貳、機率複習 (Review of Probability)

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機率函數 (probability function)

隨機變數 X 的機率函數 (probability function) 描述該變數可能產生 X 之值的機率。

- 若 X 爲離散隨機變數(discrete random variable), 則其機率函數稱爲機率質量函數(probability mass function, pmf);
- 若 X 爲連續隨機變數(continuous random variable), 則其機率函數稱爲機率密度函數(probability density function, pdf)。

I. 機率函數和機率分配 (Probability Function & Probability Distribution)

機率質量函數 (probability mass function, pmf), p.70

For a discrete random variable X with possible values $x_1, x_2, \ldots, x_n$, a probability mass function is a function such that

(1) $f(x_i) \geq 0$

(2) $\sum_{i=1}^{n} f(x_i) = 1$

(3) $f(x_i) = P(X = x_i)$

Example 3-4, p.69

This is a chance that a bit transmitted trough a digital transmission channel is received in error. Let X equal the number of bits in error in the next four bits transmitted. The possible values for X are $\{0, 1, 2, 3, 4, \}$. Based on a model for the errors that is presented in the following section, probabilities for these values will be determined. Suppose that the probabilities are

$P(X=0)=0.6561 \quad P(X=1)=0.2916 \quad P(X=2)=0.0486$

$P(X=3)=0.0036 \quad P(X=4)=0.0001$

Example 3-4 (Cont')

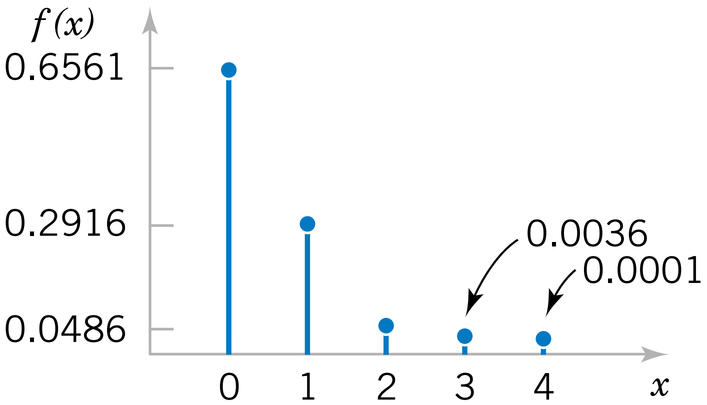


Figure 3-1 Probability distribution for bits in error.

Example (Exercise 3-14), p.71

The sample space of a random experiment is $\{a, b, c, d, e, f\}$,and each outcome is equally likely. A random variable is defined as follows:

outcome	a	b	c	d	e	f
x	0	0	1.5	1.5	2	3

Determine the probability mass function of X.

Solution

累積機率分配函數 (cumulative distribution function), p.72

The cumulative distribution function of a discrete random variable X, denoted as $F(x)$, is

$$F(x) = P(X \le x) = \sum_{x_i \le x} f(x_i)$$

For a discrete random variable X, $F(x)$ satisfies the following properties.

(1) $F(x) = P(X \le x) = \sum_{x_i \le x} f(x_i)$

(2) $0 \le F(x) \le 1$

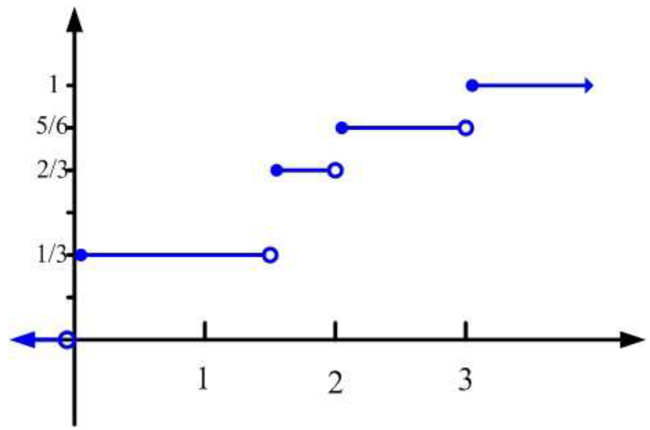
(3) If $x \le y$, then $F(x) \le F(y)$

Example

Continued from Exercise 3-14. Determine the cumulative distribution function of X.

Solution

下圖顯示此 CDF, $F(x)$.



