Sensitivity Study and Design Procedure for FRP Wrapped Reinforced Concrete Circular Columns

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Abstract: This paper presents the theoretical results of sensitivity studies performed on a simply supported FRP wrapped RC beam-column subjected to an axial load and to equal end moments. These sensitivity studies were performed to investigate the effects of the various parameters on the strength of the FRP wrapped RC beam-columns. The parameters used in the investigation include the unconfined concrete strength, the steel ratio, the thickness of FRP wraps, and the section diameter. A total of 23 cases were studied. Interaction equations were also developed in this work to provide a simplified and practical tool for engineers to evaluate the ultimate strength of the FRP wrapped columns. It is shown that the developed interaction equations for predicting the ultimate strength are reliable.

Keywords: fiber-reinforced polymers; reinforced concrete; sensitivity study; interaction diagram.

1. Introduction

Fiber Reinforced Polymers wraps have received significant attention for use in civil infrastructure due to their unique properties, such as the high strength-to-weight ratio and stiffness-to-weight ratio, corrosion and fatigue resistance, and tailorability. It has been widely accepted that FRP wraps increase both ultimate strength and ductility of reinforced concrete columns. Most of research works in this field focus on the ductility of columns, since they are concerned with the upgrade of existing RC columns in the seismic areas [1-3]. In particular, a design equation has recently been proposed to determine the optimal thickness of FRP wraps based on the needed increase of ductility [4]. In all these studies, FRP wraps are mainly placed in either the potential plastic hinge region or over the entire member with different thickness and the strength issue is not a major concern.

In the present study, the strength of RC columns wrapped with FRP wraps is investi-
gated. The scope of this work is on columns wrapped with an FRP layer of constant thickness over their entire length. The ultimate strength that defines the combinations of the axial load and the end moment that a FRP wrapped RC column can resist is of great interest in design. In order to predict such strength, a constitutive model for FRP wrapped concrete needs to be developed. Several models developed based on experimental results have been proposed by different researchers [5-7]. More recently, a numerical approach to trace the complete FRP wrapped concrete model has been developed by Spoelstra & Monti [8]. In this approach the model by Pantazopoulou & Mills’s [9], which was originally developed for unconfined concrete, is extended by introducing an iterative procedure to satisfy the continuously increasing confining pressure. Although the model suggested by Spoelstra & Monti appears to be theoretically reasonable, when compared with other models, it is not easy to use and is time-consuming. Besides, in the development of the FRP wrapped concrete model, there is no “exact” theory available. The only actual basis for comparison is the curves derived from test results. This is due to the complexities associated with the microstructure of concrete and with the crack propagation uncertainties during load application. Therefore, experimental results from different studies are used in this work to develop a suitable FRP wrapped concrete model.

This paper is intended to study the effects of design parameters on the ultimate strength and to provide a better understanding of the behavior of FRP wrapped RC columns by means of a sensitivity study. Interaction equations are subsequently proposed to provide a practical method for engineers to evaluate the ultimate strength of the FRP wrapped columns. These equations might be considered as interim guidelines in ultimate strength prediction until design code equations are developed.

2. Sensitivity studies

Sensitivity studies in terms of interaction curves have been performed to investigate the effects of design parameters on the ultimate strength of the FRP wrapped RC columns. However, to achieve this, the stress-strain models of steel and FRP wrapped concrete must first be defined. A widely used elastic-perfectly plastic constitutive model of steel is adopted. As for FRP wrapped concrete, a model is adopted which is based on experimental data obtained from different sources [7, 10-12]. This bilinear-shaped constitutive model was developed in previous work and is presented in Cheng et al. [13]. Detailed information about the \( \sigma-\varepsilon \) model for FRP wrapped concrete can be found in this reference.

\[
\sigma = \frac{(E_c - E_p)\varepsilon}{1 + \left(\frac{(E_c - E_p)\varepsilon}{f_0}\right)^n} + E_p\varepsilon
\]  

(1)

where \( E_c \) is the initial tangent modulus of unconfined concrete, \( E_p \) is the second slope (plastic slope) of the stress-strain curve, \( f_0 \) is plastic stress at the intercept of the plastic slope with the stress axis, and \( n \) is a curve-shaped parameter.

