UNIT 7

SECONDARY CONSOLIDATION

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7.1 Introduction

One-dimensional consolidation of soils is conveniently divided into three portions, viz., (1) initial compression, (2) primary consolidation, and (3) secondary consolidation. Such a classification involves the clear implication that the three effects proceed in that order. Initial one-dimensional compression of saturated clays is negligible because of the low compressibility of the pore water. Initial compression in the laboratory is likely to result from apparatus deflections, partial saturation of the soil, and improper seating of the loading head. Primary consolidation (Fig. 7.1) is the more or less S-shaped curve on a plot of settlement vs. log time and is assumed to result from the gradual dissipation of excess pore water pressures. Secondary consolidation is then taken to mean all of the compression that follows completion of primary consolidation.

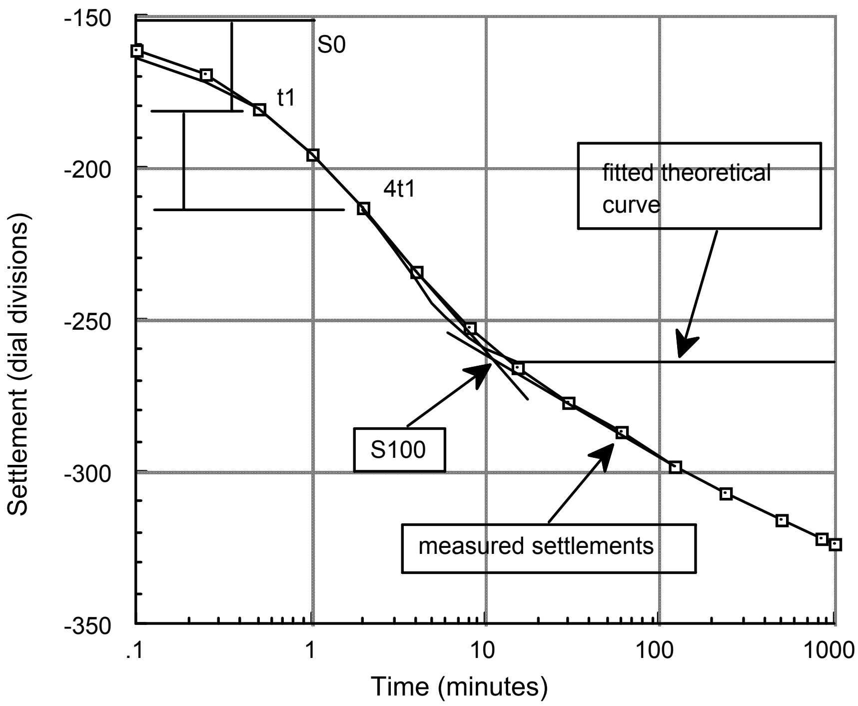

Fig. 7.1 Comparison of Measured Settlement Curve and the Fitted Primary Theoretical Curve (Terzaghi's Theory) for San Francisco Bay Mud

Such a model of behavior would appear to be over simplified because it suggests that secondary consolidation should not begin until the entire soil stratum has completed primary consolidation. A slightly more sensible assumption is that secondary consolidation in any given differential element of soil should begin as soon as that element has essentially completed primary consolidation. Thus, elements of soil adjacent to drainage boundaries should begin secondary effects almost immediately after loading. We would then expect secondary effects to appear during the supposed primary stage.

In addition, the tendency to think of secondary effects as just involving added compression, seems oversimplified. It makes just as much sense to think of secondary effects as involving some type of resistance to volume change, a resistance that occurs during primary consolidation and continues at a decreasing rate after primary consolidation has finished.

In the discussion to follow it will be convenient first to examine what is known about secondary effects that follow primary consolidation and then to consider secondary effects during primary consolidation. Such a separation is inherently unreasonable, as was indicated previously, but it is in accord with the information generally found in the literature and is convenient for our initial considerations.

We will give limited consideration to secondary consolidation theories.

7.2 Laboratory Observations

7.2.1 Introduction

We are really interested in secondary effects in the field but field observations are corrupted by a number of problems, e.g., inability to duplicate tests, and thus cannot be used to investigate the phenomena involved with secondary effects. We will give limited consideration to field observations when we discuss case histories.

7.2.2 Secondary consolidation following primary consolidation

The most obvious evidence of secondary compression is the settlement that occurs after the conclusion of primary consolidation. For example, in Fig. 1 the sample underwent a secondary settlement equal to 60% of the primary settlement. Clearly, ignoring secondary settlement can lead to substantial errors. Examination of data from numerous laboratory tests indicate that the secondary settlement may range from less than 10% of the total settlement to essentially 100%.

In order to discuss the amount of secondary compression we need some suitable parameters. We could examine secondary effects by considering the fraction of total compression that occurs in the secondary range, perhaps using the secondary compression ratio defined in the introduction. However, such ratios are not very meaningful because the amount of secondary consolidation depends on the total time under load (Fig. 7.1) and because they give no idea as to the total settlement involved. It is more convenient to measure the secondary consolidation by the slope of a curve of settlement, void ratio, or strain versus the log of time. I will follow Mesri (1973) and use the following symbols:

$$C_\alpha = de/d(\log t) \tag{7.1}$$

$$\varepsilon_\alpha = d\varepsilon/d(\log t) \tag{7.2}$$

The definition using void ratios (Eq. 7.1) should probably be preferred because void ratio is defined in terms of the constant height of solids. When linear strain is used (Eq. 7.2) it is necessary to specify which total height is used in the definition.

Linear curves of settlement versus log time have been observed in laboratory tests for periods of time up to almost seven years (Cox, 1936). Linear curves have also been observed in the field (Buisman, 1936, Thompson and Palmer, 1951; Simons, 1957; Bjerrum, Johnson, and Ostenfeld, 1957). However, our experience is that secondary slopes almost invariably tend to level out with time and only appear as straight lines when observations are made over a restricted time span. Similar observations have been made by Haefeli and Schaad (1948), Lo (1961), and many others. On the other hand, Thompson and Palmer (1951) published laboratory curves that turn down with time for a period of time and our laboratory tests often indicate such a trend for overconsolidated soils. However, it seems probable that such curves will flatten out, given enough time.

Long-term laboratory tests are difficult to perform and interpret for a variety of reasons. For example, what effect do minor building shocks have on secondary slopes? What is the effect of the growth of organic matter due to the presence of oxygen diffusing in from the atmosphere? What effect does diffusing oxygen have on the physical components of the soil, e.g., conversion of Ferris iron (gray color sample) to ferric iron (red colored sample)?

The slope of the secondary compression curve probably depends on a number of independent variables but none of these have been isolated in a definitive theory up to the present time. Thus, it is more useful to examine experimental data and look for correlation between the slope of the secondary compression curve and other variables. Some of these correlations are discussed in the following sections.

