OR(1) **Quiz 2** 2001-04

1. Consider the following problem

$$\max \quad Z = 6x_1 + x_2 + 2x_3$$

$$s.t. \quad 2x_1 + 2x_2 + \frac{1}{2}x_3 \le 2$$

$$-4x_1 - 2x_2 - \frac{3}{2}x_3 \le 3$$

$$x_1 + 2x_2 + \frac{1}{2}x_3 \le 1$$

$$x_1, x_2, x_3 \ge 0$$

Let x_4, x_5, and x_6 denote the slack variables for the respective constraints. After you apply the simplex method, a portion of the final tableau is as follows:

	x_1	x_2	x_3	x_4	x_5	x_6	RHS
Row(0)							
x_5				1	1	2	
x_3				-2	0	4	
x_1				1	0	-1	

Use the theory of simplex method to identify the missing numbers in the final tableau. Show your calculations. (20%)

2. Consider the following LP problem

$$\max \quad Z = 4x_1 + 2x_2$$

$$s.t. \quad 2x_1 \le 16 \quad \text{(resource 1)}$$

$$x_1 + 3x_2 \le 17 \quad \text{(resource 2)}$$

$$x_2 \le 5 \quad \text{(resource 3)}$$

$$x_1, x_2 \ge 0$$

(a) Solve the problem graphically. (10%)

(b) Using graphical analysis to find the shadow prices for the resources. (10%)

(c) Determine how many additional units of resource 1 would be needed to increase the optimal value of Z by 15. (5%)

3. Consider the following problem:

$$\begin{aligned}
\max \quad & Z = 2x_1 + 4x_2 - x_3 \\
s.t. \quad & 3x_2 - x_3 \le 30 && \text{(resource 1)} \\
& 2x_1 - x_2 + x_3 \le 10 && \text{(resource 2)} \\
& 4x_1 + 2x_2 - 2x_3 \le 40 && \text{(resource 3)} \\
& x_1, x_2, x_3 \ge 0
\end{aligned}$$

The simplex method is used to solve the problem and the optimal tableau is as follows, where x_4, x_5, and x_6 denote the slack variables for the respective constraints.

	x_1	x_2	x_3	x_4	x_5	x_6	RHS
Row(0)	0	0	0	1.5	0.75	0.125	57.5
x_2	0	1	0	0.5	0.25	-0.125	12.5
x_3	0	0	1	0.5	0.75	-0.375	7.5
x_1	1	0	0	0	0.25	0.125	7.5

(a) Find the shadow price for each resource. (5%)

(b) Suppose that you have a chance to buy an additional unit of resource 2 for $1. Should you do it? Why? (5%)

Suppose that the sensitivity analysis generates the following results.

Allowable range to stay optimal

Current value	Minimum	Maximum
2	1	∞
4	1	5
-1	-2	-2/3

Allowable range to stay feasible

Current value	Minimum	Maximum
30	15	∞
10	0	5
40	-20	60

(c) What is the new optimal objective value Z if c_1=2 is changed to c_1=1? (5%)

(d) Will the optimal objective value Z change if b_3=40 is changed to b_3=70? Why? (5%)

4. Use the two-phase method to solve the following LP problem. (20%)

$$\begin{aligned}
\min \quad & Z = 3x_1 + 2x_2 + 4x_3 \\
s.t. \quad & 2x_1 + x_2 + 3x_3 = 60 \\
& 3x_1 + 3x_2 + 5x_3 \ge 120 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}$$

5. Consider the following LP problem.

$$\begin{aligned} \min \quad & Z = 3x_1 - x_2 + 4x_3 \\ s.t. \quad & x_1 + x_2 - x_3 \geq -6 \\ & 3x_1 - x_2 + 2x_3 \leq 120 \\ & x_1 \geq -2,\ x_2 \text{ unrestricted},\ x_3 \geq 0 \end{aligned}$$

Transform the problem into a standard form with the objective function is to be maximized, all RHS coefficients are nonnegative, and all variables are nonnegative. (15%)