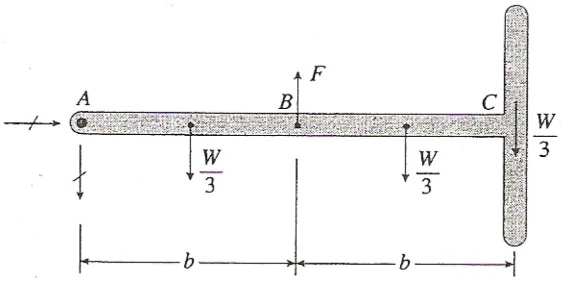
Solution 2.2-1 **T-shaped arm**

FREE-BODY DIAGRAM OF ARM



F = tensile force in the spring

$\Sigma M_A = 0$ ↺+ ↻−

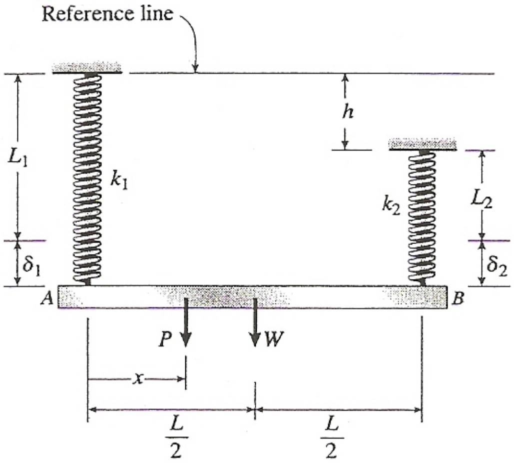
$$F(b) - \frac{W}{3}\left(\frac{b}{2}\right) - \frac{W}{3}\left(\frac{3b}{2}\right) - \frac{W}{3}(2b) = 0$$

$$F = \frac{4W}{3}$$

δ = elongation of the spring

$$\delta = \frac{F}{k} = \frac{4W}{3k} \quad \longleftarrow$$

Solution 2.2-10 **Bar supported by two springs**



$W = 25$ N

$k_1 = 300$ N/m

$k_2 = 400$ N/m

$L = 350$ mm

$h = 80$ mm

$P = 18$ N

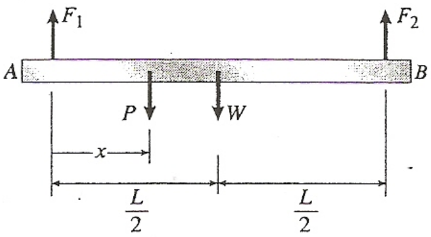
NATURAL LENGTHS OF SPRINGS

$L_1 = 250$ mm $\quad L_2 = 200$ mm

OBJECTIVE

Find distance x for bar AB to be horizontal.

FREE-BODY DIAGRAM OF BAR AB



$\Sigma M_A = 0$ ↺+ ↻−

$$F_2 L - Px - \frac{WL}{2} = 0 \qquad \text{(Eq. 1)}$$

$\Sigma F_{vert} = 0$ ↑+ ↓−

$$F_1 + F_2 - P - W = 0 \qquad \text{(Eq. 2)}$$

SOLVE EQS. (1) AND (2):

$$F_1 = P\left(1 - \frac{x}{L}\right) + \frac{W}{2} \qquad F_2 = \frac{P_x}{L} + \frac{W}{2}$$

SUBSTITUTE NUMERICAL VALUES:

UNITS: Newtons and meters

$$F_1 = (18)\left(1 - \frac{x}{0.350}\right) + 12.5 = 30.5 - 51.429x$$

$$F_2 = (18)\left(\frac{x}{0.350}\right) + 12.5 = 51.429x + 12.5$$

ELONGATIONS OF THE SPRINGS

$$\delta_1 = \frac{F_1}{k_1} = \frac{F_1}{300} = 0.10167 - 0.17143x$$

$$\delta_2 = \frac{F_2}{k_2} = \frac{F_2}{400} = 0.12857x + 0.031250$$

BAR AB REMAINS HORIZONTAL

Points A and B are the same distance below the reference line (see figure above).

