Online Appendix for the Paper "Memetic Algorithm for Real-Time Combinatorial Stochastic Simulation Optimization Problems with Performance Analysis"

Shih-Cheng Horng

Chaoyang University of Technology (schong@cyut.edu.tw)

Shin-Yeu Lin

Chang Gung University (shinylin@mail.cgu.edu.tw)

Loo-Hay Lee

National University of Singapore (iseleelh@nus.edu.sg)

Chun-Hung Chen

George Mason University (cchen9@gmu.edu)

Abstract

This document contains supplementary materials for the paper "*Memetic Algorithm for Real-Time Combinatorial Stochastic Simulation Optimization Problems with Performance Analysis*". It is organized in the following way: Section 1 contains the modeling error analysis of OFTANN; Section 2 contains the backward local structure model of ATO problem; Section 3 consists of the modeling error analysis of ONBSM, and Section 4 contains the modeling error analysis of one simulation replication required to compute $\bar{f}_i$ and σ_i for various number of simulation replications in OCBA.

This version: August 2012

Please direct communication to Shin-Yeu Lin (shinylin@mail.cgu.edu.tw)

1 Modeling Error Analysis of OFTANN

To analyze the probability distribution of the modeling errors of the OFTANN for a given arrival rate vector λ, say $\lambda^{(1)}(=[3.6,3.0,2.4,1.8,1.2]^T)$, we adopt the ordered $v_{[i]}$, $i=1,\ldots,|V'|$ and their corresponding $E[f(\lambda^{(1)},v_{[i]})]$, $i=1,\ldots,|V'|$ of the corresponding OPC presented in Fig. 3 and put the $|V'|$ points of $(v_{[i]},E[f(\lambda^{(1)},v_{[i]})])$ in Fig. 4 marked by the solid line. We let $O(\lambda^{(1)},v_{[i]})$ denote the approximate objective value of $v_{[i]}$ under the given $\lambda^{(1)}$ obtained by the OFTANN presented in Section IV.B.1. We also plot the $|V'|$ points of $(v_{[i]},O(\lambda^{(1)},v_{[i]}))$ in Fig. 4 marked by "•". Then, the *percentage modeling errors* of the OFTANN for each $v_{[i]}$ denoted by $w_{[i]}\%$ can be calculated by

$$w_{[i]}\%=\frac{O(\lambda^{(1)},v_{[i]})-E[f(\lambda^{(1)},v_{[i]})]}{E[f(\lambda^{(1)},v_{[i]})]}\times 100\% \tag{8}$$

Collecting the $w_{[i]}\%$'s for all $v_{[i]}$, $i=1,\ldots,|V'|$, we can use a histogram to represent the percentage modeling errors $w_{[i]}\%$'s of OFTANN as presented in Fig. 5. In this figure, the horizontal axis that represents the percentage modeling error $w_{[i]}\%$ is partitioned into the intervals of equal width, 0.83%, and the height of a bar represents the number of $v_{[i]}$ whose corresponding $w_{[i]}\%$ lie in the same interval. The shape of the histogram presented in Fig. 5 is of *normal distribution*, which leads us to assert the hypothesis that the percentage modeling errors $w\%$ of the OFTANN is of normal distribution with mean μ_w and variance σ_w^2 computed by

$$\mu_w=\sum_{i=1}^{|V'|}\frac{w_{[i]}\%}{|V'|} \tag{9}$$

$$\sigma_w^2=\sum_{i=1}^{|V'|}\frac{\left(w_{[i]}\%-\mu_w\right)^2}{|V'|} \tag{10}$$

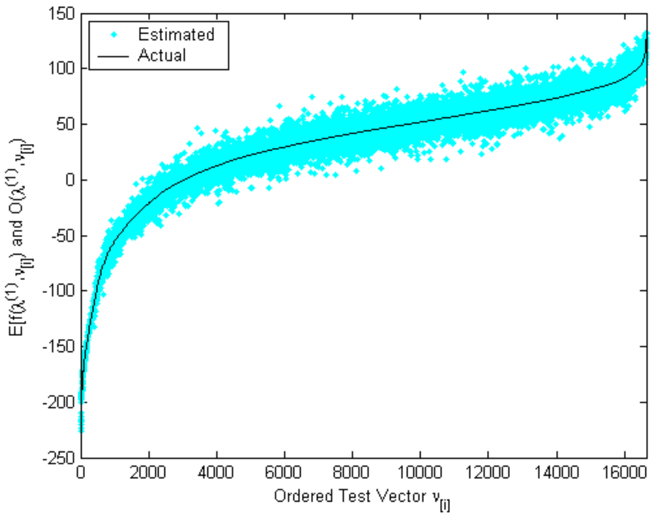

Fig. 4. $E[f(\lambda^{(1)},v_{[i]})]$ and $O(\lambda^{(1)},v_{[i]})$ for the $|V'|$ ordered solutions $v_{[i]}$.

