# UNIT 1

## ANALYSIS OF TOTAL SETTLEMENT OF AREAL FILLS

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## 1.1 Introduction

In this set of notes, consideration is restricted to the analysis of the total settlement of fills which are so wide, compared with the thickness of the compressible soils that we need not consider stress distribution. Such fills have been called **areal fills**. The application of areal fills to compressible soils typically generates pore water pressures and some of the pore water flows out of the soils leading to time-dependent volume change. Water flow and deformations are along only a vertical axis, so we can refer to the process of time dependent volume change as **one-dimensional consolidation**. This term will be applied to both swelling and compression although our interest will almost exclusively be in compression.

We will be concerned only with saturated, or nearly saturated, soils. Further, sands and gravels are generally so incompressible under one-dimensional loading that we will be concerned only with silts and clays.

Analysis of the time rates of consolidation and settlements resulting from concentrated loads will be deferred for later sets of notes.

## 1.2 Definition of Terms

In order to calculate one-dimensional settlements it is only necessary to have a one-dimensional stress-strain curve and know the initial and final stresses. Because conditions are one dimensional, we can utilize only the vertical stresses.

Measurements show that sudden application of pressure to a saturated soil, under conditions of no lateral strain, leads to a negligible immediate settlement, thus suggesting that settlements cannot be analyzed in terms of total stress. Measurements also show that the sudden application of load generates water pressures and that these water pressures dissipate as a time dependent settlement occurs. In the laboratory, when the pore water pressures have decreased back to zero, the rate of settlement diminishes to a relatively small value, i.e., the soil essentially comes to equilibrium. When the water pressure (u) is zero, the total stress($\sigma$) must be carried by the framework of mineral particles. The stress carried by the framework of mineral particles is called the effective stress, $\overline{\sigma}$ and for saturated soils it is given by (Terzaghi, 1936):

$$\overline{\sigma} = \sigma - u \qquad (1\text{-}1)$$

The stress-strain curves for soils in one-dimensional compression will always utilize the effective vertical stress.

One-dimensional (1-D) stress-strain curves can be measured in the laboratory. A 1-D $\bar{\sigma} - \varepsilon$ curve for a sample of clay which was sedimented in the laboratory is shown in Fig. 1.1a.

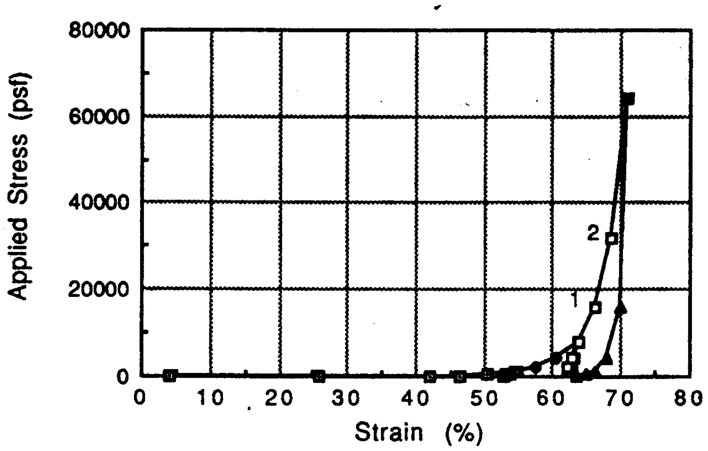


Fig. 1.1a Stress-Strain Curve for One-Dimensional Compression of Soft Clay

The $\bar{\sigma} - \varepsilon$ relationship is nonlinear and it is difficult to study behavior at low stresses while still retaining the plot at high stresses. To facilitate interpretation of the data, we decide to rotate the plot so you would move down on the curve for compression and up for swelling, just as in the field. The curve then appears as in Fig. 1.1b. The appearance of the plot is