$$\therefore\ L_1 + \delta_1 = h + L_2 + \delta_2$$

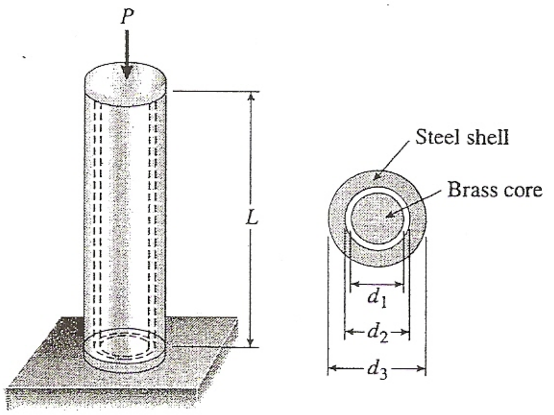
or $\quad 0.250 + 0.10167 - 0.17143\,x$
$\quad = 0.080 + 0.200 + 0.12857\,x + 0.031250$

SOLVE FOR x:

$0.300\,x = 0.040420 \qquad x = 0.1347$ m

$x = 135$ mm $\longleftarrow$

Solution 2.4-1 Cylindrical assembly in compression



$d_1 = 0.25$ in. $\quad E_b = 15 \times 10^6$ psi

$d_2 = 0.28$ in. $\quad E_s = 30 \times 10^6$ psi

$d_3 = 0.35$ in. $\quad A_s = \frac{\pi}{4}(d_3^2 - d_2^2) = 0.03464 \text{ in.}^2$

$L = 4.0$ in. $\quad A_b = \frac{\pi}{4} d_1^2 = 0.04909 \text{ in.}^2$

(a) Decrease in length ($\delta = 0.003$ in.)

Use Eq. (2-13) of Example 2-5.

$$\delta = \frac{PL}{E_s A_s + E_b A_b} \quad \text{or}$$

$$P = (E_s A_s + E_b A_b)\left(\frac{\delta}{L}\right)$$

Substitute numerical values:

$$E_s A_s + E_b A_b = (30 \times 10^6 \text{ psi})(0.03464 \text{ in.}^2) + (15 \times 10^6 \text{ psi})(0.04909 \text{ in.}^2) = 1.776 \times 10^6 \text{ lb}$$

$$P = (1.776 \times 10^6 \text{ lb})\left(\frac{0.003 \text{ in.}}{4.0 \text{ in.}}\right)$$

$$= 1330 \text{ lb} \quad \longleftarrow$$

(b) Allowable load

$\sigma_s = 22$ ksi $\qquad \sigma_b = 16$ ksi

Use Eqs. (2-12a and b) of Example 2-5.

For steel:

$$\sigma_s = \frac{PE_s}{E_s A_s + E_b A_b} \qquad P_s = (E_s A_s + E_b A_b)\frac{\sigma_s}{E_s}$$

$$P_s = (1.776 \times 10^6 \text{ lb})\left(\frac{22 \text{ ksi}}{30 \times 10^6 \text{ psi}}\right) = 1300 \text{ lb}$$

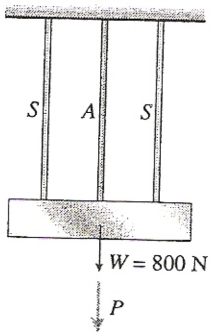
For brass:

$$\sigma_b = \frac{PE_b}{E_s A_s + E_b A_b} \qquad P_s = (E_s A_s + E_b A_b)\frac{\sigma_b}{E_b}$$

$$P_s = (1.776 \times 10^6 \text{ lb})\left(\frac{16 \text{ ksi}}{15 \times 10^6 \text{ psi}}\right) = 1890 \text{ lb}$$

Steel governs. $\quad P_{allow} = 1300 \text{ lb} \quad \longleftarrow$

Solution 2.4-10 Rigid bar hanging from three wires



STEEL WIRES

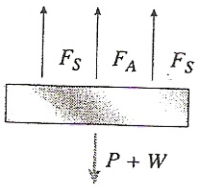
$d_s = 2\text{ mm}$ $\sigma_s = 220\text{ MPa}$ $E_s = 210\text{ GPa}$