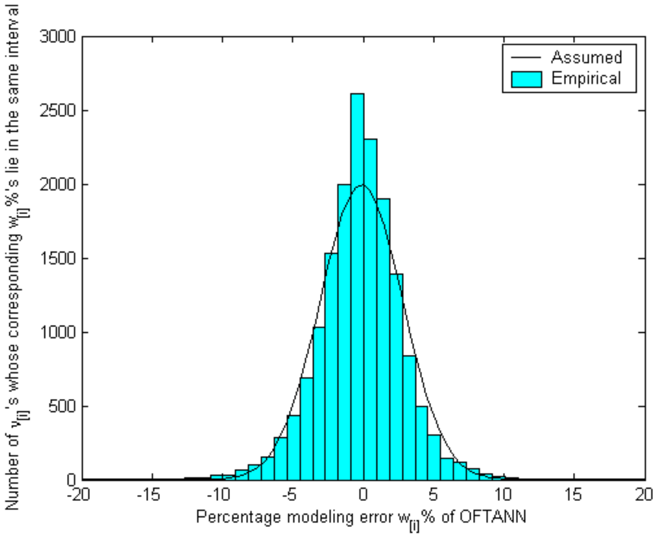

Fig. 5. Histogram of the percentage modeling errors of the OFTANN.

This model has been successfully validated by the Anderson-Darling test [23], though the details are omitted here, and the (μ_w, σ_w^2) for $\lambda^{(1)}$ is (~0%, 3.21%). Repeating the above process for each of the rest of $(|\Lambda'_d|-1)$ λ's, we can construct the probability distribution model of the percentage modeling errors of OFTANN for the corresponding λ.

2 Backward Local Structure Model of ATO problem

The purpose of the backward local structure model is to help evaluate the normalized objective value of the temporary solution resulting from the search process of the probabilistic local search method. The backward local structure model is constructed based on the forward local tier structure, which is presented in the following.

2.1 Construction of forward local tier structure

The forward local tier structure of the considered problem is formed based on the probabilistic local search method; its construction procedures can be stated below.

Step F0: Randomly select k (~100) v's from V. Repeat Steps F1-F5 for each selected v, which serves as a starting point of the probabilistic local search method.

Step F1: Let $v_{npi}, i=1,\cdots,n$ denote all the neighboring points of v, then v_{npi}, $i=1,\cdots,n$ constitute the *first-tier* of v as presented in Fig. 6. Use exact model to evaluate the objective values of v and $v_{npi}, i=1,\cdots,n$, denoted by $f(v)$ and $f(v_{npi}), i=1,\cdots,n$, respectively. Compute the percentage deviation $\eta(v_{npi})\%$ of $f(v_{npi})$ with respect to $f(v)$ for all npi by $\eta(v_{npi})\% = \frac{f(v_{npi})-f(v)}{f(v)} \times 100\%, i=1,\cdots,n$.

Step F2: Obtain approximate objective values of v and $v_{npi}, i=1,\cdots,n$ using ONBSM, and these values are used to determine the set of accepted neighboring points in the probabilistic local searches. Collect all the accepted neighboring points in this step, which are marked by square blocks in the first tier presented in Fig. 6, as the points resulting from the *first-tier search* and denote them by $v^{(1)i}, i=1,\cdots,n^{(1)}$.

Step F3: Let $v_{npj}^{(1)i}$, $j=1,\cdots,n_{(1)i}$ denote all the neighboring points of $v^{(1)i}$ for $i=1,\cdots,n^{(1)}$, then $v_{npj}^{(1)i}$, $j=1,\cdots,n_{(1)i}$, $i=1,\cdots,n^{(1)}$ constitutes the *second-tier* of the starting point v as presented in Fig. 6. Use exact model to evaluate the objective values of $v^{(1)i}$ and $v_{npj}^{(1)i}$, $j=1,\cdots,n_{(1)i}$, denoted by $f(v^{(1)i})$ and $f(v_{npj}^{(1)i})$, $j=1,\cdots,n_{(1)i}$, respectively, for $i=1,\cdots,n^{(1)}$. Compute the percentage deviation $\eta(v_{npj}^{(1)i})\%$ of $f(v_{npj}^{(1)i})$ with respect to $f(v^{(1)i})$ by $\eta(v_{npj}^{(1)i})\% = \dfrac{f(v_{npj}^{(1)i}) - f(v^{(1)i})}{f(v^{(1)i})} \times 100\%$, $j=1,\cdots,n_{(1)i}$ for $i=1,\cdots,n^{(1)}$.

