參、聯合機率分配
Joint Probability Distributions (Chapter 5)

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聯合機率分配

有時，我們有興趣的並非只是一個變數的機率分配，而是多個變數間的關係及其機率分配，因此我們必需探討多個變數間的聯合機率函數。

[p.154 (p.142)] In general, if X and Y are two random variables, the probability distribution that defines their smultaneous behavior is called a joint probability distribution.

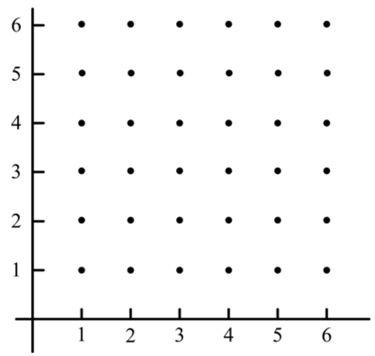
I. 聯合機率函數
(Joint Probability Mass/Density Function)

Example

投擲一個公正的骰子兩次，令

$$S = \{(x, y) | 1 \le x \le 6, 1 \le y \le 6\}$$

其中 x 代表第一次出現的點數，y 代表第二次出現的點數，如 $(3, 5)$ 代表第一次出現的點數爲 3，第二次出現的點數爲5，則共可能有 $6 \times 6 = 36$ 種不同的結果。



Example (Cont')

假設每一個可能結果發生的可能性是一樣的，則任何一個結果 (任何 x 及 y 值) 發生的機率爲 $\frac{1}{36}$，

$$f(x=3, y=5) = \frac{1}{36},$$

因此我們稱 $P(X=x, Y=y) = f(x,y)$ 爲變數 X 和 Y 的聯合機率質量函數 (joint probability mass function)。

聯合機率質量函數-兩個變數 (joint probability mass function), p.155

The joint probability mass function of the discrete random variables X and Y, denoted as $f_{XY}(x,y)$, satisfies

(1) $f_{XY}(x,y) \geq 0$

(2) $\sum_x \sum_y f_{XY}(x,y) = 1$

(3) $f_{XY}(x,y) = P(X=x, Y=y)$

聯合機率密度函數 (joint probability density function), p.167

A joint probability density function for the continuous random variables X and Y, denoted as $f_{XY}(x,y)$, satisfies the following properties:

(1) $f_{XY}(x,y) \geq 0$ for all x, y

(2) $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f_{XY}(x,y)dxdy = 1$

(3) For any region R of two-dimensional space

$$P([X,Y] \in R) = \iint_R f_{XY}(x,y)dxdy$$

Example (Cont')

已知 $f_{XY}(x,y) = 1/6, \forall x, y = 1, 2, \ldots, 6$。而 $f_X(x) = P\{X=x\}$ 稱爲 X 的邊際機率質量函數 (marginal probability mass function)，如:

$$\begin{aligned} f_X(3) = P\{X=3\} &= f(3,1) + f(3,2) \\ &+ f(3,3) + f(3,4) + f(3,5) + f(3,6) \\ &= \underbrace{\frac{1}{36} + \frac{1}{36} + \cdots + \frac{1}{36}}_{\text{共 6 個}} = \frac{1}{6} \end{aligned}$$

也就是只考慮 $X = x$ 時的機率。

邊際機率質量函數 (marginal probability mass function), p.156

If X and Y are discrete random variables with joint probability mass function $f_{XY}(x, y)$, then the marginal probability mass functions of X and Y are

$$f_X(x) = P(X = x) = \sum_y f_{XY}(x, y) \text{ and}$$

$$f_Y(y) = P(Y = y) = \sum_x f_{XY}(x, y)$$

where the first sum is over all points in the range of (X, Y) for which $X = x$ and the second sum is over all points in the range of (X, Y) for which $Y = y$.

