{1/2} = 3\)

9. \(6(10)^0 - [6(10)]^0 = 6(1) - 1 = 5\)

13. \(4^{-1/2} = \frac{1}{\sqrt{4}} = \frac{1}{2}\)

17. \(500(1.01)^{60} \approx 908.3483\)

21. \(6y^{-2}(2y^{-3})^{-3} = 6y^{-2}(2^{-3}y^{-12}) = 6\left(\frac{1}{8}\right)y^{-14} = \frac{3}{4y^{14}}\)

25. \(\frac{7x^3}{x^{-3}} = 7x^6\)

29. \(\frac{3x\sqrt{x}}{\sqrt[3]{x}} = \frac{3x\sqrt[3]{x}}{x} = 3x, \ x > 0\)

33. (a) \(\sqrt[3]{16x^8} = \sqrt[3]{(8x^4)(2x^2)} = \sqrt[3]{8x^4}\sqrt[3]{2x^2} = 2x\sqrt[3]{2x^2}\)
(b) \(\sqrt[3]{32x^3} = \sqrt[3]{16x^2z^2} = \sqrt[3]{16x^2}\sqrt[3]{z^2} = 2|x|z\sqrt[3]{2z}\)

[Note: Since \(x^3\) is under the radical, \(x\) could be positive or negative. For \(z^2\) to be under the radical, \(z\) must be positive.]

35. (a) \(\sqrt[3]{144x^3y^{-6}z^3} = \sqrt[3]{18(2x^3)(y^{-2})^3} = 2x^2y^{-2}\sqrt[3]{18z}\)

(b) \(\sqrt[3]{12(3x + 5)^7} = \sqrt[3]{3(2x^3)(3x + 5)^4} = 2(3x + 5)^{4/3}\sqrt[3]{9x + 15}\)

37. \(4x^3 - 6x = 2x(2x^2 - 3)\)

39. \(2x^{5/2} + x^{-1/2} = x^{-1/2}(2x^3 + 1) = \frac{2x^3 + 1}{x^{1/2}}\)
41. \(3(x + 1)^{3/2} - 6(x + 1)^{1/2} = 3(x + 1)^{1/2}(x(x + 1) - 2) = \frac{(x + 1)(x - 1)^2 - (x - 1)^3}{(x + 1)^2} - \frac{(x - 1)^2}{(x + 1)^2}((x + 1) - (x - 1)) = \frac{(x - 1)^2}{(x + 1)^2}\)

43. \(\frac{(x + 1)(x - 1)^2 - (x - 1)^3}{(x + 1)^2} = \frac{(x - 1)^2}{(x + 1)^2}((x + 1) - (x - 1)) = \frac{2(x - 1)^2}{(x + 1)^2}\)

45. \((x + 1)^2(x - 1)^{-1/2} + 2x(x - 1)^{1/2}(x^2 + 1) = (x^2 + 1)(x - 1)^{-1/2}((x^2 + 1) + 2x(x - 1)) = (x^2 + 1)(x - 1)^{-1/2}(3x^2 - 2x + 1) = \frac{(x^2 + 1)(3x^2 - 2x + 1)}{(x - 1)^{1/2}}\)

47. \(\sqrt{x - 1}\) is defined when \(x \geq 1\). Therefore, the domain is \([1, \infty)\).

49. \(\sqrt{x^2 + 3}\) is defined for all real numbers. Therefore, the domain is \((-\infty, \infty)\).

51. \(\frac{1}{\sqrt{x - 1}}\) is defined for all real numbers except \(x = 1\). Therefore, the domain is \((-\infty, 1) \cup (1, \infty)\).

53. The numerator is defined for \(x \geq -2\). The denominator is defined for all \(x \neq 4\). Therefore, the domain is \([-2, 4) \cup (4, \infty)\).

55. \(\sqrt{x - 1}\) is defined when \(x \geq 1\), and \(\sqrt{5 - x}\) is defined when \(x \leq 5\). Therefore, the domain of \(\sqrt{x - 1} + \sqrt{5 - x}\) is \(1 \leq x \leq 5\).