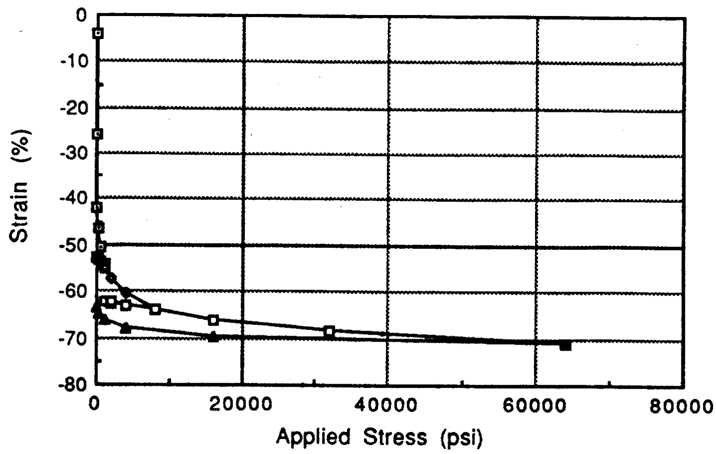


Fig. 1.1b Rotated Stress-Strain Curve for Soft Clay in One-Dimensional Compression

improved, perhaps, but the data at the low end of the stress scale still cannot be seen clearly because of the wide range of stresses used. This problem occurs in essentially all such tests. The obvious solution is to convert the stress axis to a log scale, as shown in Fig. 1.1c. Use of the log scale has also made the curves slightly more linear, although they are all still obviously curved.

To facilitate communication we will define a soil as being **normally consolidated** if the existing effective stress is the largest to which the soil has been subjected. For example, in Fig. 1.1c, the soil is normally consolidated between points 1 and 2, 3 and 4, and 5 and 6.

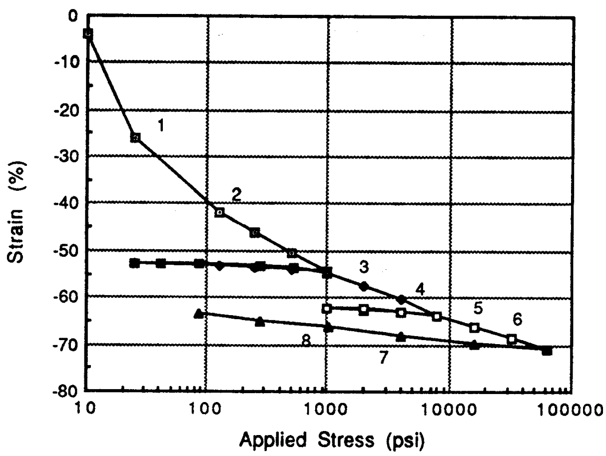


Fig. 1.1c Semi-Logarithmic Plot of Stress-Strain Curve Used to Provide More Detail at Small Stresses

The stress-strain curve is called a **virgin curve** for normally consolidated soil. If the existing effective stress is less than the maximum previous value then the soil is said to be **overconsolidated.** For example, the soil in Fig. 1c is overconsolidated in the region of point 7 to 8. The stress-strain curve is typically called a **reloading** or **recompression** curve if the effective stresses are increasing and a **swelling** or **rebound** curve if the effective stresses are decreasing.

As a side issue we may note that the definition of an **underconsolidated** clay is quite different. An underconsolidated clay is one that has finite excess pore water pressures and thus is not yet fully consolidated. An underconsolidated clay can be normally consolidated or overconsolidated.

The maximum consolidation pressure to which a soil has been subjected is denoted by $\bar{\sigma}_{\max}$. In general, $\bar{\sigma}_{\max}$ is only known in the case of laboratory tests.

## 1.3 One-Dimensional Settlement Analysis

In the case of an elastic column subject to a uniform axial stress $\Delta\bar{\sigma}$ we are used to calculating the compression ΔS, using Young's modulus (E):

$$\Delta S = L\Delta\varepsilon = L\frac{\bar{\sigma}}{E} = LC\Delta\bar{\sigma} \tag{1.2}$$

where L is the length of the column and C is the compressibility (1/E). A similar equation can be developed for soils. Define the coefficient of volume compressibility, $m_V$, as:

$$m_v = \frac{d\varepsilon}{d\bar{\sigma}} \tag{1.3}$$

Then, for a layer Δz thick, subject to a stress increase, $\Delta\bar{\sigma}$ the compression, ΔS, is:

$$\Delta S = m_V\,\Delta z\,\Delta\bar{\sigma} \tag{1.4}$$

which is analogous to Eq. 1.2.