Exercise 5-1, p.163

Show that the following function satisfies the properties of a joint probability mass function.

x	y	$f_{XY}(x, y)$
1	1	1/4
1.5	2	1/8
1.5	3	1/4
2.5	4	1/4
3	5	1/8

Solution

聯合機率分配的期望值和變異數

If the marginal probability distribution of X has the probability mass function $f_X(x)$, the

$$E(X) = \mu_X = \sum_x x f_X(x) = \sum_x x \underbrace{\left(\sum_y f_{XY}(x, y) \right)}_{= f_X(x)}$$

$$= \sum_x \sum_y x f_{XY}(x, y)$$

聯合機率分配的期望值和變異數 (Cont')

and

$$
\begin{aligned}
V(X) = \sigma_X^2 &= \sum_x (x-\mu_X)^2 f_X(x) \\
&= \sum_x (x-\mu_X)^2 \sum_y f_{XY}(x,y) \\
&= \sum_x \sum_y (x-\mu_X)^2 f_{XY}(x,y) \\
&= E(X^2) - \mu_X^2
\end{aligned}
$$

Exercise 5-1, p.163

聯合機率質量函數爲:

x	y	$f_{XY}(x,y)$
1	1	1/4
1.5	2	1/8
1.5	3	1/4
2.5	4	1/4
3	5	1/8

Determine the following probabilities:

(a) $P(X < 2.5, Y < 3)$

(b) $P(X < 2.5)$

(c) $P(Y < 3)$

(d) $P(X > 1.8, Y > 4.7)$

Exercise 5-1, p.163

X 和 Y 的聯合機率質量函數爲:

x	y	$f_{XY}(x,y)$
1	1	1/4
1.5	2	1/8
1.5	3	1/4
2.5	4	1/4
3	5	1/8

Determine $E(X)$, $E(Y)$, $V(X)$ and $V(Y)$

Solution

Solution (Cont')

條件機率質量函數 (Conditional Probability Mass Function), p.157

Given discrete random variables X and Y with joint probability mass function $f_{XY}(x, y)$ the conditional probability mass function of Y given $X = x$ is

$$f_{Y|x}(y) = \frac{f_{XY}(x, y)}{f_X(x)} \text{ for } f_X(x) > 0$$

條件機率質量函數的特性, p.157

Because a conditional probability mass function $f_{Y|x}(y)$ is a probability mass function, the following properties are satisfied:

(1) $f_{Y|x}(y) \geq 0$
(2) $\sum_y f_{Y|x}(y) = 1$
(3) $P(Y = y|X = x) = f_{Y|x}(y)$

條件期望值 (Conditional Mean) 和條件變異數 (Condition Variance), p.157

The conditional mean of Y given $X = x$, denoted as $E(Y|x)$ or $\mu_{Y|x}$, is

$$E(Y|x) = \sum_y y f_{Y|x}(y)$$

and the conditional variance of Y given $X = x$, denoted as $V(Y|x)$ or $\sigma^2_{Y|x}$, is

$$V(Y|x) = \sum_y (y - \mu_{Y|x})^2 f_{Y|x}(y) = \sum_y y^2 f_{Y|x}(y) - \mu^2_{Y|x}$$

Exercise 5-1, p.163

Continuation of Exercise 5-1. Determine

(g) The conditional probability distribution of Y given that $X = 1.5$.

(h) The conditional probability distribution of X given that $Y = 2$.

(i) $E(Y|X = 1.5)$

Solution

(g) The conditional probability distribution of Y given that $X = 1.5$.

聯合機率質量函數為:

x	y	$f_{XY}(x,y)$
1	1	1/4
1.5	2	1/8
1.5	3	1/4
2.5	4	1/4
3	5	1/8

Solution (Cont')

(h) The conditional probability distribution of X given that $Y = 2$.

聯合機率質量函數為:

x	y	$f_{XY}(x,y)$
1	1	1/4
1.5	2	1/8
1.5	3	1/4
2.5	4	1/4
3	5	1/8

Solution

(i) $E(Y|X = 1.5)$

獨立 (Independent), p.159

For discrete random variable X and Y, if any one of the following properties is true, the others are also true, and X and Y are independent.

(1) $f_{XY}(x,y) = f_X(x)f_Y(y)$ for all x and y

(2) $f_{Y|x}(y) = f_Y(y)$ for all x and y with $f_X(x) > 0$

(3) $f_{X|y}(x) = f_X(x)$ for all x and y with $f_Y(y) > 0$

(4) $P(X \in A, Y \in B) = P(X \in A)P(Y \in B)$ for any sets A and B in the range of X and Y, respectively.

Exercise 5-1, p.163

Continuation of Exercise 5-1. Determine

(j) Are X and Y independent?

