Exercise 3-2

3-14. The sample space of a random experiment is \{a, b, c, d, e, f\}, and each outcome is equally likely. A random variable is defined as follows:

<table>
<thead>
<tr>
<th>outcome</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>0</td>
<td>0</td>
<td>1.5</td>
<td>1.5</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

Determine the probability mass function of \(X\). Use the probability mass function to determine the following probabilities:

(a) \(P(X = 1.5)\)
(b) \(P(0.5 < X < 2.7)\)
(c) \(P(X > 3)\)
(d) \(P(0 \leq X < 2)\)
(e) \(P(X = 0 \text{ or } X = 2)\)

For Exercise 3-15 to 3-17 verify that the following functions are probability mass function, and determine the requested probabilities.

3-15. \(x\)

\[
f(x) = \begin{cases} 
-2 & \text{if } x = 1/8 \\
-1 & \text{if } x = 2/8 \\
0 & \text{if } x = 2/8 \\
1 & \text{if } x = 1/8 \\
2 & \text{if } x = 1/8 
\end{cases}
\]

(a) \(P(X \leq 2)\)
(b) \(P(X > -2)\)
(c) \(P(-1 \leq X \leq 1)\)
(d) \(P(X \leq -1 \text{ or } X = 2)\)

3-16. \(f(x) = (8/7)(1/2)^x, x = 1, 2, 3\)

(a) \(P(X \leq 1)\)
(b) \(P(X > 1)\)
(c) \(P(2 < X < 6)\)
(d) \(P(X \leq 1 \text{ or } X > 1)\)

3-17. \(f(x) = \frac{2x+1}{29}, x = 0, 1, 2, 3, 4\)

(a) \(P(X = 4)\)
(b) \(P(X \leq 1)\)
(c) \(P(2 \leq X < 4)\)
(d) \(P(X > -10)\)

Exercise 3-3

3-28. Determine the cumulative distribution function of the random variable is Exercise 3-14.

3-29. Determine the cumulative distribution function for the random variable in Exercise 3-15; also determine the following probabilities:

(a) \(P(X \leq 1.25)\)
(b) \(P(X \leq 2.2)\)
(c) \(P(-1.1 < X \leq 1)\)
(d) \(P(X > 0)\)

3-35. \(F(x) = \begin{cases} 
0 & \text{if } x < 1 \\
0.5 & \text{if } 1 \leq x < 3 \\
1 & \text{if } 3 \leq x 
\end{cases}\)

(a) \(P(X \leq 3)\)
(b) \(P(X \leq 2)\)
(c) \(P(1 \leq X \leq 2)\)
(d) \(P(X > 2)\)

Exercise 3-4

3-39. If the range of \(X\) is the set \{0, 1, 2, 3, 4\} and \(P(X = x) = 0.2\) determine the mean and variance of the random variable.

3-40. Determine the mean and variance of the random variable in Exercise 3-14.

3-41. Determine the mean and variance of the random variable in Exercise 3-15.

3-42. Determine the mean and variance of the random variable in Exercise 3-16.

3-43. Determine the mean and variance of the random variable in Exercise 3-17.

Supplemental Exercise

3-144. The random variable \(X\) has the following probability distribution:

<table>
<thead>
<tr>
<th>(x)</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>probability</td>
<td>0.2</td>
<td>0.4</td>
<td>0.3</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Determine the following:

(a) \(P(X \leq 3)\)
(b) \(P(X > 2.5)\)
(c) \(P(2.7 < X < 5.1)\)

3-145. Determine the probability mass function for the random variable with the following cumulative distribution function:

\[
F(x) = \begin{cases} 
0 & \text{if } x < 2 \\
0.2 & \text{if } 2 \leq x < 5.7 \\
0.5 & \text{if } 5.7 \leq x < 6.5 \\
0.8 & \text{if } 6.5 \leq x < 8.5 \\
1 & \text{if } 8.5 \leq x 
\end{cases}\]

(a) \(P(X \leq 5)\)
(b) \(P(X > 4)\)
(c) \(P(X = 1)\)