57. \(A = 10,000\left(1 + \frac{0.065}{12}\right)^{120} \approx 19,121.84\)

59. \(A = 5000\left(1 + \frac{0.055}{4}\right)^{60} \approx 11,345.46\)

61. \(T = 2\pi \sqrt{\frac{L}{32}}\)

\(= 2\pi \sqrt{\frac{4}{32}}\)

\(= 2\pi \sqrt{\frac{1}{8}}\)

\(= 2\pi \frac{1}{2}\sqrt{2}\)

\(= \frac{\pi \sqrt{2}}{2} \approx 2.22\) seconds

Section 0.4  Factoring Polynomials

1. Since \(a = 6, b = -1,\) and \(c = -1,\) we have

\[x = \frac{1 \pm \sqrt{1 - (-24)}}{12} = \frac{1 \pm 5}{12}\]

Thus, \(x = \frac{1 + 5}{12} = \frac{1}{2}\) or \(x = \frac{1 - 5}{12} = -\frac{1}{3}\)

3. Since \(a = 4, b = -12,\) and \(c = 9,\) we have

\[x = \frac{12 \pm \sqrt{144 - 44}}{8} = \frac{12 \pm 3}{8} = \frac{3}{2}\]
5. Since \(a = 1, b = 4,\) and \(c = 1,\) we have
\[
y = \frac{-4 \pm \sqrt{16 - 4}}{2} = \frac{-4 \pm \sqrt{3}}{2} = -2 \pm \sqrt{3}.
\]
7. Since \(a = 2, b = 3,\) and \(c = -4,\) we have
\[
x = \frac{-3 \pm \sqrt{9 - 4(2)(-4)}}{4} = \frac{-3 \pm \sqrt{41}}{4}
\]
9. \(x^2 - 4x + 4 = (x - 2)^2\)
11. \(4x^2 + 4x + 1 = (2x + 1)^2\)
13. \(x^2 + x - 2 = (x + 2)(x - 1)\)
15. \(3x^2 - 5x + 2 = (3x - 2)(x - 1)\)
17. \(x^2 - 4xy + 4y^2 = (x - 2y)^2\)
19. \(81 - y^4 = (9 + y^2)(9 - y^2)\)
\[= (9 + y^2)(3 + y)(3 - y)\]
21. \(x^3 - 8 = x^3 - 2^3\)
\[= (x - 2)(x^2 + 2x + 4)\]
23. \(y^3 + 64 = y^3 + 4^3\)
\[= (y + 4)(y^2 - 4y + 16)\]
25. \(x^3 - 27 = x^3 - 3^3\)
\[= (x - 3)(x^2 + 3x + 9)\]
27. \(x^3 - 4x^2 - x + 4 = x^2(x - 4) - (x - 4)\)
\[= (x - 4)(x^2 - 1)\]
\[= (x - 4)(x + 1)(x - 1)\]
29. \(2x^3 - 3x^2 + 4x - 6 = x^2(2x - 3) + 2(2x - 3)\)
\[= (2x - 3)(x^2 + 2)\]
31. \(2x^3 - 4x^2 - x + 2 = 2x^2(x - 2) - (x - 2)\)
\[= (x - 2)(2x^2 - 1)\]
33. \(x^4 - 15x^2 - 16 = (x^2 - 4)(x^2 + 1)\)
\[= (x - 2)(x + 2)(x^2 + 1)\]
35. \(x^2 - 5x = 0\)
\[x(x - 5) = 0\]
\[x = 0, 5\]
37. \(x^2 - 9 = 0\)
\[(x + 3)(x - 3) = 0\]
\[x = -3, 3\]
39. \(x^2 - 3 = 0\)
\[(x + \sqrt{3})(x - \sqrt{3}) = 0\]
\[x = \pm \sqrt{3}\]
41. \((x - 3)^2 - 9 = 0\)
\[x^2 - 6x + 9 - 9 = 0\]
\[x(x - 6) = 0\]
\[x = 0, 6\]
43. \(x^2 + x - 2 = 0\)
\[(x + 2)(x - 1) = 0\]
\[x = -2, 1\]
45. \(x^2 - 5x + 6 = 0\)
\[(x - 2)(x - 3) = 0\]
\[x = 2, 3\]
47. \(x^3 + 64 = 0\)
\[x^3 = -64\]
\[x = \sqrt[3]{-64} = -4\]
49. \(x^4 - 16 = 0\)
\[x^4 = 16\]
\[x = \pm \sqrt{16} = \pm 2\]
51. \(x^3 - x^2 - 4x + 4 = 0\)
\[x^2(x - 1) - 4(x - 1) = 0\]
\[(x - 1)(x^2 - 4) = 0\]
\[(x - 1)(x - 2)(x + 2) = 0\]
\[x = 1, \pm 2\]
53. Since \(\sqrt{x^2 - 4} = \sqrt{(x + 2)(x - 2)},\) the roots are \(x = \pm 2.\)
By testing points inside and outside the interval \([-2, 2],\)
we find that the expression is defined when \(x \leq -2\) or
\(x \geq 2.\) Thus, the domain is \((-\infty, -2] \cup [2, \infty).\)
57. \( -1 \begin{array}{cccc} 1 & -3 & -6 & -2 \\ -1 & 4 & 2 \\ \hline 1 & -4 & -2 & 0 \end{array} \)