(M1)

Solution (Cont')

(M2) 聯合機率質量函數爲:

x	y	$f_{XY}(x,y)$
1	1	1/4
1.5	2	1/8
1.5	3	1/4
2.5	4	1/4
3	5	1/8

聯合機率質量函數-多個變數 (joint probability mass function), p.159

The joint probability mass function of $X_1, X_2, \ldots, X_p$ is

$$f_{X_1,X_2,\ldots,X_p}(x_1,x_2,\ldots,x_p) = P(X_1 = x_1, X_2 = x_2, \ldots, X_p = x_p)$$

for all points $(x_1, x_2, \ldots, x_p)$ in the range of $X_1, X_2, \ldots, X_p$.

邊際機率分配-多個變數, p.160

If $X_1, X_2, X_3, \ldots, X_p$ are discrete random variables with joint probability mass function $f_{X_1,X_2,\ldots,X_p}(x_1, x_2, \ldots, x_p)$, the marginal probability mass function of any X_i is

$$f_{X_i}(x_i) = P(X_i = x_i) = \sum f_{X_1,X_2,\ldots,X_p}(x_1, x_2, \ldots, x_p)$$

where the sum is over the points in the range of $(X_1, X_2, \ldots, X_p)$ for which $X_i = x_i$

機率分配的期望值和變異數-多個變數, p.160

$$E(X_i) = \sum x_i f_{X_1,X_2,\ldots,X_p}(x_1, x_2, \ldots, x_p)$$

and

$$V(X_i) = \sum (x_i - \mu_{X_i})^2 f_{X_1,X_2,\ldots,X_p}(x_1, x_2, \ldots, x_p)$$

where the sum is over all points in the range of $X_1, X_2, \ldots, X_p$.

條件機率分配 (Conditional Probability Distributions), p.151

Conditional probability distribution can be developed for multiple discrete random variables by an an extension of the ideas used for two discrete random variables. For example, the conditional joint probability mass function of X_1, X_2, X_3, given X_4, X_5 is

$$f_{X_1,X_2,X_3|X_4,X_5}(x_1, x_2, x_3) = \frac{f_{X_1,X_2,X_3,X_4,X_5}(x_1, x_2, x_3, x_4, x_5)}{f_{X_4,X_5}(x_4, x_5)}$$

for $f_{X_4,X_5}(x_4, x_5) > 0$.

獨立 (Indpendent), p.153

Discrete random variables $X_1, X_2, \ldots, X_p$ are independent if and only if

$$f_{X_1,X_2,\ldots,X_p}(x_1, x_2, \ldots, x_p) = f_{X_1}(x_1) f_{X_2}(x_2) \cdots f_{X_p}(x_p)$$

for all $x_1, x_2, \ldots, x_p$.

Exercise 5-8(5-17), p.164

Suppose the random variables X, Y, and Z have the following joint probability distribution

x	y	z	$f(x,y,z)$
1	1	1	0.05
1	1	2	0.10
1	2	1	0.15
1	2	2	0.20
2	1	1	0.20
2	1	2	0.15
2	2	1	0.10
2	2	2	0.05

Determine the following:

(a) $P(X=2)$

(b) $P(X=1, Y=2)$

(c) $P(Z<1.5)$

(d) $P(X=1 \text{ or } Z=2)$

(e) $E(X)$

Exercise 5-8, p.164

x	y	z	$f(x,y,z)$
1	1	1	0.05
1	1	2	0.10
1	2	1	0.15
1	2	2	0.20
2	1	1	0.20
2	1	2	0.15
2	2	1	0.10
2	2	2	0.05

Determine the following:

(f) $P(X=1|Y=1)$

(g) $P(X=1, Y=1|Z=1)$

(h) $P(X=1|Y=1, Z=2)$

聯合機率密度函數 (joint probability density function), p.167

A joint probability density function for the continuous random variables X and Y, denoted as $f_{XY}(x,y)$, satisfies the following properties:

(1) $f_{XY}(x,y) \geq 0$ for all x, y

(2) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{XY}(x,y)dxdy = 1$

(3) For all region R of two-dimensional space

$$P((X,Y)\in R) = \iint_R f_{XY}(x,y)\, dxdy$$

邊際機率密度函數 (marginal probability density function), p.169

If the joint probability density function of continuous random variable X and Y is $f_{XY}(x,y)$, the marginal probability density functions of X and Y are

$$f_X(x) = \int_y f_{XY}(x,y)dy \qquad \text{and}$$

$$f_Y(y) = \int_x f_{XY}(x,y)dx$$

where the first integral is over all points in the range of (X,Y) for which $X=x$ and the second integral is over all points in the range of (X,Y) for which $Y=y$.