ALUMINUM WIRES

$d_A = 4\text{ mm}$ $\sigma_A = 80\text{ MPa}$

$E_A = 70\text{ GPa}$

FREE-BODY DIAGRAM OF RIGID BAR



EQUATION OF EQUILIBRIUM

$\Sigma F_{\text{vert}} = 0$

$$2F_s + F_A - P - W = 0 \qquad \text{(Eq. 1)}$$

EQUATION OF COMPATIBILITY

$$\delta_s = \delta_A \qquad \text{(Eq. 2)}$$

FORCE DISPLACEMENT RELATIONS

$$\delta_s = \frac{F_s L}{E_s A_s} \qquad \delta_A = \frac{F_A L}{E_A A_A} \qquad \text{(Eqs. 3, 4)}$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{F_s L}{E_s A_s} = \frac{F_A L}{E_A A_A} \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$F_A = (P + W)\left(\frac{E_A A_A}{E_A A_A + 2E_s A_s}\right) \qquad \text{(Eq. 6)}$$

$$F_s = (P + W)\left(\frac{E_s A_s}{E_A A_A + 2E_s A_s}\right) \qquad \text{(Eq. 7)}$$

STRESSES IN THE WIRES

$$\sigma_A = \frac{F_A}{A_A} = \frac{(P + W)E_A}{E_A A_A + 2E_s A_s} \qquad \text{(Eq. 8)}$$

$$\sigma_s = \frac{F_s}{A_s} = \frac{(P + W)E_s}{E_A A_A + 2E_s A_s} \qquad \text{(Eq. 9)}$$

ALLOWABLE LOADS (FROM EQS. (8) AND (9))

$$P_A = \frac{\sigma_A}{E_A}(E_A A_A + 2E_s A_s) - W \qquad \text{(Eq. 10)}$$

$$P_s = \frac{\sigma_s}{E_s}(E_A A_A + 2E_s A_s) - W \qquad \text{(Eq. 11)}$$

SUBSTITUTE NUMERICAL VALUES INTO EQS. (10) AND (11):

$$A_s = \frac{\pi}{4}(2\text{ mm})^2 = 3.1416\text{ mm}^2$$

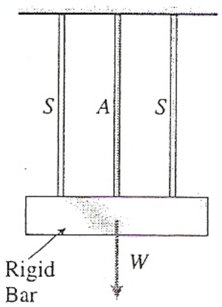
$$A_A = \frac{\pi}{4}(4\text{ mm})^2 = 12.5664\text{ mm}^2$$

$P_A = 1713\text{ N}$

$P_s = 1504\text{ N}$

Steel governs. $P_{\text{allow}} = 1500\text{ N}$ ←

Solution 2.5-3 Bar supported by three wires



S = steel $\quad$ A = aluminum

$W = 750$ lb

$d = \frac{1}{8}$ in.

$A_s = \frac{\pi d^2}{4} = 0.012272 \text{ in.}^2$

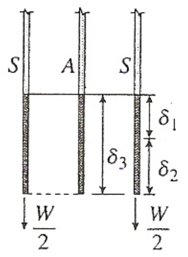
$E_s = 30 \times 10^6$ psi

$E_s A_s = 368{,}155$ lb

$\alpha_s = 6.5 \times 10^{-6}/°F$

$\alpha_a = 12 \times 10^{-6}/°F$

L = Initial length of wires



δ_1 = increase in length of a steel wire due to temperature increase ΔT

$= \alpha_s(\Delta T)L$

δ_2 = increase in length of a steel wire due to load $W/2$

$= \frac{WL}{2E_s A_s}$

δ_3 = increase in length of aluminum wire due to temperature increase ΔT

$= \alpha_a(\Delta T)L$

For no load in the aluminum wire:

$\delta_1 + \delta_2 = \delta_3$

$$\alpha_s(\Delta T)L + \frac{WL}{2E_s A_s} = \alpha_a(\Delta T)L$$

or

$$\Delta T = \frac{W}{2E_s A_s(\alpha_a - \alpha_s)} \longleftarrow$$

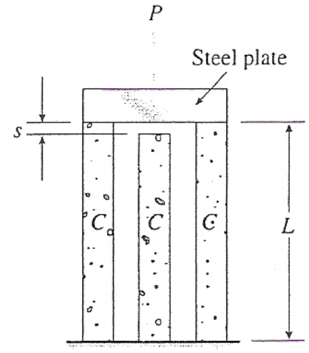
Substitute numerical values:

$$\Delta T = \frac{750 \text{ lb}}{(2)(368{,}155 \text{ lb})(5.5 \times 10^{-6}/°F)}$$

$$= 185°F \longleftarrow$$

NOTE: If the temperature increase is larger than ΔT, the aluminum wire would be in compression, which is not possible. Therefore, the steel wires continue to carry all of the load. If the temperature increase is less than ΔT, the aluminum wire will be in tension and carry part of the load.