Natural moisture content. Mesri (1973) shows that there is a rather crude correlation between ε_α and natural water content, given approximately by the following equation:

$$\varepsilon_\alpha = w/100 \tag{8.3}$$

where ε_α and w are both expressed in percentage. Values reported by Mesri (1973) lie within a factor of 3 of this relationship.

Intuitively one might expect the slope of the secondary compression curve to vary with the liquid limit or the plasticity index but such correlations do not seem to have worked out well.

Consolidation pressure. Data from the literature on the effect of consolidation pressure on the slope of the secondary compression curve are conflicting with some authors indicating a tendency for an increase in slope and others a tendency for a decrease. Mesri (1973) summarizes a number of these references.

My experience is that, for any given soil, C_α maximizes at a stress just beyond the maximum previous consolidation pressure. The slope is generally very low for stresses substantially less than the maximum previous consolidation pressure and may be essentially zero for soils that are cemented. Once the soil becomes normally consolidated there is a general tendency for reduction in C_α as the consolidation pressure increases. The range in values of C_α seem to depend on the sensitivity of the soil. For quick clays C_α becomes very large at the point where the soil structure undergoes maximum breakdown under the applied stresses, i.e., where the compressibility is highest (Fig. 7.2).

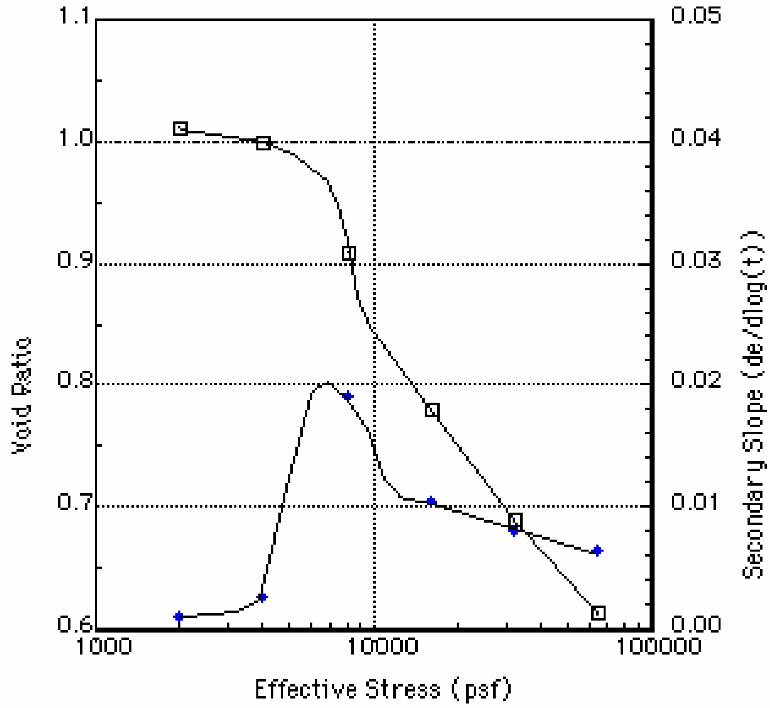

Fig. 7.2Void Ratios (upper curve) and Secondary Slopes (lower curve) for a Sample of Highly Sensitive Leda Clay from Canada

Remolding. Remolding of an undisturbed sample typically causes the secondary slope to be more-or-less independent of the consolidation pressure and to be less than the peak value encountered near the maximum previous consolidation pressure for undisturbed samples. For example, secondary slopes for the tests shown in Fig. 7.2 are plotted in Fig. 7.3. For the Leda clay, it seems apparent that secondary effects are strongly influenced by the geometric arrangement of the soil particles.

Load-increment ratio. In the standard method of performing laboratory consolidation tests the soil sample is subjected to incremental loading where each increment equals the total stress previously on the sample, i.e., the load increment ratio is one, and each load is applied for 24 hours. A few studies (Leonards and Girault, 1961) suggest that the slope of the secondary compression curve depends on the load increment ratio but most studies (Wahls, 1962; Newland and Allely, 1960) indicate no significant relationship. The relative magnitudes of primary and secondary consolidation will of course be influenced greatly by the load increment ratio.

A consolidation curve (e-log(σ)), defined at the end of primary consolidation is shown in Fig. 7.4. In the normal circumstance, we expect a significant amount of primary compression followed by secondary compression, as indicated in Fig. 7.4. However, the assumption in Fig. 7.4 is that loading occurred from a point corresponding to the end of primary consolidation under the previous load. What if we allow an amount of secondary compression, under the previous load, that exceed the primary compression under the next load? Such a circumstance could clearly develop if, say, we increased the load by 10% instead of by 100% (varied the load increment ratio). Our experience is that there will be no

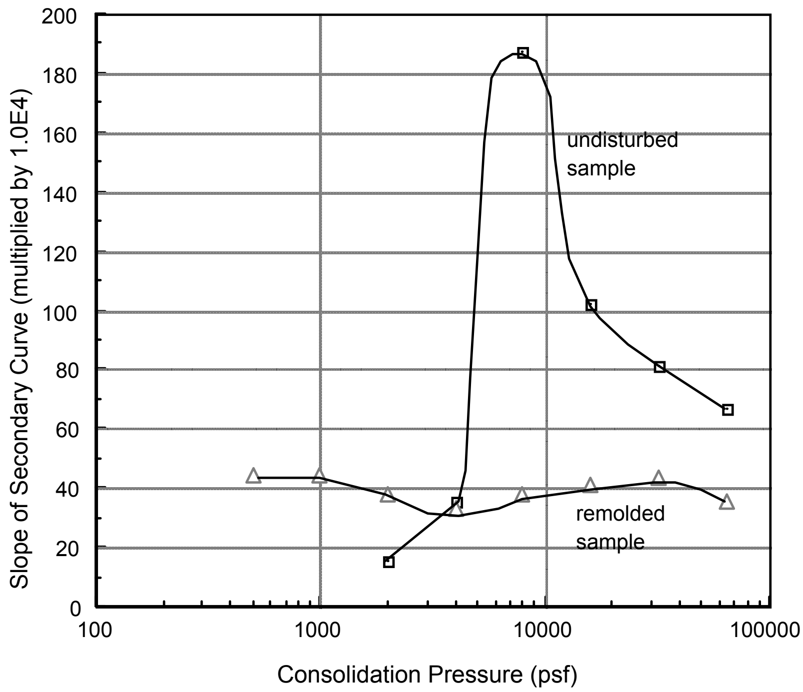

Fig. 7.3 Secondary Slopes for the Undisturbed and Remolded Samples of Leda Clay

apparent primary consolidation under a load if the amount of secondary compression under the previous load represents a major part of the primary compression under the next load.

