OR(1) **Quiz 3** 2001-05

1. Consider the following LP problem

$$\max \quad Z = -5x_1 + 5x_2 + 13x_3$$

$$s.t. \quad -x_1 + x_2 + 3x_3 \le 20$$

$$12x_1 + 4x_2 + 10x_3 \le 90$$

$$x_1, x_2, x_3 \ge 0$$

If we let x_4 and x_5 be the slack variables for the respective constraints, the simplex method yields the following final tableau:

	x_1	x_2	x_3	x_4	x_5	RHS
Row(0)	0	0	2	5	0	100
x_2	-1	1	3	1	0	20
x_5	16	0	-2	-4	1	10

Now you are to conduct sensitivity analysis by <u>independently</u> investigating each of the following changes in the original model. For each change, revise the final tableau and convert it into the proper form. Then, test this solution for feasibility and optimality. If either test fails, re-optimize to find a new optimal solution.

(a) Change the RHS of constraint 2 to $b_2 = 70$. (5%)

(b) Determine the allowable range to stay feasible for b_2. (5%)

(c) Change the coefficients of x_1 to $\begin{bmatrix} c_1 \\ a_{11} \\ a_{21} \end{bmatrix} = \begin{bmatrix} -2 \\ 0 \\ 5 \end{bmatrix}$. (5%)

(d) Change the coefficients of x_2 to $\begin{bmatrix} c_2 \\ a_{12} \\ a_{22} \end{bmatrix} = \begin{bmatrix} 6 \\ 2 \\ 5 \end{bmatrix}$. (10%)

(e) Determine the allowable range to stay optimal for c_2. (10%)

(f) Introduce a new variable x_6 with coefficients $\begin{bmatrix} c_6 \\ a_{16} \\ a_{26} \end{bmatrix} = \begin{bmatrix} 10 \\ 3 \\ 5 \end{bmatrix}$. (5%)

(g) Introduce a new constraint $2x_1 + 3x_2 + 5x_3 \le 50$. (Denote its slack variable by x_6.) (10%)

2. Construct the dual problem of the following LP problem. (15%)

$$
\begin{aligned}
\text{min} \quad & Z = 4x_1 + 2x_2 - 3x_3 \\
\textit{s.t.} \quad & 2x_1 - x_2 + 3x_3 \le 15 \\
& x_1 + 3x_2 - x_3 = 20 \\
& \quad 4x_2 + x_3 \ge 5 \\
& x_1 \text{ unrestrucited in sign }, x_2 \ge 0,\ x_3 \le 0
\end{aligned}
$$

3. Consider the following problem:

$$
\begin{aligned}
\text{max} \quad & Z = 2x_1 + 5x_2 + 3x_3 + 4x_4 + x_5 \\
\textit{s.t.} \quad & x_1 + 3x_2 + 2x_3 + 3x_4 + x_5 \le 6 \\
& 4x_1 + 6x_2 + 5x_3 + 7x_4 + x_5 \le 15 \\
& x_1, x_2, x_3, x_4, x_5 \ge 0
\end{aligned}
$$

(a) Construct the dual problem. (5%)

(b) Solve graphically the dual problem in (a). (10%)

(c) Using the results obtained in (b) and the complementary slackness theorem to identify the optimal solution for the primal problem. (10%)

4. The starting and current tableaux of a given problem are shown. Find the values of the unknowns *a* through *l*. (10%)

Starting Tableau

	x_1	x_2	x_3	x_4	x_5	RHS
Row(0)	*a*	*-1*	*3*	*0*	*0*	*0*
	b	*c*	*d*	*1*	*0*	*6*
	-1	*2*	*e*	*0*	*1*	*1*

Current Tableau

	x_1	x_2	x_3	x_4	x_5	RHS
Row(0)	*0*	*1/3*	*j*	*k*	*l*	*4*
	g	*2/3*	*2/3*	*1/3*	*0*	*f*
	h	*i*	*-1/3*	*1/3*	*1*	*3*