Some engineers are bothered by the fact that the layer thickness (Δz) changes with effective stress and prefer to define a strain scale using the height of the solid mineral grains rather than the total height. The strain scale is denoted as Δe instead of ε and:

$$\Delta e = \frac{\Delta L}{L_s} \tag{1.5}$$

where $L_S$ is the height of mineral grains in the total height L. Because there is no change in height of mineral grains, ΔL is actually a change in height of voids, $\Delta L_V$. If we substitute $\Delta L_V$ for ΔL in Eq. 1.5 and multiply numerator and denominator by any arbitrary area of interest (typically a unit area), then:

$$\Delta e = \frac{\Delta V_v}{V_s} \tag{1.6}$$

where $\Delta V_V$ is the change in volume of voids and $V_S$ is the volume of solids. The form of Eq. 1.6 is such that it seems logical to define a variable e, which has been called the **void ratio**, as:

$$e = \frac{V_V}{V_S} \tag{1.7}$$

We may now choose to plot void ratio in place of strain, converting the plot in Fig. 1.1c to the one in Fig. 1.1d.

The strain scale can be Δe or just e, as in Fig. 1.1d. We now have the option of plotting the strain axis using ε, Δe or e.

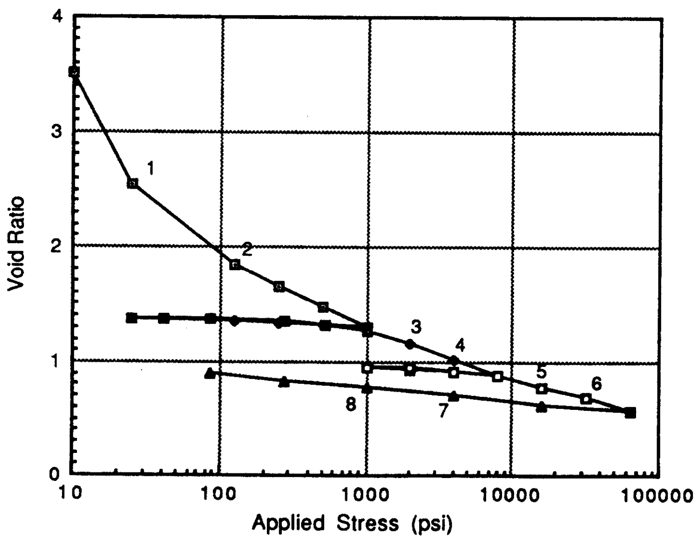


Fig. 1.1d Stress-Strain Curve for Soft Clay, Plotted using Logarithmic Stress Axis and Void Ratio for Strain

If we choose to plot e, then we can still use Eq. 1.5 to calculate a compression ΔS (ΔL in Eq. 1.5) as:

$$\Delta S = -L_S \, \Delta e \qquad (1.8)$$

The minus sign is used because we choose to have settlement increase as e decreases. It is generally preferable to replace the height of solids, $L_S$, with an expression involving the total height, L. We note that:

$$L = L_V + L_S \qquad (1.9)$$

We divide through Eq. 1.9 by $L_S$, note that $L_V/L_S = e$, and solve for $L_S$:

$$L_S = \frac{L}{1+e} \qquad (1.10)$$

Equation 1.10 is substituted into Eq. 1.8 to obtain:

$$\Delta S = -\frac{\Delta e}{1+e} L \qquad (1.11)$$

In Equation 1.11, L/(1 + e) is the constant height of solids so the e in the denominator must be defined when the layer thickness is L. Many engineers use the symbol L for the height of the zone of interest but put a sub-o on the e ($e_o$) to make it clear that this is the original void ratio. Most engineers drop the minus sign in Eq. 1.11 and define Δe to be positive.

