OR(I) **MIDTERM** 05-99

1. You are given the following nutritional and cost information regarding steak and potatoes:

Ingredient	Grams of Ingredient per serving: Steak	Grams of Ingredient per serving: Potatoes	Daily Requirement (Grams)
Carbohydrates	5	15	≥ 50
Protein	20	5	≥ 40
Fat	15	2	≥ 60
Cost per serving	\$4	\$2	

You wish to determine the number of daily servings (it may be fractional) of steak and potatoes that will meet these requirements at a minimum cost. Formulate a LP model for this problem. (No need to solve it!) (10%)

2. Use the big-M method to solve the following LP problem. (10%)

$$\begin{aligned} \textit{min} \quad & Z = 3x_1 + 2x_2 + x_3 \\ \textit{s.t.} \quad & x_1 + x_2 = 7 \\ & 3x_1 + x_2 + x_3 \geq 10 \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0 \end{aligned}$$

3. Consider the following LP problem.

$$\begin{aligned} \textit{max} \quad & Z = c_1x_1 + c_2x_2 + c_3x_3 \\ \textit{s.t.} \quad & x_1 + 2x_2 + x_3 \leq b_1 \quad \text{(resource 1)} \\ & 2x_1 + x_2 + 3x_3 \leq b_2 \quad \text{(resource 2)} \\ & x_1 \geq 0,\ x_2 \geq 0,\ x_3 \geq 0 \end{aligned}$$

Suppose now you are given the following partial optimal simplex tableau for this problem, where x_4 and x_5 are the slack variables for the first and second constraints, respectively.

	x_1	x_2	x_3	x_4	x_5	RHS
Row 0	7/10			3/5	4/5	
x_2				3/5	-1/5	1
x_3				-1/5	2/5	3

(a) Find the values of b_1 and b_2. (5%)

(b) Find the values of c_1, c_2 and c_3. (5%)

(c) Find the missing numbers in this final tableau. Show your calculations. (5%)

(d) What is the new optimal value of Z if b_1 is replaced by b_1+1? (5%)

(e) Suppose that we can purchase additional units of resource 2 at \$1 per unit. Should we do it? Why? (5%)

4. Consider the following LP problem.

$$\begin{aligned} max \quad & Z = 2x_1 + 7x_2 + 4x_3 \\ s.t. \quad & x_1 + 2x_2 + x_3 \le 10 \\ & 3x_1 + 3x_2 + 2x_3 \le 10 \\ & x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0 \end{aligned}$$

(a) Construct the dual problem. (5%)

(b) Use the dual problem to demonstrate that the optimal value of Z for the primal problem cannot exceed 24. (5%)

(c) Solve the dual problem graphically. (5%)

(d) Use the solution found in (c) to identify the optimal solution for the primal problem. (10%)

5. Consider the following LP problem.

$$\begin{aligned} max \quad & Z = -5x_1 + 5x_2 + 13x_3 \\ s.t. \quad & -x_1 + x_2 + 3x_3 \le 20 \\ & 12x_1 + 4x_2 + 10x_3 \le 90 \\ & x_1 \ge 0,\ x_2 \ge 0,\ x_3 \ge 0 \end{aligned}$$

If we let x_4 and x_5 be the slack variables for the first and second constraints, respectively, the simplex method yields the following optimal tableau.

	x_1	x_2	x_3	x_4	x_5	RHS
Row 0	0	0	2	5	0	100
x_2	-1	1	3	1	0	20
x_5	16	0	-2	-4	1	10

Now you are to conduct *sensitivity analysis* by <u>independently</u> investigating each of the following changes in the original model. For each change, revise the final tableau and convert it to proper form, if necessary. Then, test the current solution for feasibility and for optimality. If either test fails, reoptimize to find a new optimal solution.

(a) Change the RHS to $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 100 \end{bmatrix}$. (5%)

(b) Change the coefficients of x_1 to $\begin{bmatrix} c_1 \\ a_{11} \\ a_{21} \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$. (5%)

(c) Determine the allowable range to stay optimal for the coefficient c_2. (5%)

(d) Change the coefficients of x_2 to $\begin{bmatrix} c_2 \\ a_{12} \\ a_{22} \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 5 \end{bmatrix}$. (5%)

(e) Introduce a new variable x_6 with coefficients $\begin{bmatrix} c_6 \\ a_{16} \\ a_{26} \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 5 \end{bmatrix}$. (5%)

(f) Introduce a new constraint $2x_1 + 3x_2 + 5x_3 \leq 50$. (5%)