Step F4: For $i=1,\cdots,n^{(1)}$, obtain approximate objective values of $v^{(1)i}$ and $v_{npj}^{(1)i}$, $j=1,\cdots,n_{(1)i}$ using ONBSM, then these values are used to determine the set of accepted neighboring points in the probabilistic local searches. Collect all the accepted neighboring points in this step, which are marked by square blocks in the second tier presented in Fig. 6, as the points resulted from the *second-tier search*.

Step F5: Continue the above process to find the *third tier* and the points resulted from the *third-tier search* and so on until the set of points resulting from the *$(n+1)$th tier search* is an empty set. Then, the set of accepted neighboring points resulting from the *n th tier search* represents the set of local optima reached from different search paths of the probabilistic local search method starting from v, and the *n th tier* is the *last tier* of the forward local tier structure.

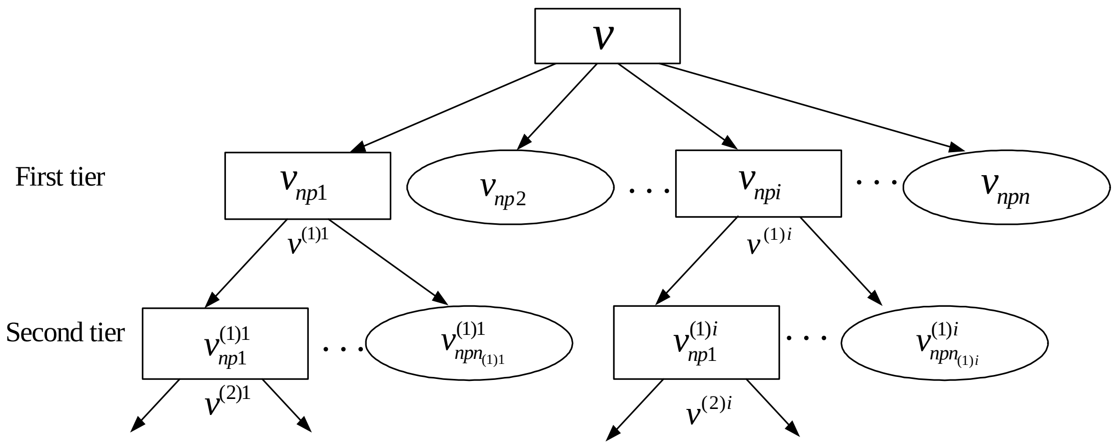

Fig. 6. Illustration of the forward local tier structure.

We have observed that different starting points will experience a different number of successful probabilistic local searches before reaching the corresponding approximate local optimum. We let n_1, n_2, …, n_k denote the *last tier* of the forward local tier structure originating from the k randomly selected points, respectively. Comparing with the k randomly selected points, the quality of the N good solutions resulting from the global search should be superior. Therefore, they will be closer to approximate local optima. Additionally, the N good solutions experience a different number of successful probabilistic local searches before reaching the corresponding local optimum. The above observations are confirmed by the test results presented in Remark 2 of Section IV.C. Since all local optima are lying in the last tier of the forward local tier structure, it is more reasonable to describe local structure model backwards. In the following, we will state the formation of *backward local structure model*.

Step B1: Collect all the percentage deviations $\eta(v_{npi})\%$'s of all points v_{npi}'s, which result from the $(n_1-1)^{\text{th}}$, the $(n_2-1)^{\text{th}}$, …, the $(n_k-1)^{\text{th}}$ tier search of the forward local tier structure corresponding to the k randomly selected points. Find the probability distribution model of these $\eta(v_{npi})\%$'s, which forms the probability distribution model of the last tier, denoted by tier LT, of the backward local structure model.

Step B2: Replace $(n_j - 1), j = 1, \cdots, k$ and LT in Step B1 with $(n_j - 2), j = 1, \cdots, k$ and $LT - 1$, respectively, and find the probability distribution model of tier $LT - 1$ of the backward local structure model.

Step B3: Continue the above process until the probability distribution model of tier $LT - [\max(n_1, n_2, \cdots, n_k) - 1]$ of the backward local structure model is formed.

Similar to the process of analyzing the probability distribution of the percentage modeling error for OFTANN, we have found that for the ATO problem, the probability distribution model of the percentage deviation of objective values in all tiers of the backward local structure model are all of normal distribution. At the arrival rates $\lambda^{(1)} = [3.6, 3.0, 2.4, 1.8, 1.2]^T$, we have $\max(n_1, n_2, \cdots, n_k) = 7$, and the pairs of mean and variance of the normal distribution model of the percentage deviation of objective values in tier $LT - i,\ i = 0, 1, \cdots, 6$ are (-0.27%, 1.53%), (-0.26%, 1.91%), (-0.29%, 2.06%), (-0.40%, 2.32%), (-0.65%, 3.36%), (-0.54%, 2.86%), and (-0.48%, 3.34%), respectively.