Equation 1.11 is a general solution similar to:

$$\Delta S = \Delta \varepsilon \, L \qquad (1.12)$$

and does not involve any assumption about the shape of the stress-strain curve. If we assume a linear e - $\bar{\sigma}$ relationship, then we can define a coefficient of compressibility, $a_V$, as:

$$a_v = -\frac{de}{d\bar{\sigma}} \qquad (1.13)$$

and convert Eq. 1.11 to:

$$\Delta S = \frac{a_v}{1+e} \Delta\bar{\sigma} \cdot \Delta z \qquad (1.14)$$

a form written to be analogous to Eq. 1.4 obviously:

$$m_V = \frac{a_V}{(1+e)} \qquad (1.15)$$

Equations 1.4 and 1.14 are written using compressibilities, $m_V$, and $a_V$. They could just as easily be developed using modulii, such as E, and engineers in some countries do prefer modulii because the equations then come out looking like ones seen in mechanics classes and encountered in structural design.

For soils, ε - $\overline{\sigma}$ or e - $\overline{\sigma}$ curves can usually be assumed to be linear for only a small change in s on a semilogarithmic scale in Figs. 1.1c and 1.1d. Linearity has been improved slightly and the new curves are easier to use in the low-stress range. For a range in s where the ε-log s curve can be considered linear, the slope (R) is:

$$R = \frac{d\varepsilon}{d(\log\overline{\sigma})} = \frac{(\varepsilon_2 - \varepsilon_1)}{(\log\overline{\sigma}_2 - \log\overline{\sigma}_1)} \qquad (1.16)$$

Thus if the soil is at point 1 in the field and is to be loaded to point 2, then from Eqs. 1.12 and 1.16:

$$\Delta S = R\, L \log\frac{\overline{\sigma}_2}{\overline{\sigma}_1} \qquad (1.17)$$

In cases where the soil is consolidated through a reloading curve and out onto the virgin curve, the ε-log s curve may be approximated as two straight lines. For example, in Fig. 1.2, curve 1-2 can be drawn tangent to the reloading curve and 2-3 tangent to the virgin curve.

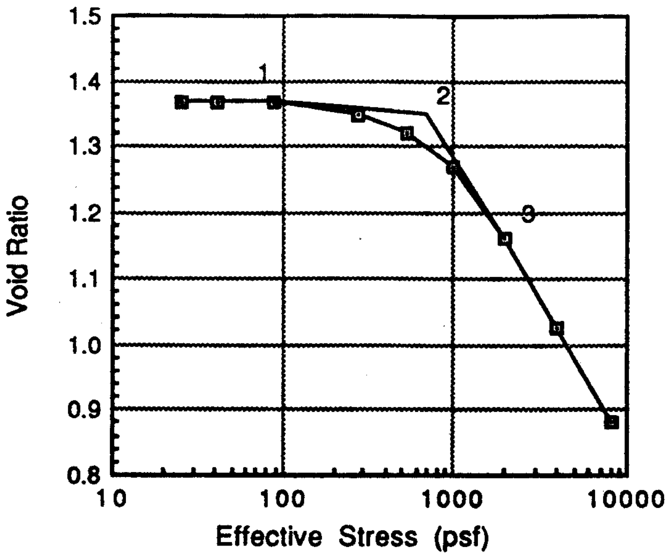


Fig. 1.2 Use of Bilinear Stress-Strain Curve to Approximate Actual Curve

If the reloading curve is drawn to start at the overburden stress, $s_o$, and ends at the final field stress, $s_f$, with the two straight lines intersecting at $s_i$ (same as point 2 in Fig. 1.2). Then:

$$\Delta S = R_r\, L \log \frac{\overline{\sigma}_i}{\overline{\sigma}_0} + R_c\, L \log \frac{\overline{\sigma}_f}{\overline{\sigma}_i} \qquad (1.18)$$

where $R_r$ and $R_c$ are the slopes (Eq. 1.16) of the reloading and virgin curves, respectively, and are termed the **recompression ratio** and **compression ratio** respectively. Note that the stress at the intersection of the two straight lines, $\overline{\sigma}_i$, is not the maximum previous consolidation pressure, $\overline{\sigma}_{max}$.