Solution 2.5-16 Plate supported by three posts



s = size of gap = 1.0 mm

L = length of posts = 2.0 m

A = 40,000 mm^2

σ_{allow} = 20 MPa

E = 30 GPa

C = concrete post

DOES THE GAP CLOSE?

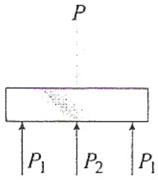
Stress in the two outer posts when the gap is just closed:

$$\sigma = E\varepsilon = E\left(\frac{s}{L}\right) = (30\text{ GPa})\left(\frac{1.0\text{ mm}}{2.0\text{ m}}\right)$$

$$= 15\text{ MPa}$$

Since this stress is less than the allowable stress, the allowable force P will close the gap.

EQUILIBRIUM EQUATION



$$2P_1 + P_2 = P \qquad \text{(Eq. 1)}$$

COMPATIBILITY EQUATION

δ_1 = shortening of outer posts

δ_2 = shortening of inner post

$$\delta_1 = \delta_2 + s \qquad \text{(Eq. 2)}$$

FORCE-DISPLACEMENT RELATIONS

$$\delta_1 = \frac{P_1 L}{EA} \quad \delta_2 = \frac{P_2 L}{EA} \qquad \text{(Eqs. 3, 4)}$$

SOLUTION OF EQUATIONS

Substitute (3) and (4) into Eq. (2):

$$\frac{P_1 L}{EA} = \frac{P_2 L}{EA} + s \quad \text{or} \quad P_1 - P_2 = \frac{EAs}{L} \qquad \text{(Eq. 5)}$$

Solve simultaneously Eqs. (1) and (5):

$$P = 3P_1 - \frac{EAs}{L}$$

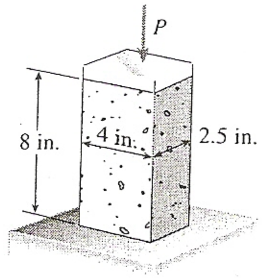
By inspection, we know that P_1 is larger than P_2. Therefore, P_1 will control and will be equal to $\sigma_{allow} A$.

$$P_{allow} = 3\sigma_{allow} A - \frac{EAs}{L}$$

$$= 2400\text{ kN} - 600\text{ kN} = 1800\text{ kN}$$

$$= 1.8\text{ MN} \quad \longleftarrow$$

Solution 2.6-3 Standard brick in compression



A = 2.5 in. × 4.0 in. = 10.0 in.^{2}

Maximum normal stress:

$$\sigma_x = \frac{P}{A}$$

Maximum shear stress:

$$\tau_{max} = \frac{\sigma_x}{2} = \frac{P}{2A}$$

σ_{ult} = 3600 psi $\qquad \tau_{ult}$ = 1200 psi

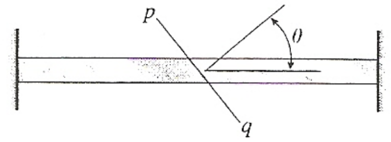
Because τ_{ult} is less than one-half of σ_{ult}, the shear stress governs.

$$\tau_{max} = \frac{P}{2A} \quad \text{or} \quad P_{max} = 2A\tau_{ult}$$

$$P_{max} = 2(10.0\text{ in.}^2)(1200\text{ psi})$$

$$= 24{,}000\text{ lb} \quad \longleftarrow$$

Solution 2.6-12 Copper bar between rigid supports



$\alpha = 17 \times 10^{-6}/°\mathrm{C}$

$E = 120$ GPa

Plane pq: $\theta = 55°$

Allowable stresses on plane pq:

$\sigma_{allow} = 60$ MPa (Compression)

$\tau_{allow} = 30$ MPa (Shear)

(a) Maximum permissible temperature rise ΔT

$\sigma_\theta = \sigma_x \cos^2\theta \quad -60 \text{ MPa} = \sigma_x (\cos 55°)^2$

$\sigma_x = -182.4$ MPa

$\tau_\theta = -\sigma_x \sin\theta \cos\theta$

$30 \text{ MPa} = -\sigma_x (\sin 55°)(\cos 55°)$

$\sigma_x = -63.85$ MPa

Shear stress governs. $\sigma_x = -63.85$ MPa

Due to temperature increase ΔT:

$\sigma_x = -E\alpha(\Delta T)$ (See Eq. 2-18 in Section 2.5)