Further, if there is major secondary compression under one load, the following S-$\sqrt{t}$ curve often starts out relatively flat and then slowly turns downwards into the usual linear portion. If an even larger amount of secondary compression occurred under the previous load, the following S-$\sqrt{t}$ curve may have an shape similar to a typical S-log(t) curve.

<u>Temperature</u>. It seems to be the consensus of researchers that the slope of the secondary compression curve increases as the temperature increases (Gray, 1936; Lo, 1961). Most people believe that secondary compression is a creep phenomenon and thus its amount (or rate) increases as temperature increases because of the reduction in the apparent viscosity ofthe contacts between particles.

However, most of the data in existence were obtained using regular consolidation cells where temperature changes cause volume change of the ring itself

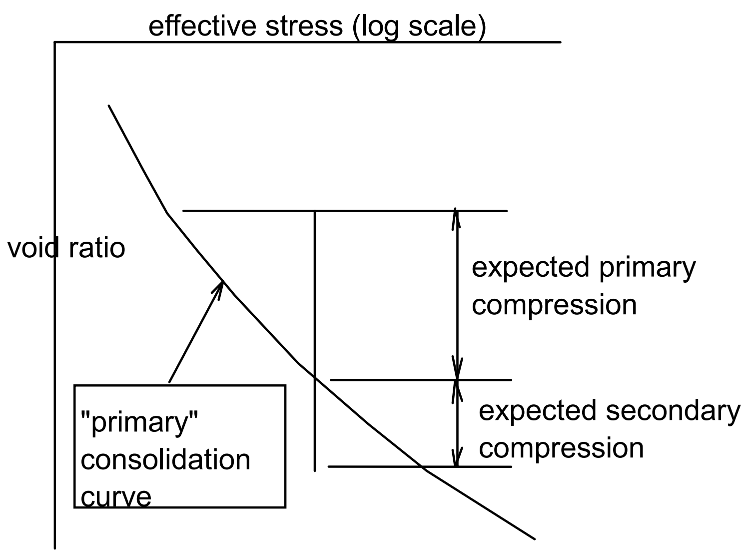

Fig. 7.4 Primary and Secondary Compression under an Applied Stress

and thus introduce extraneous effects. For example, increasing the temperature can cause expansion of the ring, a reduction of the normal stresses between the soil and the ring and thus a reduction in ring friction with an attendant increase in applied stress to the soil and thus more consolidation. Similarly, a reduction in temperature may increase ring friction and cause a reduction in the apparent rate of secondary compression.

Gray (1936) showed that the position of the virgin consolidation curve is a function of temperature, having lower void ratios at higher temperatures. Thus changes in temperature cause primary consolidation or a tendency to rebound and such effects are superimposed on top of apparent secondary compression curves. Thus tests of the obvious type cannot be used to determine the effect of temperature on secondary slopes.

A more definitive study was performed by Habibagahi (1969) who found that the effect of temperature on secondary slope was smaller than the normal scatter in his data. Until better data become available it appears that temperature is not a significant variable and therefore engineers need not be concerned about the fact that laboratory temperature are often significantly different form the temperature of the soil in the field, at least so far as secondary effects are concerned.

Organic matter. The consensus of the geotechnical engineering field seems to be that essentially all consolidation of highly organic soils is of the secondary type with very steep secondary compression curves. The steep slopes of secondary compression curves for organic soils are accounted for indirectly using Mesri's correlation with water content because the water content of highly organic soils are among the largest of any encountered in the field. Thus, water contents of highly organic soils range from about 100% to 4000%, which are beyond the water contents typically encountered with inorganic soils.

State of stress. We have not collected definite data on this point but I have had the impression that secondary slopes are significantly lower for a sample subjected to isotropic stresses as opposed to say a one-dimensional state of stress. One-dimensional states of stress involve application of a gross (as opposed to microscopic) shearing stress and this shearing stress may influence secondary compression. We have tried to investigate the influence of applied shearing stresses by consolidating samples in a triaxial apparatus with separate control of the axial stress using the loading piston. Unfortunately, normally trivial errors due to piston friction and membrane leakage caused intolerable errors for the very small secondary slopes we were encountering (we used sedimented samples of kaolinite). The tests could be repeated with mercury as the cell fluid and an essentially frictionless loading system but we have never had time to run the tests.

7.2.3 Secondary effects occur during consolidation

A commonly used measure of the applicability of Terzaghi's theory of consolidation to laboratory data is the precision with which his theory can be fit to the so-called "primary" part of the settlement-time curve. Although a poor fit between theory and measurement would indicate an inadequate theory, a good fit is not definitive because a similarly good fit could be obtained using relatively simple mathematical functions that could not, for example, be used to predict the effect of drainage distance on consolidation times. Further, it is hard to imagine that no secondary effects occur during primary consolidation when secondary was occurring under the previous load and is occurring after primary on this load, thus suggesting that it is a continuous phenomenon. Further, even if secondary effects follow primary consolidation, primary effects should disappear near drainage boundaries almost at once and thus secondary effects should start in some parts of the soil sample before primary has been completed everywhere in the sample.

Probably the simplest way of estimating the accuracy of Terzaghi's theory is look for a variable in his theory that is subject to an independent measurement and which controls the time rate of settlement. That variable is clearly the coefficient of permeability. Terzaghi's original consolidation apparatuses (1921, 1925a, 1925b) were designed in such a way that permeability tests could be performed with the soil under load. Terzaghi (1923) reported the following comparison between measured and computed coefficients of permeability for a clay with the void ratio of 0.86:

measured coefficient of permeability = 1.65×10^{-7} cm/min
computed coefficient of permeability = 1.69×10^{-7} cm/min

Casagrande and Fadum (1944, p. 480) reported: "Using the semilogarithmic plot for time curves, the writers have always been able to check, with satisfactory accuracy, the results of direct permeability tests with the k-values computed from the time curves, provided that the slope of the secondary compression branch was not so steep as to obliterate the bend in the curve indicating the transition from primary to secondary compression." They neglected to define the meaning of "satisfactory accuracy".

A number of comparisons of measured and computed permeabilities were made at MIT between about 1940 and 1960. Taylor (1942) worked with remolded samples of Boston Blue Clay. He had a considerable amount of scatter in his data but generally found that the measured coefficient of permeability was somewhat higher than the value calculated with Terzaghi's theory when he used a load increment ratio of one, e.g., in one test at a pressure of 1 kg/cm^2 he measured 1.05×10^{-8} cm/sec whereas the value computed with Terzaghi's theory was 0.85×10^{-8} cm/sec. O'Neil (1954) used undisturbed samples of Boston Blue Clay and found that the measured coefficient of permeability was as much as ten times the computed value at large void ratios but the error diminished as the void ratio was reduced. All work at MIT involved use of the square root of time fitting method.