If the engineer chooses to plot void ratios instead of strains, then the slope of the consolidation curve is:

$$C = -\frac{de}{d(\log \overline{\sigma})} \tag{1.19}$$

For a range in void ratio where the e-log s relationship can be approximated as linear, Eq. 1.19 can be inserted into Eq. 1.11 to obtain:

$$\Delta S = \frac{C}{1+e} L \log \frac{\overline{\sigma}_2}{\overline{\sigma}_1} \tag{1.20}$$

For a reloading curve C is replaced with $C_r$, the **recompression index**, whereas for a virgin curve C is replaced with $C_c$, the **compression index**. For a bilinear curve:

$$\Delta S = \frac{C_r}{1+e} L \log \frac{\overline{\sigma}_i}{\overline{\sigma}_0} + \frac{C_c}{1+e} L \log \frac{\overline{\sigma}_f}{\overline{\sigma}_i} \tag{1.21}$$

It should be noted that most texts in soil mechanics use H in place of L but they also use H for a different dimension in time-rate-of-settlement analyses and thus may cause confusion.

In cases where the soil is overconsolidated but the applied load will consolidate the soil out onto the virgin curve, the analysis could be further simplified by assigning $R_r$ (or $C_r$) a value of zero so there is no reloading settlement and treating the soil as normally consolidated with $\overline{\sigma}_0$ set equal to $\overline{\sigma}_i$. The calculations are so simple, however, that use of the two straight lines (Fig. 1.2) with reasonable slopes seems proper.

Note that there is no reason why the designer shouldn't just approximate the whole curve as a series of short straight lines and rewrite equations such as Eqs. 1.18 and 1.21 into forms such as:

$$\Delta S = \sum_{j=1}^{N} R_j L \log(\frac{\overline{\sigma}_j}{\overline{\sigma}_{j-1}}) \tag{1.22}$$

and

$$\Delta S = \sum_{j=1}^{N} \frac{C_j}{1+e} L \log(\frac{\overline{\sigma}_j}{\overline{\sigma}_{j-1}}) \tag{1.23}$$

where i just denotes the point number on the stress-strain curve, with point 0 at the original field stress and point N being at the final field stress. A computer program would logically be used to perform the calculations. The designer can then track any desired curve.

An alternative, and preferable approach, is to input the coordinates of points on the stress-strain curve, and have the computer program find the void ratios or strains corresponding to the initial and final field stresses, by interpolation, and then apply Eqs. 1.11 or 1.12. We use this latter approach.

The designer clearly has a variety of options available. The use of the bilinear curves goes back to a time when slide rules were used for analyses and it would have been unreasonably laborious to try to track the entire stress-strain curve. With modern microcomputers, it makes little sense not to track the whole stress-strain curve.

## 1.4 Examples

**Site in Louisiana**

If there is only a single consolidation curve ($\bar{\sigma}$ - ε , $\bar{\sigma}$ - e) for a layer of soil then it is usually easiest to use the non-linear stress-strain curve directly in the analysis. For example, suppose that the stress-strain curves shown in Fig. 1.1 are the actual field curves, rather than laboratory curves. Suppose that the layer is five feet thick, the average initial effective stress is 135 psf, and we will apply a stress of 2000 psf. From the stress-strain curve in Fig. 1c, the initial strain is 42.2% and the final strain (assumed to be at 2135 psf) is 57.6% (both strains found by interpolation using the source data for the plots) so the settlement is (0.576-0.422) (5) = 0.77 feet.

**Site in New Hampshire**

As a further example of settlement analyses, consider a site in New Hampshire where the soil profile consists of three layers of compressible soils overlying a relatively incompressible sand. The properties of the compressible soils are as follows:

| Layer No. | Soil Description | Thickness ft. | γ' pcf | $R_r$ | $R_c$ | $\bar{\sigma}_i$ psf |
|---|---|---|---|---|---|---|
| 1 | fibrous peat | 10 | 4 | 0.06 | 0.50 | 450 |
| 2 | amorphous peat | 10 | 14 | 0.05 | 0.45 | 300 |
| 3 | organic silt | 5 | 31 | 0.05 | 0.30 | 1000 |