條件機率密度函數 (conditional probability density function), p.170

Given continuous random variables X and Y with joint probability density function $f_{XY}(x,y)$ the conditional probability density function of Y given $X=x$ is

$$f_{Y|x}(y) = \frac{f_{XY}(x,y)}{f_X(x)} \qquad \text{for } f_X(x) > 0$$

條件機率密度函數的特性, p.157

Because a conditional probability density function $f_{Y|x}(y)$ is a probability density function, the following properties are satisfied:

(1) $f_{Y|x}(y) \geq 0$

(2) $\int f_{Y|x}(y)dy = 1$

(3) $P(Y \in B | X = x) = \int_B f_{Y|x}(y)dy$ for any set B in the range of Y.

條件平均數及變異數 (conditional mean and variance), p.171

The conditional mean of Y given $X=x$, denoted as $E(Y|x)$ or $\mu_{Y|x}$ is

$$E(Y|x) = \int y f_{Y|x}(y)dy$$

and the conditional variance of Y given $X=x$, denoted as $V(Y|x)$ or $\sigma^2_{Y|x}$, is

$$V(Y|x) = \int (y-\mu_{Y|x})^2 f_{Y|x}(y)dy$$

$$= \int y^2 f_{Y|x}(y)dy - \mu^2_{Y|x}$$

獨立 (independent), p.172

For continuous random variables X and Y, if any one of the following properties is true, the others are also true, and X and Y are said to be independent.

(1) $f_{XY}(x,y) = f_X(x)f_Y(y)$ for all x and y

(2) $f_{Y|x}(y) = f_Y(y)$ for all x and y with $f_X(x) > 0$

(3) $f_{X|y}(x) = f_X(x)$ for all x and y with $f_Y(y) > 0$

(4) $P(X \in A, Y \in B) = P(X \in A)P(Y \in B)$ for any sets A and B in the range of X and Y, respectively.

Exercise 5-17, p. 177

Determine the value of c such that the function $f(x, y) = cxy$ for $0 < x < 3$ and $0 < y < 3$ satisfies the properties of a joint probability density function. Determine

(a) $P(X < 2, Y < 3)$

(b) $P(X < 2.5)$

(c) $P(1 < Y < 2.5)$

(d) $P(X > 1.8, 1 < Y < 2.5)$

(e) $E(X)$

(f) $P(X < 0, Y < 4)$

Solution

Exercise 5-17 (Cont.), p. 177

(g) Marginal probability distribution of the random variable X

(h) Conditional probability of Y given that $X = 1.5$

(i) $E(Y|X = 1.5)$

(j) $P(Y < 2|X = 1.5)$

(k) Conditional probability distribution of X given that $Y = 2$

Solution (Cont')

Solution (Cont')

II. 共變數和相關係數 (Covariance and Correlation)

Solution (Cont')

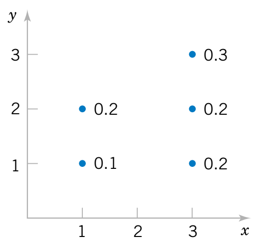
函數的期望值 (Expected Value of a Function), p.179

$$E[h(X,Y)] = \begin{cases} \sum\sum h(x,y) f_{XY}(x,y) & X, Y \text{ discrete} \\ \iint h(x,y) f_{XY}(x,y)\, dxdy & X, Y \text{ continuous} \end{cases}$$

Example 5-24, p.179

For the joint probability distribution of the two random variables in Fig. 5-12, calculated $E[(X-\mu_X)(Y-\mu_Y)]$.

(Sol.)