A number of comparisons of measured and computed permeabilities were made at the Universities of Illinois and Texas over a period of years. For soils remolded at high water contents or sedimented from suspension, correlations between computed and measured permeabilities were always about as shown in Fig. 7.5. For consolidation on the virgin curve the measured permeabilities are generally about 20% to 40% higher than the values computed using Terzaghi's theory. These numbers seem to be in keeping with a substantial number of other observations. For stresses less than the maximum previous consolidation pressure the computed permeabilities are considerably less than the measured values in all cases with the ratio of measured to computed permeabilities generally being in the range of about 2 to 10 but with values as high as 100 possible. Clearly, Terzaghi's theory should only be applied to normally consolidated clays.

Load-increment ratio Taylor (1942) reported a series of one-dimensional consolidation tests on samples of remolded Boston Blue Clay in which the load increment ratio was varied from 0.2 to 2.4. As an example of his data, the average calculated coefficient of permeability varied from 0.8×10^{-9} cm/sec at a load increment ratio of 0.2 up to 12.0×10^{-9} cm/sec at a

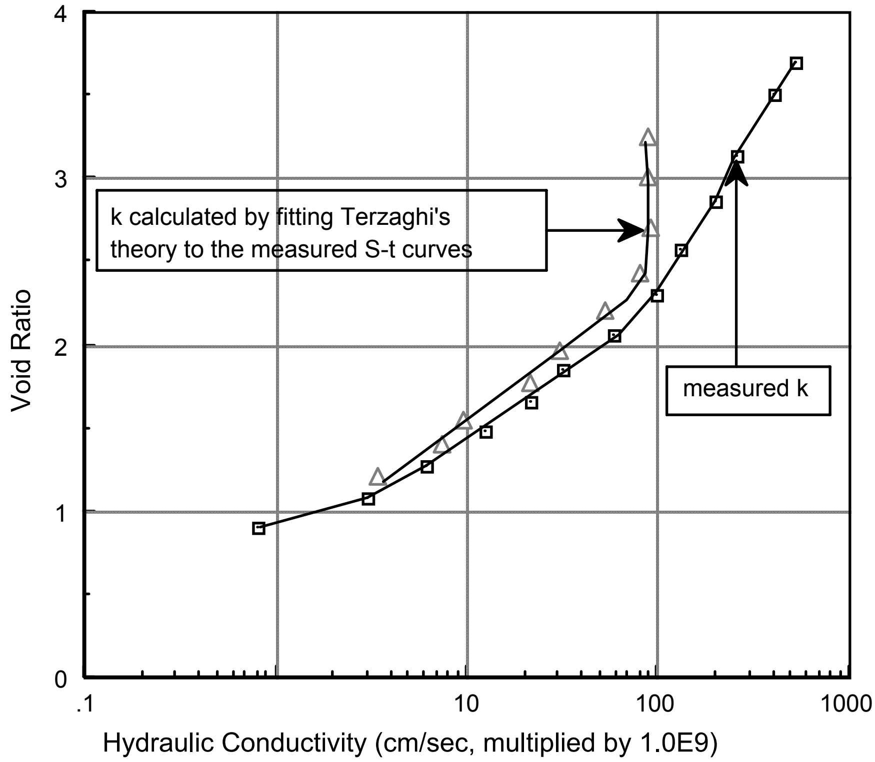

Fig.7.5 Comparison of Measured and Computed (Terzaghi's theory) Hydraulic Conductivities for an Illitic Clay Prepared by Remolding at a Water Content of 150% (LL=86%)

load increment ratio of 2.4, a ratio of 15 times, for loading from a single pressure. The measured coefficient of permeability averaged about 10.5 x 10^{-9} cm/sec.

If it is assumed that the void ratio at the point of 100% primary consolidation on any loading is independent of the load increment ratio (Casagrande, 1944; Casagrande and Fadum, 1944; Leonards and Ramaiah, 1959; Hamilton and Crawford, 1959) then the secondary compression that occurs under one load increment reduces the amount of primary consolidation on the next increment as noted previously. Thus, part of the effective stress increase for the new loading corresponds to virgin consolidation and part corresponds to compression along a reloading curve where previous data have shown a substantial difference between measured and computed coefficients of permeability. Thus, significant amounts of secondary compression on one load increment or, alternatively, use of a small load increment ratio to reduce the primary compression on the next load, effectively introduces a degree of overconsolidation and results in an increased difference between measured and computed coefficients of permeability.

7.3 Effects of Secondary Compression on Subsequent Consolidation Behavior

Truly definitive data do not exist to show the effects of secondary compression on subsequent consolidation behavior. However, data referenced earlier suggests that secondary compression on any given loading reduces the amount of primary compression on the next loading and had an effect similar to overconsolidation. An interesting set of observations was published by Lake (1961) using remolded peat. He performed consolidation tests on samples with total thicknesses ranging from 0.5 to 4 inches. Data for the extreme cases (0.5 and 4 inch thicknesses) have been replotted in Fig. 7.6 (numbers were scaled from Lake's plots so they may differ slightly from the original data).

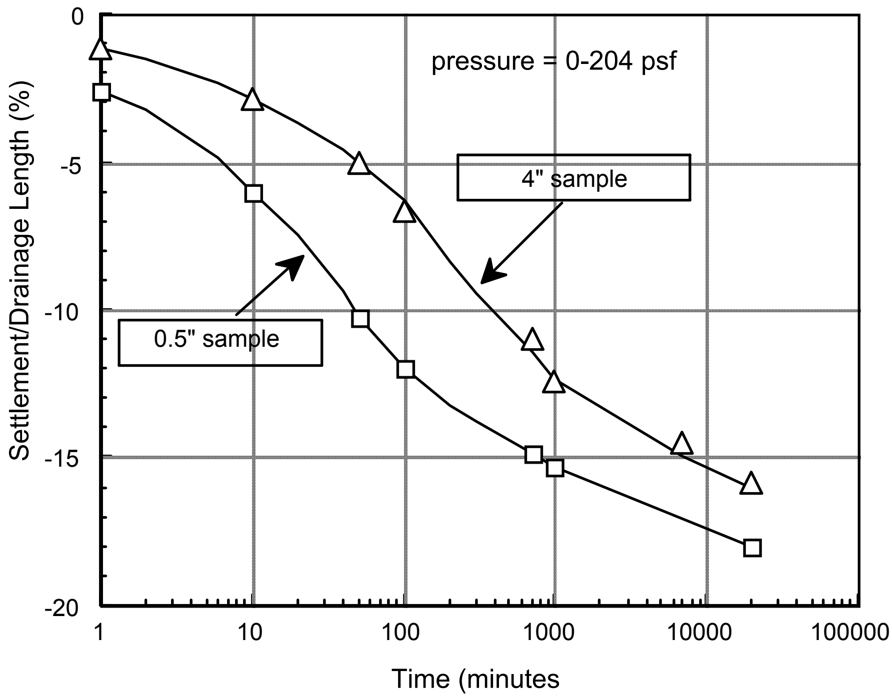

Fig. 7.6a Initial Application of Pressure (Lake, 1961)