The site is to be covered with a wide fill and the top of the fill is to be at an elevation 20 feet above the elevation of the original ground surface, after consolidation is complete. The water table is at the elevation of the original ground surface and remains at that elevation. The fill will have total and submerged unit weights of 125 pcf and 70 pcf, respectively. The initial effective stresses at the centers of three layers are 20 psf, 110 psf, and 257.5 psf. The stress applied by the fill is (20) (125) = 2500 psf. Equation 18 is applied. For layer 1:

$$\Delta S_1 = (0.06)\,(10)\,\log\left(\frac{450}{20}\right) + (0.50\,(10)\,\log\left(\frac{2520}{450}\right) = 0.81 + 3.74 = 4.55 \text{ feet}$$

For layers 2 and 3 the compressions (ΔS) are 4.45 feet and 0.81 feet, so the surface settlement is 9.81 feet. Of course, there are now two changes. First, the final surface elevation is 10.19 feet, not 20 feet. If we assume that filling will continue to force the final surface elevation to be 20 feet, then more fill must be applied. Second, 9.81 feet of the fill is submerged. A second calculation is performed with 20 feet of fill above the water table and 9.81 feet below the water table. The settlement is then 10.93 feet. This process is repeated (the solution is an *iterative* one) until an acceptable level of accuracy is achieved. For the next two iterations the calculated settlements are 11.05 feet and 11.06 feet. The thickness of the fill is thus 31 feet.

In the above analysis, the assumption was made that the conditions in each layer can be represented by conditions at the center of the layer. To check the accuracy of this approach without engaging in an excessive amount of calculations in these notes, consider 20 feet of fill (γ = 125 pcf) applied to a layer of normally consolidated clay (L = 20 feet, γ' = 50 pcf, $R_c$

= 0.25) overlain by an incompressible layer that applies 100 psf. If the whole layer is used in the analysis:

S = (0.25) (20) log (3100/600) = 3.57 feet

To save work here we will not iterate to correct for submergence or addition of more fill. If the layer is broken into two 10-foot layers:

S = (0.25) (10) log (2850/350) + (0.25) (10) log (3350/850)
= 3.77 feet

As the layers are subdivided further the apparent accuracy increases.

A solution obtained by integration (infinite number of layers), but based on the assumption that the initial void ratio is independent of depth, is:

$$S = \frac{R}{\gamma'}(\bar{\sigma}_{to}\log\bar{\sigma}_{to} - \bar{\sigma}_{bo}\log\bar{\sigma}_{bo} - \bar{\sigma}_{tf}\log\bar{\sigma}_{tf} + \bar{\sigma}_{bf}\log\bar{\sigma}_{bf}) \quad (1.22)$$

where subscripts t and b denote the top and bottom of the layer, respectively, and o and f denote the original and final conditions, respectively. For the above example problem $\bar{\sigma}_{t0}$ = 100 psf, $\bar{\sigma}_{b0}$ = 1100 psf, $\bar{\sigma}_{tf}$ = 2600 psf, and $\bar{\sigma}_{bf}$ = 3600 psf. The theoretically exact settlement is 3.89 feet. The "exact" solution is exact only in the sense of analytical accuracy. In a real field problem the accuracy is controlled by other than analytical aspects, e.g., by uncertainties in applied loads, location of the water table, thicknesses and properties of the various layers, secondary effects, and doubtless other sources of error as well. Consequently, the function of the exact solution is just to indicate the effects of subdividing a layer analytically.

As an aside it may be noted that it is impossible to have a non-zero value of R and $\bar{\sigma}_{t0} = 0$ because then application of any positive stress would lead to infinite settlements. The e-log s curve must either start out with R = 0 or, more likely, there must be an original effective stress caused by internal attractive forces (Jakobson, 1953, reported field evidence of such attractive forces). Note also that Equation 22 can be used with e-log s curves by replacing R with C/(l+e), that it can be applied for bilinear curves by applying the equation separately to the reloading and virgin curves, and it can be applied iteratively to account for settlement dependent submergence and effects of added fill.