3 Modeling Error Analysis of ONBSM

3.1 Modeling error analysis for given (λ, v)

To proceed with the modeling error analysis of the ONBSM for a given (λ, v), we will perform n_s simulation replications to calculate $\bar{f}(\lambda, v) = \frac{1}{n_s}\sum_{l=1}^{n_s} f_l(\lambda, v)$. We let $e^{n_s}\%$ denote the percentage modeling error of ONBSM for (λ, v), then

$$e^{n_s}\% = \frac{\bar{f}(\lambda, v) - E[f(\lambda, v)]}{E[f(\lambda, v)]} \times 100\% \tag{11}$$

Repeating the above process for arbitrarily large number of times, say L, then we can obtain L $e^{n_s}\%$ denoted by $e^{n_s}_{[i]}\%$, $i = 1, \cdots, L$, for the given (λ, v). Similar to the analysis of the probability distribution model of the percentage modeling errors of OFTANN, the probability distribution model of the percentage modeling errors $e^{n_s}\%$ for any selected (λ, v) is also a normal distribution with mean $\mu_{e^{n_s}}(\lambda, v)$ and variance $\sigma^2_{e^{n_s}}(\lambda, v)$ computed by

$$\mu_{e^{n_s}}(\lambda, v) = \sum_{i=1}^{L} \frac{e^{n_s}_{[i]}\%}{L} \tag{12}$$

$$\sigma^2_{e^{n_s}}(\lambda, v) = \sum_{i=1}^{L} \frac{\left(e^{n_s}_{[i]}\% - \mu_{e^{n_s}}\right)^2}{L} \tag{13}$$

The $(\mu_{e^{n_s}}(\lambda, v), \sigma^2_{e^{n_s}}(\lambda, v))$ for $\lambda = \lambda^{(1)} = [3.6, 3.0, 2.4, 1.8, 1.2]^T$ and $v = [5, 2, 3, 7, 5, 5, 4, 2]^T$ is (~0%, 1.08%).

3.2 Approximate modeling error of ONBSM

Since $\sigma_{e^{n_s}}(\lambda, v)$ varies with different (λ, v), to avoid evaluating $\sigma_{e^{n_s}}(\lambda, v)$ for any (λ, v)

on-line, we will derive an approximate $\bar{\sigma}$ to apply to any (λ, v) encountered in the probabilistic local search process. Let $\sigma_{e^{n_s}}^{[j]}(\lambda, v)$ be the standard deviation of the modeling error of ONBSM of the j th (λ, v) in $\Lambda'_d \times V'$. We then set

$$\bar{\sigma} = \sum_{i=1}^{|\Lambda'_d|\cdot|V'|} \frac{\sigma_{e^{n_s}}^{[j]}}{|\Lambda'_d|\cdot|V'|} \tag{14}$$

For the considered ATO problem, $\bar{\sigma}$ =1.1341%.

3.3 Appropriateness of approximate modeling error of ONBSM

To justify the appropriateness of $\bar{\sigma}$, which will be applied to any (λ, v), we calculate the variance of the $|\Lambda'_d|\cdot|V'|$ $\sigma_{e^{n_s}}^{[j]}(\lambda, v)$ s with respect to $\bar{\sigma}$, and the resulting variance is 0.0910%, which is small enough, and we can claim that the approximation is appropriate.

4 Modeling Error Analysis of One Simulation Replication Required to Compute $\bar{f}_i$ and σ_i for Various Number of Simulation Replications in OCBA

For a given (λ, v), the probability distribution model of the percentage modeling errors of one simulation replication denoted by $e^1\%$ can be obtained in the same way as that for ONBSM by simply replacing n_s by 1. The result is also a normal distribution with mean 0 and standard deviation $\sigma_{e^1}(\lambda, v)$ for the given (λ, v). The approximate standard deviation denoted by $\bar{\sigma}_{e^1}$ that can be applied to all $(\lambda, v) \in \Lambda'_d \times V'$ can also be derived in a similar way as that used to derive $\bar{\sigma}$ in Section III.B. The $\bar{\sigma}_{e^1}$ is 2.53%, and the appropriateness of this approximation has also been justified by a way similar to that used in Section III.C.

Reference

[23] J.Q.C. Wang, “Sample distribution function based goodness-of-fit test for complex surveys,” *Comput. Stat. Data Anal.*, vol. 56, no. 3, pp. 664-679, Mar. 2012.