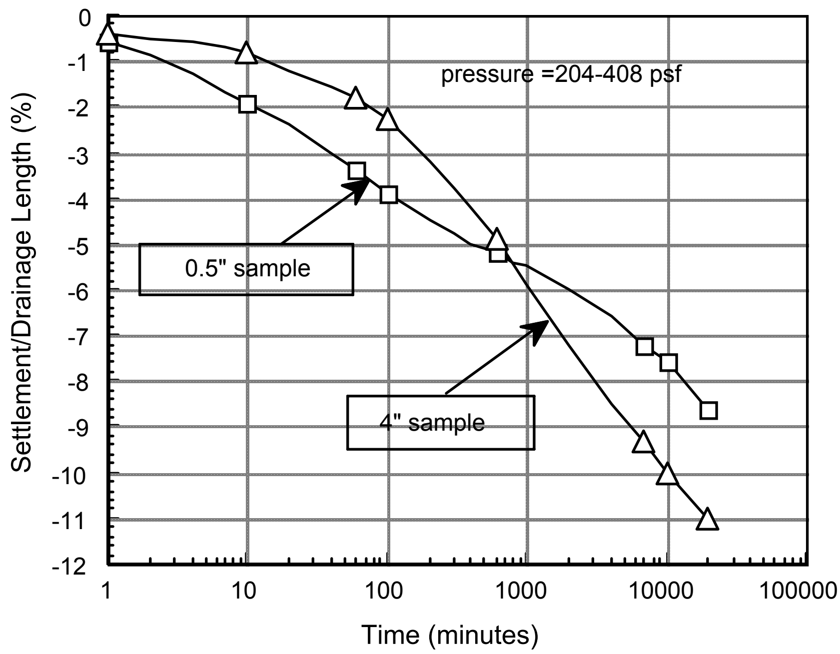

Fig. 7.6b Second Loading

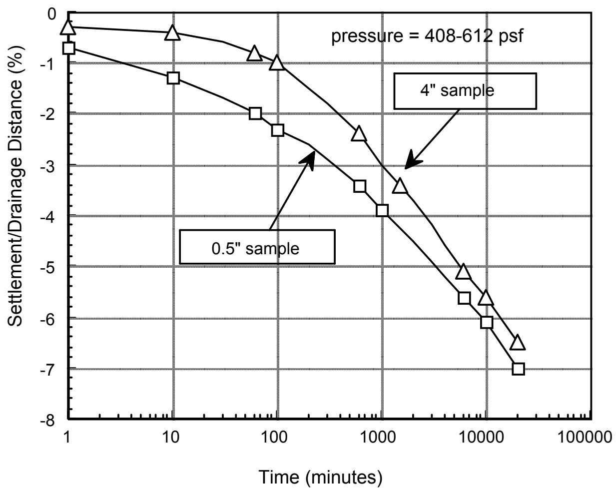

Fig. 7.6c Third Loading (Lake, 1961)

Under the first load it seems clear that both samples underwent primary consolidation. Further, the times for any given strain are larger for the thicker sample, as would be expected from primary consolidation theory. However, the primary times should be in the ratio of $4^2/0.5^2 = 64$ but the actual ratio seems to be closer to about eight so primary theory didn't work precisely. Primary and secondary effects are less easily separated on the second load and disappear altogether for all subsequent loads.

I selected the data for the sample with a thickness of 1 inch and replotted the data in Fig. 7.7. Primary and secondary consolidation were separable on the first two loads and the compression curve presumably starts at an effective stress of zero because of the fact that the peat was remolded. Thus, a virgin consolidation curve can be sketched in as shown. Under the first load the soil was not subject to any prestress prior to loading and thus the primary consolidation curve was well defined. For the second load, 50% of the total compression was secondary compression under the first load and thus the primary compression curve is badly defined. For the third load, it appears that about 60% to 70% of the primary compression was removed by secondary compression on the previous load and primary and secondary effects cannot be distinguished (Fig. 7.6). For higher pressures it does not appear possible to separate primary and secondary compression.

Although most of the Lake's data indicate that the peat is undergoing exclusively secondary consolidation, it seems probable that had the final pressure been applied in a single increment there would have been a well defined primary curve with secondary compression following it. I believe that most studies of consolidation of peat have involved similar distortions of the data and thus the conclusions are not necessarily relevant to practical problems. The obvious conclusion is that consolidation tests with highly organic soils might best be performed by applying the final consolidation pressure in one increment (unless such loading would result in extrusion of the soil), or apply loads until the primary curve is defined and then load at once rather than allowing substantial amounts of secondary consolidation to occur.

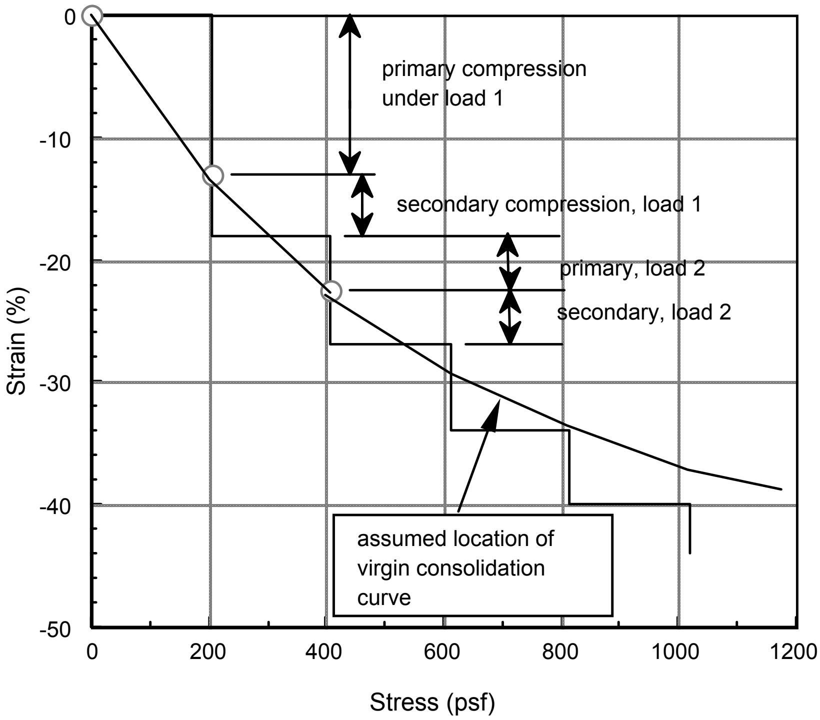

Fig.7.7 My Interpretation of the Consolidation Data Collected by Lake (1961) for a Sample with a Total Height of 1.0 inch

7.4 Causes of Secondary Compression

It seems clear that the common view of secondary effects as being simply settlement following primary settlement is an oversimplification. Secondary effects occur during primary consolidation, as indicated by permeability data, as well as afterwards.

