1. Use the graphical method to solve this problem. (20\%)

$$
\begin{array}{ll}
\min & Z=15 x_{1}+20 x_{2} \\
\text { s.t. } & x_{1}+2 x_{2} \geq 10 \\
& 2 x_{1}-3 x_{2} \leq 6 \\
& x_{1}+x_{2} \geq 6 \\
& x_{1}, x_{2} \geq 0
\end{array}
$$

2. The enclosed area in the following graph represents the feasible region of a LP problem whose objective function is to be maximized. (10\%)


Label each of the following statements True or False.
(a) If $(3,3)$ produces a larger value of the objective function than $(0,2)$ and $(6,3)$, then $(3,3)$ must be an optimal solution.
(b) If $(3,3)$ is an optimal solution and multiple optimal solutions exist, then either $(0,2)$ or $(6,3)$ must also be an optimal solution.
(c) The point $(0,0)$ cannot be an optimal solution.
3. John is an elemental school teacher who also raises pigs for supplemental income.

He is trying to decide what to feed his pigs. He is considering using a combination of pig feeds available from local suppliers. He would like to feed his pigs at minimum cost while making sure each pig receive an adequate supply of calories and vitamins. The cost, calorie content, and vitamin content of each feed is given as follows:

| Contents | Feed type $A$ | Feed type $B$ |
| :--- | :--- | :--- |
| Calories (per pound) | 800 | 1000 |
| Vitamins (per pound) | 140 units | 70 units |
| Cost (per pound) | $\$ 0.4$ | $\$ 0.8$ |

Each pig requires at least 8000 calories per day and at least 700 units of vitamins. A further constraint is that no more than one-third of the diet (by weight) can consists of Feed type A, since it contains an ingredient which is toxic if consumed in too large a quantity.
(a) Formulate a LP model for this problem. (20\%)
(b) Use the graphical method to solve this model. What is the resulting daily cost per day? (20\%)
4. Use the simplex method (in tableau form) to solve the following problem. (30\%)

$$
\begin{array}{ll}
\max & Z=4 x_{1}+3 x_{2}+6 x_{3} \\
\text { s.t. } & 3 x_{1}+x_{2}+3 x_{3} \leq 30 \\
& 2 x_{1}+2 x_{2}+3 x_{3} \leq 40 \\
& x_{1}, x_{2}, x_{3} \geq 0
\end{array}
$$