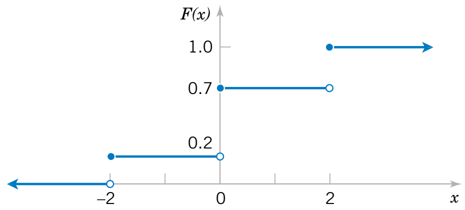
Example 3-7, p.72

Determine the probability mass function of X from the following cumulative distribution function:

$$F(x) = \begin{cases} 0 & x < -2 \\ 0.2 & -2 \le x < 0 \\ 0.7 & 0 \le x < 2 \\ 1 & 2 \le x \end{cases}$$

Solution

下圖顯示此 CDF, $F(x)$.



機率密度函數 (probability density function), p.111

For a continuous random variable X, a probability density function is a function such that

(1) $f(x) \geq 0$

(2) $\int_{-\infty}^{\infty} f(x)dx = 1$

(3) $P(a \leq X \leq b) = \int_a^b f(x)dx =$ area under $f(x)$ from a to b for any a and b

連續隨機變數的機率 (Probability of a continuous random variable)

If X is a continuous random variable, for any x_1 and x_2,

$$\begin{aligned} P(x_1 \leq X \leq x_2) &= P(x_1 < X \leq x_2) \\ &= P(x_1 \leq X < x_2) \\ &= P(x_1 < X < x_2) \end{aligned}$$

Example 4-1, p. 112

Let the continuous random variable X denote the current measured in a thin copper wire in milliamperes. Assume that the range of X is $[0, 20mA]$, and assume that the probability density function of X is $f(x) = 0.05$ for $0 \leq x \leq 20$. What is the probability that a current measurement is less than 10 milliamperes?

Example 4-1(Cont'), p. 112

The probability density function is shown in Fig. 4-4. It is assumed that $f(x) = 0$ wherever it is not specifically defined. The probability requested is indicated by the shaded area in Fig. 4-4.

$$P(X < 10) = \int_0^{10} f(x)dx = \int_0^{10} 0.05dx = 0.5$$

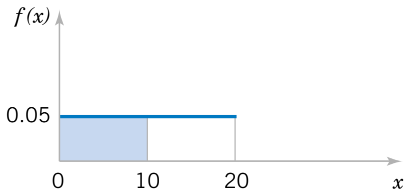


Figure 4-4 Probability density function for Example 4-1.

II. 平均數 (期望值) 和變異數
(Mean (Expected Value) and Variance)

累積機率分配函數 (cumulative distribution function), p.114

The cumulative distribution function of a continuous random variable X is

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(u)\, du$$

for $-\infty < x < \infty$.

平均數 (期望值) 和變異數–離散隨機變數, p.74

The mean or expected value of the discrete random variable X, denoted as μ or $E(X)$, is

$$\mu = E(X) = \sum_x xf(x)$$

The variance of X, denoted as σ^2 or $V(X)$, is

$$\sigma^2 = V(X) = E(X-\mu)^2 = \sum_x (x-\mu)^2 f(x) = \sum_x x^2 f(x) - \mu^2$$

The standard deviation of X is $\sigma = \sqrt{\sigma^2}$.

平均數 (期望值) 和變異數–離散隨機變數, p.77

If X is a discrete random variable with probability mass function $f(x)$,

$$E[h(X)] = \sum_x h(x)f(x)$$

Example: 令 X 爲一離散隨機變數, 且令 $h(X) = X^2$, 則

$$E[h(X)] = E[X^2] = \sum_x x^2 f(x)$$

此即所熟知 X^2 之期望值。所以 $\sigma^2 = E[X^2] - \mu^2$。

Example (Exercise 3-39, p.77)

If the range of X is the set $\{0, 1, 2, 3, 4\}$ and $P(X = x) = 0.2$ determine the mean and variance of the random variable.

Solution (Cont')

平均數 (期望值) 和變異數–連續隨機變數, p.117

Suppose X is a continuous random variable with probability density function $f(x)$.
The mean or expected value of X, denoted as μ or $E(X)$, is

$$\mu = E(X) = \int_{-\infty}^{\infty} x f(x) dx$$

The variance of X, denoted as $V(X)$ or σ^2, is

$$\sigma^2 = V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x) dx = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$$

The standard deviation of X is $\sigma = \sqrt{\sigma^2}$.

平均數 (期望值) 和變異數–連續隨機變數, p.117

If X is a continuous random variable with probability mass function $f(x)$,

$$E[h(X)] = \int_{-\infty}^{\infty} h(x)f(x)$$

xample (Exercise 4-23, p. 118)

Suppose $f(x) = 0.25$ for $0 < x < 4$. Determine the mean and variance of X.