共變數 (covariance), p.179

The covariance between the random variables X and Y, denoted as $\text{cov}(X,Y)$ or σ_{XY}, is

$$\sigma_{XY}=E[(X-\mu_X)(Y-\mu_Y)]=E(XY)-\mu_X\mu_Y$$

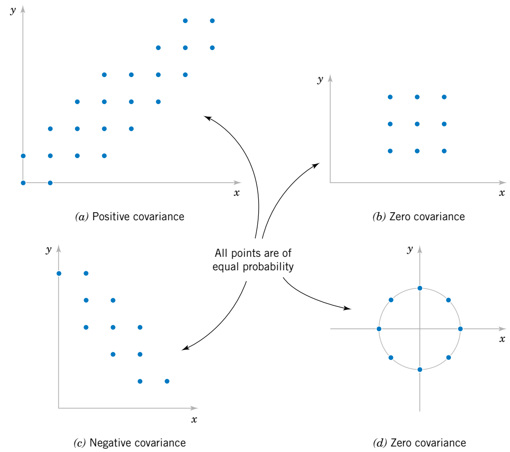
Moreover,

$$\sigma_{XY}=\text{cov}(X,Y)=\text{cov}(Y,X)=\sigma_{YX}$$

共變數的特性, p.181

共變數 (Covariance) 可以顯示變數之間的線性關係 (linear relationship), 若變數間的關係並非線性, 則共變數 (covariance) 無法顯示變數間的關係, 如圖 5-13(a) 所示, 當 X 和 Y 之間的 covariance 爲正值時 (positive), X 和 Y 之間爲正相關, 也就是變數 Y 值會隨著 X 值的增加而增加, 相對地, 亦會隨著 X 值的減少而減少。而圖 5-13(c) 所呈現的爲 X 和 Y 之間爲負相關, 其 covariance 爲負值 (negative), 也就是當 X 值增加時, Y 確相對地減少, 而當 X 值減少時, Y 值確相對地增加。而圖 (b) 和 (d) 中 X 和 Y 並無顯著的直線相關性, 故 X 和 Y 爲零 (zero) 相關。

Figure 5-13 Joint probability distribution and the sign of the covariance between X and Y, p.181



相關係數 (correlation), p.181

The correlation between random variables X and Y, denoted as ρ_{XY}, is

$$\rho_{XY} = \frac{\text{cov}(X, Y)}{\sqrt{V(X)V(Y)}} = \frac{\sigma_{XY}}{\sigma_X \sigma_Y}$$

For any two random variables X and Y

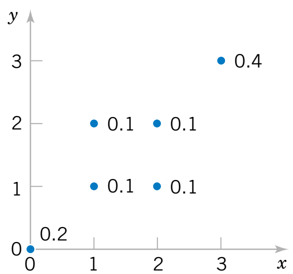
$$-1 \leq \rho_{XY} \leq +1$$

聯合機率函數 共變數和相關係數 變數的線性組合

Example 5-26, p.182

For the discrete random variables X and Y with the joint distribution shown in Fig. 5-14, determine σ_{XY} and ρ_{XY}.

(Sol.)



聯合機率函數 共變數和相關係數 變數的線性組合

Solution (Cont')

聯合機率函數 共變數和相關係數 變數的線性組合

III. 變數的線性組合
(Linear Combinations of Random Variables)

變數的線性組合 (linear combination of random variables)

令 X_1 和 X_2 爲兩個隨機變數, c_1 和 c_2 爲兩個常數, 則

$$Y = c_1X_1 + c_2X_2$$

稱爲 X_1 和 X_2 的線性組合 (linear combination)。若 $Y = c_1X_1 + c_2X_2$ 爲 X_1 和 X_2 的線性組合 (linear combination), 則 Y 亦爲一個隨機變數, 期望值爲

$$E(Y) = E(c_1X_1 + c_2X_2) = c_1E(X_1) + c_2E(X_2)$$

Y 的變異數爲

$$\begin{aligned}
\sigma_Y^2 = V(Y) &= E[(Y-\mu_Y)^2] \\
&= E\{[(c_1X_1 + c_2X_2) - (c_1\mu_{X_1} + c_2\mu_{X_2})]^2\} \\
&= E\{[(c_1X_1 - c_1\mu_{X_1}) + (c_2X_2 - c_2\mu_{X_2})]^2\} \\
&= E\{c_1^2(X_1-\mu_{X_1})^2 + c_2^2(X_2-\mu_{X_2})^2 \\
&\qquad + 2c_1c_2(X_1-\mu_{X_1})(X_2-\mu_{X_2})\} \\
&= c_1^2E[(X_1-\mu_{X_1})^2] + c_2^2E[(X_2-\mu_{X_2})^2] \\
&\qquad + 2c_1c_2E[(X_1-\mu_{X_1})(X_2-\mu_{X_2})] \\
&= c_1^2V(X) + c_2^2V(Y) + 2c_1c_2\mathrm{cov}(X_1, X_2)
\end{aligned}$$