Secondary effects probably result from different mechanisms in different soils. Some simple mechanisms include:

1. Soils have void spaces of widely differing sizes. In some soils, water may drain from the larger voids in accord with primary theory and then water may more slowly squeeze out of smaller voids, producing a secondary effect.

2. In organic soils containing plant matter, water may similarly squeeze out of the voids in accord with primary theory and then water may squeeze slowly out of the individual plant cells, through the cell walls, at a slow rate, producing a secondary effect.

3. In more granular soils, the shearing stresses between particles may be of a viscous nature. When primary consolidation is occurring, the rate of movement between particles is a maximum and the shearing stresses will then cause a maximum resistance to volume change. The retarded rate of compression explains the observation of lower fitted hydraulic conductivity compared to the measured values. The fitted value must be lower than the real value to explain the retarded rate of consolidation. Creep following primary consolidation would also be expected.

4. Some clay particles may be surrounded by water that is adsorbed onto the surfaces by local electrical effects. This adsorbed water may grade imperceptibly outwards into normal liquid water. As particles are pressed more closely together during primary consolidation, there would be expected to be a viscous resistance to volume change developed, which might produce apparent secondary effects.

5. Field cases that show large secondary effects sometimes involve the settlement of comparatively narrow embankments with low factors of safety against failure. In some cases, some part of the time-dependent settlement may be due to mass movement of subsoil out from under the embankment due to the high shearing stresses and overall soil viscosity.

6. Some case histories of settlement of wide embankments involve a shallow highly compressible soil and deeper less compressible soils. Apparent secondary settlement may actually represent delayed primary consolidation of the relatively incompressible soil which cannot drain until the overlaying, more compressible layer, has consolidated somewhat.

7. In the case of some organic soils, the hydraulic conductivity of the soil decreases by more than an order of magnitude during consolidation under a given load. Consolidation naturally proceeds more rapidly initially but then at a decreasing rate because of the reduction in hydraulic conductivity, thus producing an apparent secondary effect.

8. Highly non-linear stress-strain curves can produce settlement-time behavior that looks like primary consolidation followed by secondary consolidation.

It is unknown whether, or to what extent, any of the above mechanisms control secondary effects. Secondary effects have been measured with dry sands, on a sample composed of cloth rags, on samples made up of packed threads, and on various other materials, so we need not expect any simple mechanism to explain the observations.

7.5 Secondary Consolidation Theories

Under the circumstances of the foregoing discussion of secondary mechanisms, it seems unlikely that a truly quantitative secondary theory, comparable to the primary consolidation theory, can be developed. Instead, we may seek some sort of analytical formulation that will yield predictions that can be fit to observations and perhaps extended to field conditions. A number of theories have been proposed. The simplest theory containing combined primary and secondary effects, and the only one that we will examine, was developed by Gibson and Lo (1961)

7.5.1 Gibson and Lo's theory

Gibson and Lo (1961) used a rheologic model composed of a spring in series with a combination of a spring and dashpot (Fig. 7.8). In the model, the effective stress is applied to the top of the primary spring with a resulting instantaneous compression of that primary spring (compressibility = "a"). For a linearly elastic body (the spring is then called a Hookean element), the compressibility of the primary spring becomes m_V if we define compressibility using total height, or a_V if we use the height of solids. The load in the

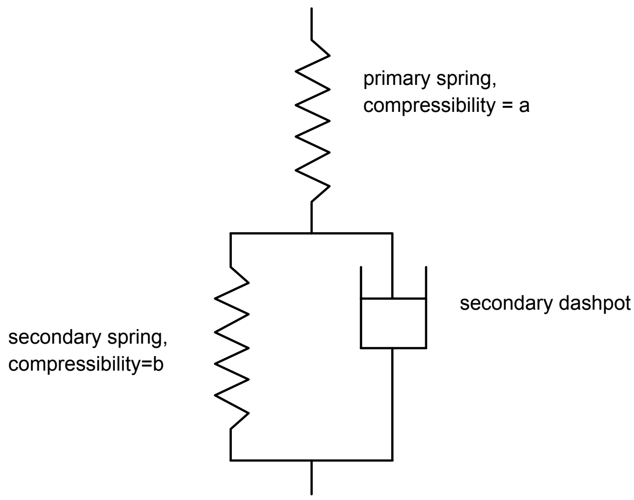

Fig. 7.8 Secondary Consolidation Model of Gibson and Lo (1961)

primary spring is also transferred to the secondary spring and dashpot (a Kelvin body). Instantaneously, the load is entirely carried in the dashpot because it is incompressible. However, fluid escapes from the dashpot and it compresses, thus causing the secondary spring to compress, and thus take load. When the secondary spring takes load, that amount of load is removed from the dashpot, when then compresses more slowly. Thus, the load is gradually transferred from the dashpot to the spring and, at time infinity, all of the load is in the spring. Secondary compression is then the compression of the Kelvin body.

The force carried by the dashpot is a function of the loading rate. You can think of the dashpot as having viscosity. If the dashpot viscosity is infinite, then no secondary effects are visible in a finite measurement time, i.e., no load is ever transferred from the dashpot to the secondary spring. If the viscosity is zero, then the rheologic model reduces to two springs in series and we would simply observe primary compression (remember that dissipation of pore water pressure and generation of effective stress is covered separately, e.g. by Terzaghi's theory). For intermediate values of dashpot viscosity, we would expect the dashpot to creep slowly, transferring load to the spring "b", and involving a time dependent compression.

You should think of the model as representing the behavior of any differential element of soil so the behavior of the whole mass would be found by adding up the responses of all of the elements, i.e., by integration.

To develop the theory, let the primary compressibility be represented by a linear spring with a compressibility defined using "a":

$$a = \frac{\varepsilon_a}{\sigma} \tag{7.4}$$

where ε_a is primary strain and σ is effective stress (thus, "a" is the same as m_v).

In a similar fashion, the compressibility of the secondary spring is b:

$$b = \frac{\varepsilon_b}{\bar{\sigma}_b} \tag{7.5}$$

where $\bar{\sigma}_b$ is used to denote the stress applied to the secondary spring.

For the dashpot, assume that the strain rate is a linear function of the applied stress (a Newtonian dashpot)

$$\frac{d\varepsilon_b}{dt} = \lambda\, \bar{\sigma}_\lambda \tag{7.6}$$

where $\bar{\sigma}_\lambda$ is the stress in the dashpot. With this formulation, the strain rate increases as λ increases so λ is like an inverse viscosity.