## 1.5 Approximate Stress-Strain Curves

Occasionally an engineer may wish to make approximate analyses of total settlement in cases where no consolidation data are available. It then becomes necessary to estimate values for $R_r$ (or $C_r$), $\bar{\sigma}_{max}$, and $R_c$ (or $C_c$).
A number of papers exist in which the authors attempt to correlate $R_c$ (or $C_c$) with an index property. For example, Skempton (1944) used samples of remolded clay and found:

$$C_c = 0.007\,(LL - 10) \quad (1.23)$$

where LL is the liquid limit in percent. Terzaghi and Peck (1948, p. 66) concluded that for undisturbed soils:

$$C_c = 0.009\ (LL - 10) \qquad (1.24)$$

For organic deposits, Moran et al. (1958, p. 111) found that:

$$C_c = w/100 \qquad (1.25)$$

where w is the water content in percent. For the Motley clays from Sao Paulo, Cozzolino (1961) found that:

$$C_c = 0.256 + 0.00106\ (LL - 65) + 0.32\ (e - 0.84) \qquad (1.26)$$

with a range of about ±0.063. Nishidu (Moran et al., 1958) suggested that:

$$C_c = 0.54\ (e_o - 0.35) \qquad (1.27)$$

Kapp et al. (1966) showed that for marsh deposits in the New York City metropolitan area:

$$C_c = 0.6\ (e_o - 1) \quad \text{for } e_o < 6 \qquad (1.28a)$$

$$= 0.85\ (e_o - 2) \quad \text{for } 6 \le e_o \le 14 \qquad (1.28b)$$

An engineer can often develop the ability to estimate compressibilities with acceptable accuracy. For example, San Francisco Bay Mud often has $R_c$ of about 0.25 with values of up to 0.4 for peaty bay mud and down to 0.1 for inorganic bay mud.

For highly overconsolidated or cemented clays the compressibility is often considered negligible. When an estimate of reloading settlement is required, the reloading slope may be estimated as the estimated virgin slope divided by a factor which may be as low as 1.1 for some bentonitic clays to more than 10 for highly sensitive clays and cemented clays with values in the range of 2 1/2 to 4 common.

For lightly overconsolidated clays, the main source of error is in estimating a reasonable value of $s_i$. Perhaps the best approach is to assume that $s_i$ is $s_{max}$ and that the undrained shearing strength, $c_u$, is unchanged during a small reduction in stress. Thus:

$$\bar{\sigma}_{max} = c_u/(c/p) \qquad (1.29)$$

where $c_u$ is measured and the c/p ratio is estimated as (Skempton, 1957):

$$c/p = 0.11 + 0.0037 I_w \qquad (1.30)$$

where $I_w$ is the plasticity index in percent.

## 1.6 References

Cozzolino, E.V. M. (1961), "Statistical Forecasting of Compression Index," Proc., Fifth Intern. Conf. on Soil Mech. and Found. Engr., Vol. 1, pp. 51-53.

Jakobson, B. (1953), "Origin Cohesion of Clay," Proc., Third Intern. Conf. on Soil Mech. and Found. Engr., Vol. 1, pp. 35-37.

Kapp, M.S., D.L. York, A. Aronowitz, and H. Sitomer (1966), "Construction on Marshland Deposits," Highway Research Record, No. 133, pp. 1-22.

Moran, Proctor, Meuser, and Rutledge (1958), "Study of Deep Soil Stabilization by Vertical Sand Drains," U.S. Dept. of Commerce, Office of Technical Services, No. PB151692.

Peck, R.B., and W.C. Reed (1954), "Engineering Properties of Chicago Subsoils," Univ. of Illinois Engr. Exp. Sta. Bull. No. 423, 62 pp.

Skempton, A.W. (1944), "Notes on the Compressibility of Clays," Quart, Jour. Geol. Soc., London, Vol. C, pp. 119-135.

Skempton, A.W. (1957), "Discussion: The Planning and Design of the New Hong Kong Airport," Proc., Instn. of Civil Engineers, London, Vol. 7, pp. 305-307.

Terzaghi, K. (1936), "The Shearing Resistance of Saturated Soils," Proc., Intern. Conf. on Soil Mech. and Found. Engr., Cambridge, Mass., Vol. 1, pp. 54-56.