Therefore, the factorization is
\[ x^3 - 3x^2 - 6x - 2 = (x + 1)(x^2 - 4x - 2) \]

61. Possible rational zeros: \( \pm 8, \pm 4, \pm 2, \pm 1 \)

Using synthetic division for \( x = -1 \), we have
\[ \begin{array}{cccc} -1 & 1 & -10 & -8 \\ \hline 1 & -2 & -8 & 0 \end{array} \]

Therefore,
\[ x^3 - x^2 - 10x - 8 = 0 \]
\( (x + 1)(x^2 - 2x - 8) = 0 \]
\( (x + 1)(x + 2)(x - 4) = 0 \]

Therefore,
\[ x = -1, -2, 4 \]

65. Possible rational zeros: \( \pm 6, \pm 3, \pm 2, \pm 1, \pm \frac{1}{2}, \pm \frac{1}{3}, \pm \frac{1}{4}, \pm \frac{1}{6} \)

Using synthetic division for \( x = 3 \), we have
\[ \begin{array}{cccc} 3 & 6 & -11 & -19 & -6 \\ \hline 18 & 72 & 2 \end{array} \]

Therefore,
\[ 6x^3 - 11x^2 - 19x - 6 = 0 \]
\( (x - 3)(6x^2 + 7x + 2) = 0 \]
\( (x - 3)(3x + 2)(2x + 1) = 0 \]

Therefore,
\[ x = 3, -\frac{2}{3}, -\frac{1}{2} \]

69. \( 0.0003x^2 - 1200 = 0 \)

\[ \begin{align*}
0.0003x^2 &= 1200 \\
x^2 &= 4,000,000 \\
x &= 2000 \text{ units}
\end{align*} \]

59. \( 1 \begin{array}{cccc} 2 & -1 & -2 & 1 \\ 2 & 1 & -1 & 0 \\ \hline 2 & 1 & -1 & 0 \end{array} \)

Therefore, the factorization is
\[ 2x^3 - x^2 - 2x + 1 = (x - 1)(2x^2 + x - 1) \]

63. Possible rational roots: \( \pm 1, \pm 2, \pm 3, \pm 6 \)

Using synthetic division for \( x = 1 \), we have the following.
\[ \begin{array}{cccc} 1 & 1 & -6 & 11 & -6 \\ \hline 1 & -5 & 6 \end{array} \]

Therefore, we have
\[ x^3 - 6x^2 + 11x - 6 = 0 \]
\( (x - 1)(x^2 - 5x + 6) = 0 \]
\( (x - 1)(x - 2)(x - 3) = 0 \]

Therefore, \( x = 1, 2, 3 \).

67. Possible rational roots: \( \pm 1, \pm 2, \pm 4 \)

Using synthetic division for \( x = 4 \), we have the following.
\[ \begin{array}{cccc} 4 & 1 & -3 & -3 & -4 \\ \hline 4 & 4 & 4 \end{array} \]

Therefore, we have
\[ x^3 - 3x^2 - 3x - 4 = 0 \]
\( (x - 4)(x^2 + x + 1) = 0 \).

