Differentiation
微分

之五

以公式法求函數的微分

隱函數的微分

Implicit function

Implicit functions
隱函數

當函數可以表達為y=f(x)的形式，f(x)叫做顯函數。

但是有些式子，y仍然是x的函數，但卻無法簡化為y=f(x)的形式。

也就是說，沒辦法讓y單獨留在等號的一側而另一側只有x。

例如：$x+2xy^2-y=0$

這類的函數，y仍然是x的函數，但卻不能化成y=f(x)的形式，稱作隱函數。

隱函數的微分法則

$$f(x,y)=0$$

$$\frac{d}{dx}f(x,y)=0$$

Example 4.19 隱函數的微分

Find dy/dx for $f(x,y)=y^5-2y-x=0$

$$f(x,y)=y^5-2y-x=0$$

$$\frac{d}{dx}f(x,y)=0$$

$$\frac{d}{dx}f(x,y)=\frac{dy^5-2y-x}{dx}=\frac{dy^5}{dx}-2\times\frac{dy}{dx}-\frac{dx}{dx}$$

$$=5y^4\frac{dy}{dx}-2\frac{dy}{dx}-1=\left(5y^4-2\right)\frac{dy}{dx}-1=0$$

$$\frac{dy}{dx}=\frac{1}{5y^4-2}$$

Exercisese 隱函數的微分

65. $x^2 + y^2 = 4$ Find $\frac{dy}{dx}$

$$f(x, y) = x^2 + y^2 - 4 = 0$$

$$\frac{d}{dx} f(x, y) = \frac{d}{dx}(x^2 + y^2 - 4) = 2x + \frac{dy^2}{dx} - 0 = 2x + 2y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{2x}{2y} = -\frac{x}{y}$$

Exercisese 隱函數的微分

66. $y^3 + 3x + x^2 - 1 = 0$ Find $\dfrac{dy}{dx}$

$$f(x, y) = y^3 + 3x + x^2 - 1 = 0$$

$$\frac{d}{dx} f(x, y) = \frac{d}{dx}(y^3 + 3x + x^2 - 1) = 3y\frac{dy}{dx} + \frac{d(3x)}{dx} + \frac{dx^2}{dx}$$

$$= 3y^2\frac{dy}{dx} + 3 + 2x = 0$$

$$\frac{dy}{dx} = \frac{-2x-3}{3y^2}$$

Exercise 隱函數的微分

67. $x = y \ln xy$

Find $\dfrac{dy}{dx}$

$$f(x, y) = x - y \ln xy = 0$$

$$\frac{d}{dx} f(x, y) = \frac{d}{dx}(x - y \ln xy) = \frac{dx}{dx} - \frac{d(y \ln xy)}{dx}$$

$$= 1 - (y \frac{d \ln xy}{dx} + \ln xy \frac{dy}{dx})$$

$$= 1 - \left[y \times \frac{1}{xy} \times \frac{d(xy)}{dx} + \ln xy \frac{dy}{dx} \right]$$

$$= 1 \text{ - } \left[\frac{1}{x} \times \left(x \frac{dy}{dx} + y \frac{dx}{dx} \right) + \ln xy \frac{dy}{dx} \right]$$

$$= 1 - \left(\frac{dy}{dx} + \frac{y}{x} + \ln xy \frac{dy}{dx} \right)$$

$$= 1 - \frac{y}{x} - (1 + \ln xy) \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{1 - \frac{y}{x}}{(1 + \ln xy)} = \frac{x - y}{x(1 + \ln xy)}$$

Exercise 隱函數的微分

68. $y^2+\frac{2}{y}-x^2y^2+3x+2=0$ Find $\frac{dy}{dx}$

$$f(x,y)=y^2+\frac{2}{y}-x^2y^2+3x+2=0$$

$$\frac{d}{dx}f(x,y)=\frac{d}{dx}(y^2+\frac{2}{y}-x^2y^2+3x+2)$$

$$=\frac{dy^2}{dx}+2\frac{dy^{-1}}{dx}-\frac{dx^2y^2}{dx}+3$$

$$=2y\times\frac{dy}{dx}-2y^{-2}\frac{dy}{dx}-(x^2\frac{dy^2}{dx}+y^2\frac{dx^2}{dx})+3$$

$$=2y\times\frac{dy}{dx}-(2y^{-2})\times\frac{dy}{dx}-(2x^2y)\frac{dy}{dx}-(2xy^2)+3=0$$

$$\frac{dy}{dx}=\frac{2xy^2-3}{2y-\frac{2}{y^2}-2x^2y}$$