$-63.85 \text{ MPa} = -(120 \text{ GPa})(17 \times 10^{-6}/°\mathrm{C})(\Delta T)$

$\Delta T = 31.3°\mathrm{C}$ ←

(b) Stresses on plane pq

$\sigma_x = -63.85$ MPa

$\sigma_\theta = \sigma_x \cos^2\theta = (-63.85 \text{ MPa})(\cos 55°)^2$

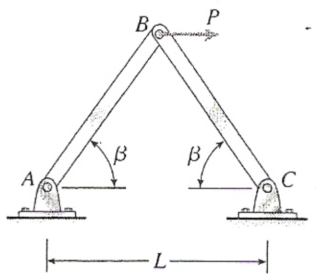
$= -21.0$ MPa (Compression) ←

$\tau_\theta = -\sigma_x \sin\theta \cos\theta$

$= -(-63.85 \text{ MPa})(\sin 55°)(\cos 55°)$

$= 30.0$ MPa (Counter clockwise) ←

Solution 2.7-6 Truss subjected to a load P



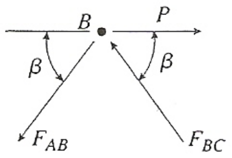
$\beta = 60°$

$L_{AB} = L_{BC} = L$

$\sin\beta = \sqrt{3}/2$

$\cos\beta = 1/2$

Free-body diagram of joint B



$\Sigma F_{vert} = 0 \quad \uparrow + \quad \downarrow -$

$-F_{AB}\sin\beta + F_{BC}\sin\beta = 0$

$F_{AB} = F_{BC}$ (Eq. 1)

$\Sigma F_{horiz} = 0 \quad \xrightarrow{+} \quad \leftarrow$

$-F_{AB}\cos\beta - F_{BC}\cos\beta + P = 0$

$$F_{AB} = F_{BC} = \frac{P}{2\cos\beta} = \frac{P}{2(1/2)} = P \quad \text{(Eq. 2)}$$

Axial forces: $N_{AB} = P$ (tension)

$N_{BC} = -P$ (compression)

(a) Strain energy of truss (Eq. 2-40)

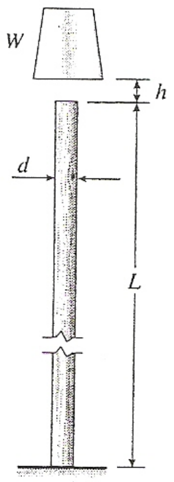
$$U = \sum \frac{N_i^2 L_i}{2E_i A_i} = \frac{(N_{AB})^2 L}{2EA} + \frac{(N_{BC})^2 L}{2EA}$$

$$= \frac{P^2 L}{EA} \quad \leftarrow$$

(b) Horizontal displacement of joint B (Eq. 2-42)

$$\delta_B = \frac{2U}{P} = \frac{2}{P}\left(\frac{P^2 L}{EA}\right) = \frac{2PL}{EA} \quad \leftarrow$$

Solution 2.8-7 **Weight falling on a wood pole**



$W = 4500$ lb $\qquad d = 12$ in.

$L = 15$ ft $= 180$ in.

$$A = \frac{\pi d^2}{4} = 113.10 \text{ in.}^2$$

$E = 1.6 \times 10^6$ psi

$\sigma_{allow} = 2500$ psi $(= \sigma_{max})$

Find h_{max}

STATIC STRESS

$$\sigma_{st} = \frac{W}{A} = \frac{4500 \text{ lb}}{113.10 \text{ in.}^2} = 39.79 \text{ psi}$$

MAXIMUM HEIGHT h_{max}

Eq. (2-59): $\quad \sigma_{max} = \sigma_{st}\left[1 + \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}\right]$

or

$$\frac{\sigma_{max}}{\sigma_{st}} - 1 = \left(1 + \frac{2hE}{L\sigma_{st}}\right)^{1/2}$$

Square both sides and solve for h:

$$h = h_{max} = \frac{L\sigma_{max}}{2E}\left(\frac{\sigma_{max}}{\sigma_{st}} - 2\right) \quad \longleftarrow$$

SUBSTITUTE NUMERICAL VALUES:

$$h_{max} = \frac{(180 \text{ in.})(2500 \text{ psi})}{2(1.6 \times 10^6 \text{ psi})}\left(\frac{2500 \text{ psi}}{39.79 \text{ psi}} - 2\right)$$

$$= 8.55 \text{ in.} \quad \longleftarrow$$