Since \( x^2 + x + 1 \) has no real solutions, \( x = 4 \) is the only real solution.

71. \( 1.8 \times 10^{-5} = \frac{x^2}{1.0 \times 10^{-4} - x} \)

\[ 1.8 \times 10^{-9} - 1.8 \times 10^{-5}x = x^2 \]
\[ x^2 + 1.8 \times 10^{-5}x - 1.8 \times 10^{-9} = 0 \]

By the Quadratic Formula:
\[ x = \frac{-1.8 \times 10^{-5} \pm \sqrt{(1.8 \times 10^{-5})^2 + 4 \times 1.8 \times 10^{-9}}}{2} \]
\[ = \frac{-1.8 \times 10^{-5} \pm \sqrt{7.524 \times 10^{-9}}}{2} \]
\[ = 3.437 \times 10^{-5} \text{[H+]}} \]
Section 0.5  Fractions and Rationalization

1. \[
\frac{5}{x - 1} + \frac{x}{x - 1} = \frac{5 + x}{x - 1} = \frac{x + 5}{x - 1}
\]

3. \[
\frac{2\sqrt{x}}{x^2 + 2} - \frac{1 - 3x}{x^2 + 2} = \frac{2x - (1 - 3x)}{x^2 + 2} = \frac{5x - 1}{x^2 + 2}
\]

5. \[
\frac{2}{x^2 - 4} - \frac{1}{x - 2} = \frac{2}{(x - 2)(x + 2)} - \frac{1}{x - 2}(x + 2)
\]

= \[
\frac{2 - (x + 2)}{(x - 2)(x + 2)}
\]

= \[
\frac{-x}{x^2 - 4}
\]

= \[
\frac{x}{4 - x^2}
\]

7. \[
\frac{5}{x - 3} + \frac{3}{3 - x} = \frac{5}{x - 3} + \frac{-3}{x - 3} = \frac{2}{x - 3}
\]

9. \[
\frac{A}{x + 1} + \frac{B}{(x + 1)^2} + \frac{C}{x - 2} = \frac{A(x + 1)(x - 2) + B(x - 2) + C(x + 1)^2}{(x + 1)^2(x - 2)}
\]

= \[
\frac{A(x^2 - x - 2) + B(x - 2) + C(x^2 + 2x + 1)}{(x + 1)^2(x - 2)}
\]

= \[
\frac{Ax^2 - Ax - 2A + Bx - 2B + Cx^2 + 2Cx + C}{(x + 1)^2(x - 2)}
\]

= \[
\frac{(A + C)x^2 - (A - B - 2C)x - (2A + 2B - C)}{(x + 1)^2(x - 2)}
\]

11. \[
\frac{A}{x - 6} + \frac{Bx + C}{x^2 + 3} = \frac{A(x^2 + 3) + (Bx + C)(x - 6)}{(x - 6)(x^2 + 3)}
\]

= \[
\frac{(A + B)x^2 + (C - 6B)x + 3A - 6C}{(x - 6)(x^2 + 3)}
\]

13. \[
\frac{1}{x} + \frac{2}{x^2 + 1} = \frac{-x^2 + 2x}{x(x^2 + 1)}
\]

= \[
\frac{-x^2 + 2x - 1}{x(x^2 + 1)}
\]

= \[
\frac{-x^2 - 2x + 1}{x(x^2 + 1)}
\]

= \[
\frac{-(x - 1)^2}{x(x^2 + 1)}
\]

15. \[
\frac{1}{x^2 - x - 2} - \frac{x}{x^2 - 5x + 6} = \frac{1}{(x + 1)(x - 2)} - \frac{x}{(x - 2)(x - 3)}
\]

= \[
\frac{(x - 3) - x(x + 1)}{(x + 1)(x - 2)(x - 3)}
\]

= \[
\frac{-x^2 - 3}{(x + 1)(x - 2)(x - 3)}
\]

= \[
\frac{x^2 + 3}{(x + 1)(x - 2)(x - 3)}
\]