With the known constitutive models, the interaction curves for FRP wrapped columns have been developed. The parameters used in sensitivity studies include the unconfined concrete strength, the steel ratio, the thickness of FRP wraps, and the section diameter. A simply supported FRP wrapped RC column subjected to an axial load \( P \) and to equal end moments \( M \) has been considered. The detailed configuration of the cross section of the member is shown in Figure 1. The basic properties of the member, for the so-called the standard case herein, have been chosen as follows:
1. Unconfined concrete strength: $f'_c = 34.5$ MPa (5 ksi).
2. Steel ratio: $\rho = 3\%$.
3. Thickness of FRP wraps: $t = 7.62 \times 10^{-3}$ m (0.30 in).
4. Section diameter: $D = 0.915$ m (36 in).
5. Column length: $L = 5.5$ m (216 in).

![Diagram of FRP wrapped RC beam-column](image)

**Figure 1.** A simply supported FRP wrapped RC beam-column and its configuration of the cross section

The tensile strength, the elastic modulus, and the thickness of the FRP wrap have been taken as 1035 MPa (150 ksi), 69000 MPa (10000 ksi), and $1.27 \times 10^{-3}$ m/layer (0.05 in/layer), respectively. These properties are consistent with the product SCH 41S/Tyfo S Epoxy (Carbon Fiber) made by TYFE Company. Therefore, six layers are used in the standard case. Table 1 gives the various cases studied in the form of a matrix.

### 2.1. Results

Figures 2 through 5 show the interaction diagrams for the various design parameters considered in this study. Results for FRP wrapped columns are shown with solid lines, while the corresponding unwrapped cases are shown with dashed lines. In each case, one design parameter is varied, while the other parameters are assumed to be constant.

![Interaction Curves](image)

**Figure 2.** Interaction Curves for Different Concrete Strength (under $\rho = 3\%$, $t = 7.62 \times 10^{-3}$ m, $D = 0.915$ m)
Table 1. Sensitivity study matrix

<table>
<thead>
<tr>
<th>Variables</th>
<th>20.7</th>
<th>27.6</th>
<th>34.5</th>
<th>41.4</th>
</tr>
</thead>
<tbody>
<tr>
<td>f’c (MPa): f</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ρ (%): s</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>t (×10^-3 m): t</td>
<td>1.27</td>
<td>2.54</td>
<td>3.81</td>
<td>5.08</td>
</tr>
<tr>
<td>D (m): d</td>
<td>0.610</td>
<td>0.915</td>
<td>1.220</td>
<td>1.525</td>
</tr>
</tbody>
</table>

Note: The bold numbers represent the standard case.

It is found in all figures that the ultimate strength of the unwrapped member increases significantly by applying FRP wraps. This finding has been widely accepted and is reconfirmed here. From Figure 2, it can be seen that the ratio between the strength of a FRP wrapped RC column and the strength of its unwrapped counterpart increases as concrete strength increases. For example, for concrete strength of 20.7 MPa (3 ksi), a 23 % increase in the pure axial load is found, while a 34 % increase is observed for concrete strength of 41.4 MPa (6 ksi). However, it should be noted that this finding is only valid for normal strength concrete, not for high strength concrete, since the test data used in the development of the FRP wrapped concrete model are only valid for normal strength concrete.

Figure 3 shows the effect of varying the steel ratio. As expected, for a fixed number of layers of FRP wraps, the ultimate strength increases as the steel ratio increases. It is also found that the rate of increase in strength is lower for higher levels of steel ratio.

Figure 4 shows the effect of number of layers of FRP wraps. As expected, the ultimate strength of the member increases as the number of layers of FRP wraps increases. It is also observed that the rate of increase is fairly constant. This is because of the increase in confinement, which is proportional to the thickness of FRP wraps. Approximately, a 4 % increase in strength is found by increasing one layer of the FRP wraps.

Figure 3. Interaction Curves for Different Steel Ratios (under f’c = 34.5 MPa, t = 7.62×10^-3 m, D = 0.915 m)
The effect of the cross section on the ultimate strength of a FRP wrapped column is shown in Figure 5. It should be noted that in all cases, while the section diameter is changed, the slenderness ratio (L/D) is maintained. The ultimate strength of a FRP wrapped member increases as the section diameter increases even though the confinement decreases for a member with a larger section. This is because the decrease on the ultimate strength due to the confinement reduction is offset by the increase in strength due to the increase in section diameter. Ultimately, it is found that the rate of increase in strength with respect to the unwrapped counterpart is smaller for a larger section diameter.