若 X_1 和 X_2 爲 independent 時, 則 $\mathrm{cov}(X_1, X_2) = 0$, 所以

$$V(c_1X_1 + c_2X_2) = c_1^2V(X) + c_2^2V(X)$$

線性組合 (Linear Combination), p.188

Given random variables $X_1, X_2, \ldots, X_p$ and constants $c_1, c_2, \ldots, c_p$,

$$Y = c_1X_1 + c_2X_2 + \cdots + c_pX_p$$

is a linear combination of $X_1, X_2, \ldots, X_p$. The mean(平均數) of linear combination

$$E(Y) = c_1E(X_1) + c_2E(X_2) + \cdots + c_pE(X_p)$$

線性組合的變異數 (variance), p.189

If $X_1, X_2, \ldots, X_p$ are random variables, and $Y = c_1X_1 + c_2X_2 + \cdots + c_pX_p$, then in general

$$\begin{aligned}
V(Y) = c_1^2V(X_1) + c_2^2V(X_2) + \cdots + c_p^2V(X_p) \\
+ 2\sum_{i<j}\sum c_ic_j cov(X_i, X_j)
\end{aligned}$$

If $X_1, X_2, \ldots, X_p$ are independent,

$$V(Y) = c_1^2V(X_1)c_2^2V(X_2) + C_p^2V(X_p)$$

Exercise 5-54, p.191

X and Y are independent, normal random variables with $E(X) = 0$, $V(X) = 4$, $E(Y) = 10$, and $V(Y) = 9$. Determine $E(2X + 3Y)$ and $V(2X + 3Y)$.

常態變數 (Normal Random Variable), p.122

A random variable X with probability density function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \qquad -\infty < x < \infty$$

is a normal random variable with parameters μ, where $-\infty < \mu < \infty$, and $\sigma > 0$. Also,

$$E(X) = \mu \quad \text{and } V(X) = \sigma^2$$

and the notation $\mathcal{N}(\mu, \sigma^2)$ is used to denote the distribution. The mean and variance of X are shown to equal μ and σ^2, respectively.

標準常態隨機變數 (standard normal random variable), p.123

A normal random variable with

$$\mu = 0 \text{ and } \sigma^2 = 1$$

is call a standard normal random variable and is denoted as Z.
The cumulative distribution function of a standard normal random variable is denoted as

$$\Phi(z) = P(Z \leq z)$$

常態變數標準化 (Standarding), p.125

If X is a normal random variable with $E(X) = \mu$ and $V(X) = \sigma^2$, the random variable

$$Z = \frac{X - \mu}{\sigma}$$

is a normal random variable with $E(Z) = 0$ and $V(Z) = 1$. That is, Z is a standard normal random variable.

常態變數的機率, p.126

Suppose X is a normal random variable with mean μ and variance σ^2. Then,

$$P(X \leq x) = P\left(\frac{X-\mu}{\sigma} \leq \frac{x-\mu}{\sigma}\right) = P(Z \leq z)$$

where Z is a standard normal random variable, and $z = \frac{x-\mu}{\sigma}$ is the z-**value** obtained by stnadardizing X.

常態變數的累積機率, p.126

654 APPENDIX A STATISTICAL TABLES AND CHARTS

$$\Phi(z) = P(Z \leq z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2}\, du$$