17. \[
\frac{-x}{(x + 1)^{3/2}} + \frac{2}{(x + 1)^{3/2}} = \frac{-x + 2(x + 1)}{(x + 1)^{3/2}}
\]

= \[
\frac{x + 2}{(x + 1)^{3/2}}
\]

19. \[
\frac{2 - t}{2\sqrt{1 + t}} - \sqrt{1 + t} = \frac{2 - t}{2\sqrt{1 + t}} - \frac{\sqrt{1 + t}}{1} \cdot \frac{2\sqrt{1 + t}}{2\sqrt{1 + t}}
\]

= \[
\frac{(2 - t) - 2(1 + t)}{2\sqrt{1 + t}}
\]

= \[
\frac{-3t}{2\sqrt{1 + t}}
\]
21. \( \left( 2\sqrt{x^2 + 1} - \frac{x^3}{\sqrt{x^2 + 1}} \right) \div (x^2 + 1) = \frac{2x(x^2 + 1) - x^3}{\sqrt{x^2 + 1}} \cdot \frac{1}{x^2 + 1} \)
\[= \frac{x^3 + 2x}{\sqrt{x^2 + 1}(x^2 + 1)} \]
\[= \frac{x(x^2 + 2)}{(x^2 + 1)^{3/2}} \]

23. \( \frac{(x^2 + 2)^{1/2} - x^2(x^2 + 2)^{-1/2}}{x^2} = \frac{(x^2 + 2)^{-1/2}[(x^2 + 2) - x^2]}{x^2} \)
\[= \frac{2}{x^2 \sqrt{x^2 + 2}} \]

25. \( \frac{\sqrt{x + 1} - \sqrt{x}}{2(x + 1)} \)
\[= \frac{(x + 1) - x - 1}{2(x + 1)} \cdot \frac{1}{2(x + 1)} \]
\[= \frac{1}{2 \sqrt{x(x + 1)^{1/2}}} \]

29. \( \frac{-x^2}{(2x + 3)^{3/2}} + \frac{2x}{(2x + 3)^{1/2}} = \frac{-x^2}{(2x + 3)^{3/2}} + \frac{2x}{(2x + 3)^{1/2}} \cdot \frac{2x + 3}{2x + 3} \)
\[= \frac{-x^2 + 2x(2x + 3)}{(2x + 3)^{3/2}} \]
\[= \frac{3x^2 + 6x}{(2x + 3)^{3/2}} \]
\[= \frac{3x(x + 2)}{(2x + 3)^{3/2}} \]

31. \( \frac{3}{\sqrt{6}} = \frac{3}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{6}}{2} \)

33. \( \frac{x}{\sqrt{x - 4}} = \frac{x}{\sqrt{x - 4}} \cdot \frac{\sqrt{x - 4}}{\sqrt{x - 4}} = \frac{x\sqrt{x - 4}}{x - 4} \)

35. \( \frac{49(x - 3)}{\sqrt{x^2 - 9}} = \frac{49(x - 3)}{\sqrt{x^2 - 9}} \cdot \frac{\sqrt{x^2 - 9}}{\sqrt{x^2 - 9}} \)
\[= \frac{49(x - 3)\sqrt{x^2 - 9}}{(x + 3)(x - 3)} \]
\[= \frac{49\sqrt{x^2 - 9}}{x + 3}, \quad x \neq 3 \]

37. \( \frac{5}{\sqrt{14} - 2} = \frac{5}{\sqrt{14} - 2} \cdot \frac{\sqrt{14} + 2}{\sqrt{14} + 2} \)
\[= \frac{5(\sqrt{14} + 2)}{14 - 4} \]
\[= \frac{\sqrt{14} + 2}{2} \]

39. \( \frac{2x}{5 - \sqrt{3}} = \frac{2x}{5 - \sqrt{3}} \cdot \frac{5 + \sqrt{3}}{5 + \sqrt{3}} \)
\[= \frac{2x(5 + \sqrt{3})}{25 - 3} \]
\[= \frac{x(5 + \sqrt{3})}{11} \]