**Figure 4.** Interaction Curves for Different Number of Layers of FRP Wraps (under $f'_c = 34.5$ MPa, $\rho = 3\%$, $D = 0.915$ m)

**Figure 5.** Interaction Curves for Different Section Diameters (under $f'_c = 34.5$ MPa, $\rho = 3\%$, $t = 7.62 \times 10^{-3}$ m)

3. **Interaction equations**

Due to the nonlinearity of their moment-curvature relationships, the analysis of FRP wrapped RC columns requires an iterative procedure. As a result, numerical methods are needed to obtain the solutions. An analysis based on Newmark’s method to de-
termine the ultimate strength of FRP wrapped columns has been formulated, verified, and implemented in a computer program. However, such an analysis is time-consuming and cumbersome, and it requires a computer program to be used. Therefore, it is desirable to develop interaction equations that are able to predict the ultimate strength with high accuracy and with minimum effort.

The normalized variables used in this work are defined as

\[ m = \frac{M}{M_0}; \quad \phi = \frac{\phi}{\phi_0}; \quad p = \frac{P}{P_0} \]  \hfill (2)

where \( M_0 \) is the ultimate bending moment without the presence of axial force (i.e., pure bending moment); \( \phi_0 \) is the ultimate curvature corresponding to \( M_0 \); and \( P_0 \) is the ultimate axial force without the presence of bending moment (i.e., pure axial force).

### 3.1. Basic assumptions

A method based on the average flow moment is adopted in the development of the interaction equations. This method has been successfully applied to concrete-filled steel tubular beam-columns by Chen & Rentschler [14]. The following two assumptions are made in the implementation of this method:

1. The moment-curvature relationship of an FRP wrapped RC column is assumed to be elastic-perfectly plastic, as shown in Figure 6.

2. The stress-strain models for FRP wrapped concrete and steel are assumed to be elastic-perfectly plastic, as shown in Figure 7.

The main concept of average flow moment is to approach the exact solution by setting the upper and lower bound values. A typical non-dimensional moment-curvature relationship, as the one shown in Figure 6., can be divided by two regions, i.e., linear elastic region and elastic-plastic region. These two regions are separated by a point which corresponds to an initial yield moment \( m_{yc} \) and an initial yield curvature \( \phi_{yc} \).

![Figure 6. Idealization of nondimensional moment-curvature relationship](image)
The ultimate point \((m_{yc}, \varphi_{pc})\), called plastic limit moment and plastic limit curvature, defines the state that the concrete fiber farthest from the neutral axis reaches its maximum compressive strain at the time of fracture of the FRP wraps. Therefore, the ultimate point represents the failure state of the cross section of the FRP wrapped RC column. If \(m_{yc}\) is used for the idealized flow moment, the ultimate strength of the column will be lower than the actual one. This represents a lower bound solution. On the other hand, if \(m_{pc}\) is used as the flow moment, the solution will be higher than the actual one, thus providing an upper bound solution. Therefore, a satisfactory average flow moment \(m_{mc}\), which must fall somewhere between \(m_{yc}\) and \(m_{pc}\), has to be chosen so that the ultimate strength of the column can be predicted. This average flow moment and can be expressed as follows.

\[
m_{mc} = m_{pc} - f(m_{pc} - m_{yc})
\]

where \(f\) is a parameter function. As \(f = 0\), \(m_{mc} = m_{pc}\), which provides the upper bound solution, while \(f = 1\), \(m_{mc} = m_{yc}\), which gives the lower bound solution. The parameter function \(f\) is considered as a function of the axial force, the design variables, and the boundary conditions of the column. It can be further expressed as

\[
f = f_1(p, L, t, f'_y, D) \cdot f_2(\text{B.C.})
\]

where \(p\) is the normalized axial force. The abbreviation B.C. stands for Boundary Conditions. The function \(f_1\) accounts for the effects of \(p\), \(L\), \(t\), \(f'_y\), and \(D\), while the function \(f_2\) considers the effects of the boundary conditions.

On the other hand, to simplify the derivation of the interaction equations, the assumption of elastic-perfectly plastic stress-strain models for both FRP wrapped concrete and steel is made in this method, as shown in Figure 7, where \(f'_y\) is the steel yield stress and \(\zeta f'_{cu}\) is the modified maximum stress of FRP wrapped concrete. It should be noted that \(\zeta\) is a factor used to account for the second ascending part of the real stress-strain model of FRP wrapped concrete. Once these assumptions are made, the ultimate strength of the FRP wrapped column can be easily computed by means of a linear elastic analysis. The proposed design procedure based on the developed interaction curves is discussed next.