The effective stress, $\bar{\sigma}$, in the primary spring is thus equal to the sum of the stresses in the secondary spring and the dashpot:

$$\bar{\sigma} = \bar{\sigma}_b + \bar{\sigma}_\lambda = \frac{\varepsilon_b}{b} + \frac{d\varepsilon_b/dt}{\lambda} \tag{7.7}$$

The solution of Eq. 7.7 is found from:

$$\varepsilon_b = \int_0^t \bar{\sigma} \exp\left(-\frac{\lambda}{b}(t-\tau)\right) d\tau \tag{7.8}$$

In Eq. 7.8, τ has been used in place of the original t so we can use t as an upper limit of integration.

The total strain, ε, is the sum of the primary and secondary strains:

$$\varepsilon = \varepsilon_a + \varepsilon_b = a\bar{\sigma} + \int_0^t \bar{\sigma} \exp\left(-\frac{\lambda}{b}(t-\tau)\right) d\tau \tag{7.9}$$

We can return to the original derivation of Terzaghi's differential equation and put it into the same symbolism used in Eq. 7.9, to obtain:

$$\frac{\partial \varepsilon}{\partial t} = \frac{k}{\gamma_w} \frac{\partial^2 q}{\partial z^2} \tag{7.10}$$

The differential equation governing both primary and secondary compression is then:

$$\frac{k}{\gamma_w} \frac{\partial^2 q}{\partial z^2} = a \frac{\partial \bar{\sigma}}{\partial t} + \lambda \bar{\sigma} - \frac{\lambda^2}{b} \int_0^t \bar{\sigma} \exp\left(-\frac{\lambda}{b}(t-\tau)\right) d\tau \tag{7.11}$$

Gibson and Lo applied the boundary conditions that $\bar{\sigma}\,(z=0,t) = q_o$ and $\frac{\partial \bar{\sigma}}{\partial t} = 0$ at $z=H$, i.e., there is a constant effective stress at the top boundary and there is an impervious lower boundary. For a single step loading, Gibson and Lo applied Laplace transforms, the convolution theory, and contour integrals, to obtain the following solution:

$$S = (a+b)q_oH \{(1 + \frac{8}{\pi^2} \sum_{n=odd}^{\infty} \frac{1}{n^2} [(\frac{K-x_1}{x_1-x_2} \exp(-x_2t) - (\frac{K-x_2}{x_1-x_2})\exp(-x_1t))]) \quad (7.12)$$

where S is the time dependent settlement, q_0 is the instantly applied load, H is the drainage distance, t is time since loading,

$$K = \frac{a}{a+b} K_1 \quad (7.13)$$

$$K_1 = \frac{n^2\pi^2c_v}{4h^2} \quad (7.14)$$

$$c_v = \frac{k}{a\gamma_w} \quad (7.15)$$

$$x_1 = \frac{(\alpha+K_1)+\sqrt{(\alpha+K_1)^2-4\beta K_1}}{2} \quad (7.16)$$

$$x_2 = \frac{(\alpha+K_1)-\sqrt{(\alpha+K_1)^2-4\beta K_1}}{2} \quad (7.17)$$

$$\alpha = \lambda (\frac{1}{a} + \frac{1}{b}) \quad (7.18)$$

$$\beta = \frac{\lambda}{b} \quad (7.19)$$

Settlement now depends on the drainage distance H and time t, and on the soil properties "a", b, k (hydraulic conductivity), and λ. We could perform analyses immediately using these parameters. However, it may be more convenient to introduce the dimensionless parameters:

$$M = 1 + \frac{b}{a} = \frac{\text{total compressibility}}{\text{primary compressibility}} \quad (7.20)$$

$$x_1 = \frac{(4MN+n^2\pi^2)+\sqrt{(4MN+n^2\pi^2)^2-16Nn^2\pi^2}}{2} \quad (7.21a)$$

$$x_2 = \frac{(4MN+n^2\pi^2)-\sqrt{(4MN+n^2\pi^2)^2 -16Nn^2\pi^2}}{2} \quad (7.21b)$$

$$N = \frac{\lambda}{b} \frac{H^2}{c_v} \quad (7.22)$$

$$T = \frac{c_v t}{H^2} \tag{7.23}$$

$$U = 1 + \frac{8}{\pi^2} \sum_{n=odd}^{\infty} \frac{\left(\frac{n^2\pi^2}{M} - x_1\right)\exp\left(\frac{-x_2 T}{4}\right) - \left(\frac{n^2\pi^2}{M} - x_2\right)\exp\left(\frac{-x_1 T}{4}\right)}{n^2(x_1 - x_2)} \tag{7.24}$$

Equation 7.24 is defined in such a way that:

$$S = S_u U \tag{7.25}$$

Note that Gibson and Lo used "a" in place of m_v so primary compression is just $m_v \Delta\bar{\sigma} H$ and total compression is given by:

$$S_u = (a+b)\Delta\bar{\sigma} H \tag{7.26}$$

I performed a set of analyses for a laboratory sample that is doubly drained and is one inch thick. The sample has c_v=0.1 sq.ft./day, m_v(primary)=1.0E-5 sq.ft./lb, m_v(secondary)=4.0E-6 sq.ft./lb, and a range in values of λ from 0.0 (the secondary dashpot never allows any outflow) to 1.0 (the secondary dashpot has essentially no resistance). Representative U-log(t) curves are shown in Fig. 7.9.

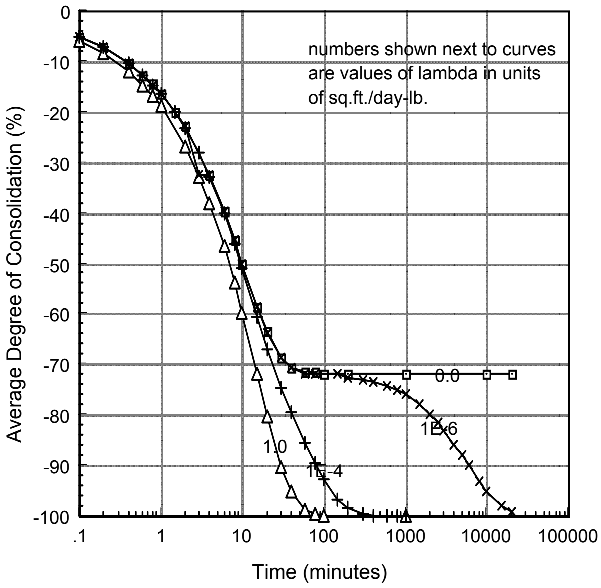

Fig. 7.9 Settlement-Log(Time) Curves for a Sample with a Thickness of 1.0 Inch, with Double Drainage, in Accord with the Theory of Gibson and Lo (1961)

The shape of the curves in Fig. 7.9 is not precisely what we sometimes observe. However, note that:

1. if the secondary dashpot doesn't allow any outflow, we get a curve that looks like primary consolidation.