41. \( \frac{1}{\sqrt{6} + \sqrt{3}} = \frac{1}{\sqrt{6} + \sqrt{3}} \cdot \frac{\sqrt{6} - \sqrt{3}}{\sqrt{6} - \sqrt{3}} \)
\[= \frac{\sqrt{6} - \sqrt{3}}{6 - 3} \]
\[= \sqrt{6} - \sqrt{3} \]
43. \[
\frac{2}{\sqrt{x} + \sqrt{x - 2}} = \frac{2}{\sqrt{x} + \sqrt{x - 2}} \cdot \frac{\sqrt{x} - \sqrt{x - 2}}{\sqrt{x} - \sqrt{x - 2}}
\]
\[
= \frac{2(\sqrt{x} - \sqrt{x - 2})}{x - (x - 2)}
\]
\[
= \sqrt{x} - \sqrt{x - 2}
\]

45. \[
\frac{\sqrt{4 - x^2}}{x^4} - \frac{2}{4 - x^2}
\]
\[
= \frac{(4 - x^2) - 2x^2}{x^4(4 - x^2)^{3/2}}
\]
\[
= \frac{4 - 3x^2}{x^4(4 - x^2)^{3/2}}
\]

47. \(P = 10,000, \ r = 0.14, \ N = 60\)

\[
M = 10,000 \left[ \frac{0.14/12}{1 - \left( \frac{1}{(0.14/12) + 1} \right)^{60}} \right] \approx \$232.68
\]


**Practice Test for Chapter 0**

1. Determine whether $\sqrt[3]{81}$ is rational or irrational.

2. Determine whether the given value of $x$ satisfies the inequality $3x + 4 \leq x/2$.
   (a) $x = -2$   (b) $x = 0$   (c) $x = -\frac{6}{5}$   (d) $x = -6$

3. Solve the inequality $3x + 4 \geq 13$.

4. Solve the inequality $x^2 < 6x + 7$.

5. Determine which of the two given real numbers is greater, $\sqrt{19}$ or $\frac{13}{2}$.

6. Given the interval $[-3, 7]$, find (a) the distance between $-3$ and 7 and (b) the midpoint of the interval.

7. Solve the inequality $|3x + 1| \leq 10$.

8. Solve the inequality $|4 - 5x| > 29$.

9. Solve the inequality $\left|3 - \frac{2x}{5}\right| < 8$.

10. Use absolute values to describe the interval $[-3, 5]$.

11. Simplify $\frac{12x^3}{4x^2}$.

12. Simplify $\left(\frac{\sqrt{3}/\sqrt{x^3}}{x}\right)^0$, $x \neq 0$.

13. Remove all possible factors from the radical $\sqrt[3]{32x^4y^3}$.

14. Complete the factorization: $\frac{2}{5}(x + 1)^{-1/3} + \frac{1}{3}(x + 1)^{2/3} = \frac{1}{5}(x + 1)^{-1/3}( )$

15. Find the domain: $\frac{1}{\sqrt{5} - x}$

16. Factor completely: $3x^2 - 19x - 14$

17. Factor completely: $25x^2 - 81$

18. Factor completely: $x^3 + 8$

19. Use the Quadratic Formula to find all real roots of $x^2 + 6x - 2 = 0$. 

20. Use the Rational Zero Theorem to find all real roots of \(x^3 - 4x^2 + x + 6 = 0\).

21. Combine terms and simplify: \(\frac{x}{x^2 + 2x - 3} - \frac{1}{x - 1}\)

22. Combine terms and simplify: \(\frac{3 - x}{2\sqrt{x} + 5} + \sqrt{x} + 5\)

23. Combine terms and simplify: \(\frac{\sqrt{x} + 2}{\sqrt{x}} \div \frac{\sqrt{x} + 2}{2(x + 2)}\)

24. Rationalize the denominator: \(\frac{3y}{\sqrt{y^2 + 9}}\)

25. Rationalize the numerator: \(\frac{\sqrt{x} + \sqrt{x + 7}}{14}\)

Graphing Calculator Required

26. Use a graphing calculator to find the real solutions of \(x^3 - 5x^2 + 2x + 8 = 0\) by graphing \(y = x^3 - 5x^2 + 2x + 8\) and finding the \(x\)-intercepts.