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**Figure 7.** Idealized stress-strain relations for FRP wrapped concrete and steel: (a) FRP wrapped concrete, (b) steel
3.2. Design procedure

1. Calculation of $P_0$:

$P_0$ is the ultimate axial force of FRP wrapped RC columns without the presence of the bending moment and is expressed as follows.

$$P_0 = f_{cu}^* A_c + f_y A_s$$  \hspace{1cm} (5)

where $A_s = \pi (r_2^2 - r_3^2)$, $A_c = \pi r_1^2 - A_s$, and $f_{cu}^*$ is the ultimate compressive stress of FRP wrapped concrete. $A_c$ and $A_s$ are areas of the concrete section and longitudinal steel, respectively. The quantity $r_i$ is defined as illustrated in Figure 8(a).

2. Calculation of $m_{pc}$ and $M_0$:

A typical FRP wrapped RC circular column section is shown in Figure 8. It consists of three components: FRP wrap, concrete, and steel. The FRP wrap serves mainly as confinement for concrete and its effects on concrete have been implicitly accounted for in the stress-strain model of concrete. Thus, in this study, the equivalent section shown in Figure 8(a) does not show the FRP wraps. Furthermore, for simplicity, the longitudinal reinforcement is replaced by a ring of the same material with the same area as the longitudinal reinforcement. Therefore, the equivalent section (Figure 8(a)) replaces the original section and is referred herein as the composite section. Figure 8 shows the composite section and its decomposed parts. Each of these component sections is a solid circular section and can be represented by Figure 8. The shaded area $A_i$ is the compressive area of the section. $\varepsilon_i$ refers to the location of the neutral axis, while $\bar{\varepsilon}_i$ represents the centroid of the compressive area of the section.

The value of $\varepsilon_i$ is measured from point 0 with sign consideration. The absolute value of $\varepsilon_i$ should always be less than or equal to the corresponding value of $r_i$, the radius of the section, i.e., $|\varepsilon_i| \leq r_i$.

As both FRP wrapped concrete and steel are fully yielded over the cross section, the axial load $P$ and the plastic limit moment $M_{pc}$ can be expressed by

$$P = \zeta f_{cu}^* A_1 - \zeta f_{cu}^* A_2 - f_y (A_{cc} - 2A_2) + f_y (A_{ss} - 2A_3) + \zeta f_{cu}^* A_3$$  \hspace{1cm} (6a)

$$M_{pc} = \zeta f_{cu}^* A_1 \bar{\varepsilon}_1 - \zeta f_{cu}^* A_2 \bar{\varepsilon}_2 + 2f_y A_2 \bar{\varepsilon}_2 - 2f_y A_3 \bar{\varepsilon}_3 + \zeta f_{cu}^* A_3 \bar{\varepsilon}_3$$  \hspace{1cm} (6b)

where

$$A_{cc} = \pi r_2^2 \quad , \quad A_{ss} = \pi r_3^2 \quad , \quad A_i = r_i^2 \left( \theta_i - \frac{1}{2} \sin(2\theta_i) \right)$$  \hspace{1cm} (6c)

$$\bar{\varepsilon}_i = \frac{1}{2} \left[ \sin \theta_i - \frac{3}{4} \sin(3\theta_i) \right]$$  \hspace{1cm} (6d)

and

$$\zeta = \begin{cases} 0.81 & \text{for } |\varepsilon_i| \leq 0.96r_i \\ (-0.21r^2 + 0.03r + 1)^{0.5} \geq 0.81 & \text{for } 0.96r_i < |\varepsilon_i| \leq r_i \end{cases}$$  \hspace{1cm} (6e)

and where $r = r_1 - \frac{|\bar{\varepsilon}_i|}{\bar{\varepsilon}_i - \bar{\varepsilon}_2}$, $\theta_i = \cos^{-1} \left( \frac{\varepsilon_i}{r_i} \right)$.
It should be noted that $P$ and $M_{pc}$ are functions only of $e_i$, i.e., of the location of the neutral axis. After the determination of $M_0$ and $P_0$, $m_{pc}$ and $p$ are then calculated using Eqs. (2) and (6).

3. Determination of $\phi_0$:

$\phi_0$ is the ultimate curvature corresponding to $M_0$. Based on the moment-curvature relations developed, the expression of $\phi_0$ is found to be a function of $M_0$, $t$, and $f_c'$. Therefore, it is assumed to be expressed as follows.

$$\phi_0 = (10.404 M_0^{-0.51}) \left(\frac{t}{0.00762}\right) \left(\frac{f_c'}{34.5}\right)^{-1.39} \quad (7)$$