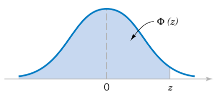


Table II Cumulative Standard Normal Distribution (*continued*)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.500000	0.503989	0.507978	0.511967	0.515953	0.519939	0.532922	0.527903	0.531881	0.535856
0.1	0.539828	0.543795	0.547758	0.551717	0.555760	0.559618	0.563559	0.567495	0.571424	0.575345
0.2	0.579260	0.583166	0.587064	0.590954	0.594835	0.598706	0.602568	0.606420	0.610261	0.614092
0.3	0.617911	0.621719	0.625516	0.629300	0.633072	0.636831	0.640576	0.644309	0.648027	0.651732
0.4	0.655422	0.659097	0.662757	0.666402	0.670031	0.673645	0.677242	0.680822	0.684386	0.687933
0.5	0.691462	0.694974	0.698468	0.701944	0.705401	0.708840	0.712260	0.715661	0.719043	0.722405
0.6	0.725747	0.729069	0.732371	0.735653	0.738914	0.742154	0.745373	0.748571	0.751748	0.754903
0.7	0.758036	0.761148	0.764238	0.767305	0.770350	0.773373	0.776373	0.779350	0.782305	0.785236
0.8	0.788145	0.791030	0.793892	0.796731	0.799546	0.802338	0.805106	0.807850	0.810570	0.813267
0.9	0.815940	0.818589	0.821214	0.823815	0.826391	0.828944	0.831472	0.833977	0.836457	0.838913
1.0	0.841345	0.843752	0.846136	0.848495	0.850830	0.853141	0.855428	0.857690	0.859929	0.862143
1.1	0.864334	0.866500	0.868643	0.870762	0.872857	0.874928	0.876976	0.878999	0.881000	0.882977
1.2	0.884930	0.886860	0.888767	0.890651	0.892512	0.894350	0.896165	0.897958	0.899727	0.901475
1.3	0.903199	0.904902	0.906582	0.908241	0.909877	0.911492	0.913085	0.914657	0.916207	0.917736
1.4	0.919243	0.920730	0.922196	0.923641	0.925066	0.926471	0.927855	0.929219	0.930563	0.931888
1.5	0.933193	0.934478	0.935744	0.936992	0.938220	0.939429	0.940620	0.941792	0.942947	0.944083
1.6	0.945201	0.946301	0.947384	0.948449	0.949497	0.950529	0.951543	0.952540	0.953521	0.954486
1.7	0.955435	0.956367	0.957284	0.958185	0.959071	0.959941	0.960796	0.961636	0.962462	0.963273
1.8	0.964070	0.964852	0.965621	0.966375	0.967116	0.967843	0.968557	0.969258	0.969946	0.970621
1.9	0.971283	0.971933	0.972571	0.973197	0.973810	0.974412	0.975002	0.975581	0.976148	0.976705
2.0	0.977250	0.977784	0.978308	0.978822	0.979325	0.979818	0.980301	0.980774	0.981237	0.981691
2.1	0.982136	0.982571	0.982997	0.983414	0.983823	0.984222	0.984614	0.984997	0.985371	0.985738

Exercise 4-45, p.129

Assume X is normal distributed with a mean of 10 and a standard deviation of 2. Determine $P(X < 13)$, $P(X > 9)$, and $P(6 < X < 14)$.

(Sol.)

平均數的期望值和變異數, p.190

If $\bar{X} = \frac{(X_1+X_2+\cdots+X_p)}{p}$ with $E(X_i) = \mu$ for $i = 1, 2, \ldots, p$

$$E(\bar{X}) = \mu$$

if $X_1, X_2, \ldots, X_p$ are also independent with $V(X_i) = \sigma^2$ for $i = 1, 2, \ldots, p$,

$$V(\bar{X}) = \frac{\sigma^2}{p}$$

常態變數線性組合的特性, p.190

If $X_1, X_2, \ldots, X_p$ are independent, normal random variables with $E(X_i) = \mu_i$ and $V(X_i) = \sigma_i^2$, for $i = 1, 2, \ldots, p$

$$Y = c_1X_1 + c_2X_2 + \cdots + c_pX_p$$

is a normal random variable with

$$E(Y) = c_1\mu_1 + c_2\mu_2 + \cdots + c_p\mu_p$$

and

$$V(Y)c_1^2\sigma_1^2 + c_2^2\sigma_2^2 + \cdots + c_p^2\sigma_p^2$$

Exercise 5-54, p.191

X and Y are independent, normal random variables with $E(X) = 0$, $V(X) = 4$, $E(Y) = 10$, and $V(Y) = 9$. Determine $P(2X + 3Y < 30)$ and $P(2X + 3Y < 40)$.

(Sol.)