2. if the secondary dashpot has no resistance, we again get a curve looking like a primary curve but we get the compression of both the primary and secondary springs.
3. the model indicates a finite amount of total compression at large times (a limiting total settlement), in accord with measurements.
4. if we had an incompressible primary spring, the curves would yield a curve of S-log(t) that would start out relatively flat and then turn downwards, in accord with many observations.
3. many of the laboratory measurement of linear secondary slopes involve less than a log cycle of time. For such a small range of time, even Gibson and Lo's curves might be approximated as linear.

7.5.2 Laboratory observations

Lo (1961) reported that measured secondary effects generally end within three weeks of loading in the laboratory but noted problems with building vibrations (his laboratory was near an elevated train track in London), temperature fluctuations, and ring friction. Dhowian and Edil (1980) performed long-term tests with peat and concluded that secondary effects tend to disappear after about 300,000 minutes (200 days).

Lo (1961) noted that the b/a ratio (ratio of secondary compression to primary compression) was generally small for remolded clays, of the order of 5% to 15%, for the clays he tested. For

undisturbed samples, he found that b/a ranged from 15% to 25% for the normally consolidated clay but was more nearly in the range of 40% to 150% for stresses near those where the stress-strain curve suddenly steepens (the "maximum previous consolidation pressure").

7.5.3 Field application of Gibson and Lo's theory

The goal of a consolidation analysis is to predict field behavior. We performed an analysis using Gibson and Lo's theory using properties that Gibson and Lo obtained for the Grangemouth clay. The properties were:

$a = 3.96E\text{-}5$ sq.in./lb
$c_v = 0.1$ sq.ft/day
$b = 3.26E\text{-}6$ sq.in./lb
$\lambda = 1.0E\text{-}6$ sq.ft./(day-lb)

An analysis was performed for a laboratory sample with double drainage and a total thickness of one inch, and a field case involving double drainage and a total thickness of twenty feet. In order to compare the results, settlement was replaced by the average degree of consolidation (Eq. 7.25). The U-log(t) curves are compared in Fig. 7.10.

The behavior of the laboratory sample and the field layer are apparently different in that there is a clear secondary effect in the laboratory but none in the field. The explanation is as

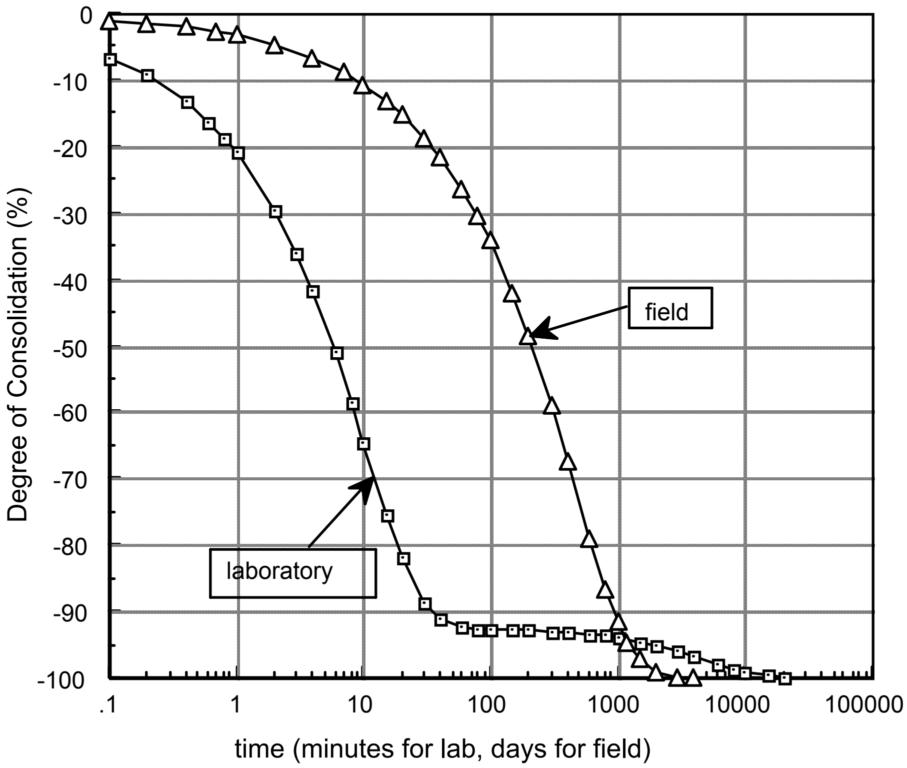

Fig. 7.10 Comparison of Laboratory and Field U-Log(Time) Curves. The lab sample was 0.1-inch thick and the field layer was 20 feet thick, but with the same soil properties.

follows. In the field, primary consolidation is slow because of the layer thickness. As a result, effective stresses build up so slowly on most soil elements that negligible resistance develops in the dashpot and thus the secondary delay is eliminated.

The analysis leads to the assumption that we should calculate field settlements using the largest settlement measured in the laboratory rather than the 100% primary settlements.

Terzaghi's theory can be fit to the calculated lab and field curves. For the lab curve, primary consolidation ends at about U=92% so U is actually about 46% when primary consolidation is 50% completed (remember that I used Eq. 25 here to define U). The time for 50% primary consolidation is about 4.9 minutes and the backcalculated value of c_v is 0.100 sq.ft./day. The secondary dashpot apparently had little influence on the primary curve. If we had measured a field curve like the calculated one, we would be forced to assume that only primary consolidation occurred in the field. Accordingly, t_{50} is about 115 weeks=805 days and the fitted value of c_v is $(0.197)(10^2)/805 = 0.0245$ sq.ft./day. The lowered field value cannot be due to the dashpot causing delay because it had less influence in the field than in the lab. Accordingly, the cause must be the inclusion of the secondary compression as if it were part of the primary compression.

7.5.4 Problems with Gibson and Lo's theory

In the same way that we could list problems with Terzaghi's theory, we can list problems with the theory of Gibson and Lo. For example:

1. the theory should be expanded to include time-dependent loading, stratified soil deposits, large strains, etc.
2. we need a way of obtaining the parameters from laboratory tests.
3. In the same way that there are problems applying Terzaghi's theory when the stress-strain curve is non-linear and c_v varies, we also have the potential that the two springs in the Gibson and Lo model are also non-linear, and the dashpot response can be non-linear as well.

7.5.5 Other theories

We won't cover other theories. You can find a variety of theories in the literature, formulated with different rheological models. We should see potential application for some of these theories when we discuss case histories.

7.6 References on Secondary Consolidation

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