4. Determination of $m_{yc}$ and $\varphi_{yc}$:

As stated previously, the point $(m_{yc}, \varphi_{yc})$ divides the moment-curvature curve into the elastic region and elastic-plastic region. It has been found that the axial load has a major influence on the moment capacity and on the initial stiffness of the moment-curvature curve. Therefore, for simplicity, it is appropriate to consider $m_{yc}$ and $\varphi_{yc}$ as functions of the axial load $p$. The proposed expressions are shown as follows.

$$m_{yc} = \frac{m_{pc}}{\beta}, \quad \beta = \begin{cases} 44.184 p^3 - 21.759 p^2 + 4.766 p + 1.215 & \text{for } p \leq 0.6 \\ 15973 p^3 - 40313 p^2 + 31992 p - 75.507 & \text{for } 0.6 \leq p \leq 1.0 \end{cases} \quad (8)$$

$$\varphi_{yc} = \begin{cases} 0.583 p^3 - 0.421 p^2 + 0.0527 p + 0.0301 & \text{for } p \leq 0.4 \\ 0.201 p^3 - 0.418 p^2 + 0.324 p - 0.0547 & \text{for } 0.4 \leq p \leq 1.0 \end{cases} \quad (9)$$

5. Calculation of parameter function $f$:

The parameter function $f$ is considered as a function of the axial force, the design variables, and the boundary conditions, and is expressed by Eq. 4. The proposed function has the following form:

$$f_1 = -0.172 + 2.229 \left(\frac{p}{5.5}\right)^{0.3} - 0.001 \left(\frac{L}{5.5}\right)^5 + 1.198 \left(\frac{p}{0.00762}\right)$$

$$- 2.357 \left(\frac{f_c'}{34.5}\right) - 0.434 \left(\frac{D}{0.915}\right)$$

for $L/D \leq 20$
Figure 8. FRP wrapped RC cross section characteristics
where $0 \leq f_1 \leq 1$

The $f_2$ function is determined based on the boundary conditions. Chen & Rentschler have shown that $f_2 = 1$ for a hinge-ended column and $f_2 = 0.5$ for a fixed-ended column [14].

4. **Verification**

To validate the proposed interaction equations Eqs. (3~10), a verification study is carried out. The cases listed in Table 1 are once again considered. Approximate solutions obtained by the proposed interaction equations are shown in Figure(9) ~ Figure(13) with discrete dots, while the solutions obtained by the developed program are plotted using solid lines for comparison. The latter is referred to as the exact solution. It is found that the proposed interaction equations predict well the ultimate strength of the FRP wrapped RC columns.

It should be mentioned that Figure 13 shows the predicted and exact solutions for the column with 6 different L/D ratios. According to the ACI code, the slenderness effect of a column needs to be considered if $(kL/r) > 34-12$ $(M_1/M_2)$, where $k$ is an effective length factor, $r$ is the radius of gyration, and $M_1/M_2$ is the ratio of the end moments. For the case of a simply supported column with equal end moments as the one studied herein, $(kL/r) > 22$ or $(L/D) > 5.5$ holds for a slender column. Therefore, all columns but two fall into the group of slender columns.

5. **Conclusions**

Sensitivity studies are performed to investigate the effects of the various parameters on the behavior of the FRP wrapped RC columns. The parameters include unconfined concrete strength, steel ratio, thickness of FRP wraps, and section diameter. A total of 23 cases are studied. Interaction equations are also proposed to provide a simplified and practical tool for engineers to evaluate the ultimate strength of the FRP wrapped RC columns. The conclusions drawn from this study can be summarized as follows.

1. FRP wraps significantly increase the ultimate strength of RC columns.
2. In terms of the effect of concrete strength on the ultimate strength of an FRP wrapped RC column, it has been found that the rate of increase in strength with is higher as the concrete strength is increased. However, in terms of the effect of the steel ratio, it has been found that the rate of increase in strength is lower for higher levels of steel ratio.
3. In terms of the effect of the thickness of the layer of FRP wraps, it has been found that the rate of increase in strength increases proportionally to the increase in FRP layer thickness. This is due to the increase in confinement, which is proportional to the thickness of FRP wraps. As for the effect of the cross section on the ultimate strength, the increase ratio in strength with respect to its unwrapped counterpart is smaller for a larger section diameter.
4. The proposed interaction equations have been verified for a simply supported FRP wrapped RC column subjected to equal end moments. It has been found that these equations for predicting the ultimate strength are reliable.
Figure 9. Interaction curves for different concrete strengths

Figure 10. Interaction curves for different steel ratios
Figure 11. Interaction curves for different number of layers of FRP wraps

Figure 12. Interaction curves for different section diameters
Figure 13. Interaction curves for different column lengths

References


