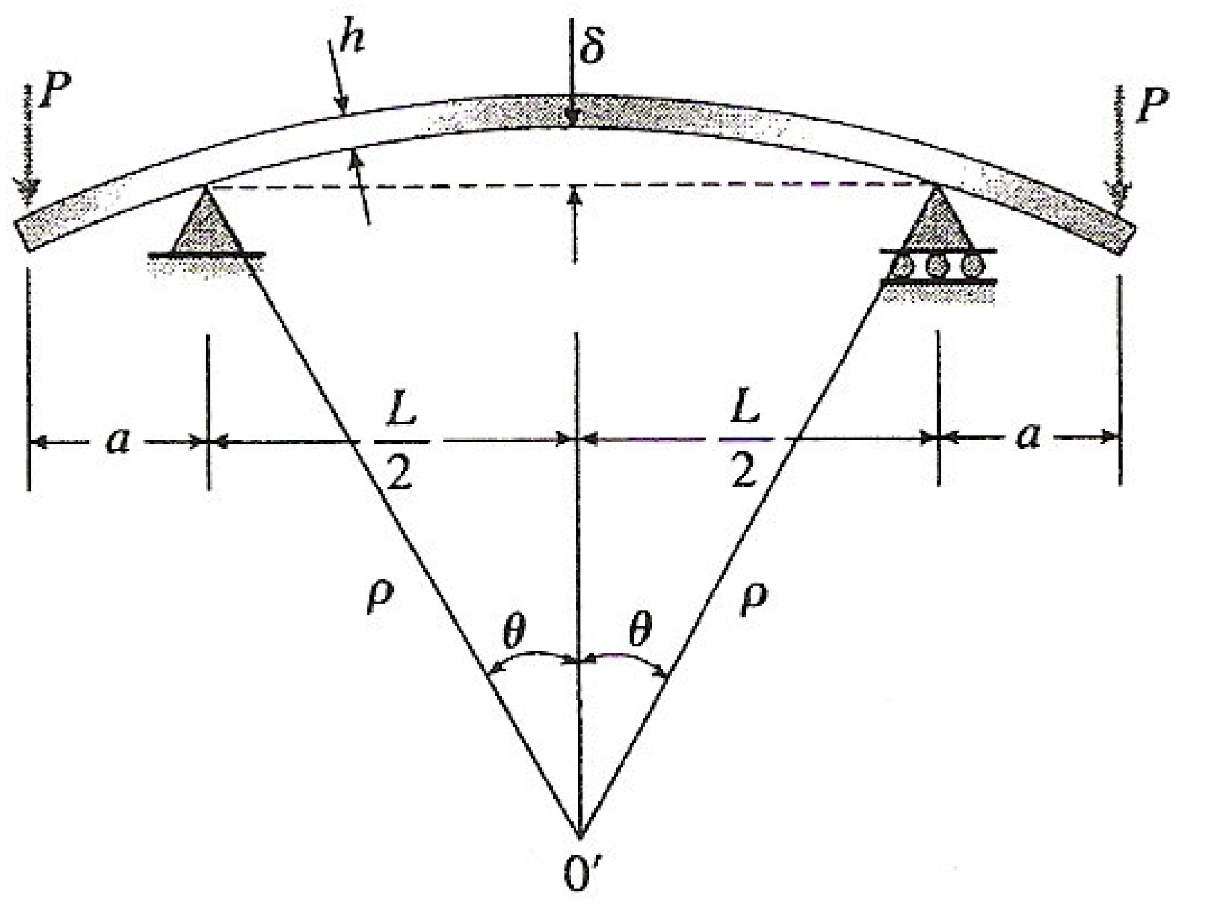
Solution 5.4-6 **Bar of rectangular cross section**



$L = 1.2$ m $\quad h = 100$ mm $\quad \delta = 3.6$ mm

Note that the deflection curve is nearly flat ($L/\delta = 333$) and θ is a very small angle.

$$\sin\theta = \frac{L/2}{\rho}$$

$$\theta = \frac{L/2}{\rho} \text{ (radians)}$$

$$\delta = \rho\,(1 - \cos\theta) = \rho\left(1 - \cos\frac{L}{2\rho}\right)$$

Substitute numerical values (ρ = meters):

$$0.0036 = \rho\left(1 - \cos\frac{0.6}{\rho}\right)$$

Solve numerically: $\rho = 50.00$ m

NORMAL STRAIN

$$\varepsilon = \frac{y}{\rho} = \frac{h/2}{\rho} = \frac{50 \text{ mm}}{50{,}000 \text{ mm}} = 1000 \times 10^{-6} \quad \longleftarrow$$

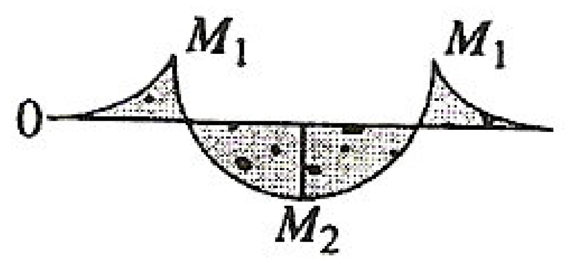
(Elongation on top; shortening on bottom)

Solution 5.5-10 **Railroad tie (or sleeper)**

DATA $\quad P = 175$ kN $\quad b = 300$ mm $\quad h = 250$ mm
$\quad L = 1500$ mm $\quad a = 500$ mm

$$q = \frac{2P}{L + 2a} \qquad S = \frac{bh^2}{6} = 3.125 \times 10^{-3}\ \text{m}^3$$

BENDING-MOMENT DIAGRAM



$$M_1 = \frac{qa^2}{2} = \frac{Pa^2}{L + 2a}$$

$$M_2 = \frac{q}{2}\left(\frac{L}{2} + a\right)^2 - \frac{PL}{2}$$

$$= \frac{P}{L + 2a}\left(\frac{L}{2} + a\right)^2 - \frac{PL}{2}$$

$$= \frac{P}{4}(2a - L)$$

Substitute numerical values:

$M_1 = 17{,}500$ N · m $\quad M_2 = -21{,}875$ N · m

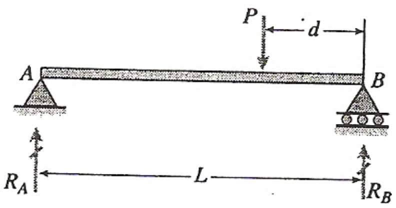
$M_{max} = 21{,}875$ N · m

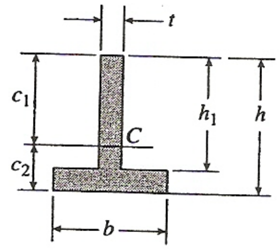
MAXIMUM BENDING STRESS

$$\sigma_{max} = \frac{M_{max}}{S} = \frac{21{,}875 \text{ N} \cdot \text{m}}{3.125 \times 10^{-3}\ \text{m}^3} = 7.0 \text{ MPa} \quad \longleftarrow$$

(Tension on top; compression on bottom)

Solution 5.5-16 Simple beam of T-section





$P = 5.4$ kN $\quad L = 3.0$ m

$b = 75$ mm $\quad t = 25$ mm

$d = 1.2$ m $\quad h = 100$ mm $\quad h_1 = 75$ mm

PROPERTIES OF THE CROSS SECTION

$A = 3750 \text{ mm}^2$

$c_1 = 62.5$ mm $\quad c_2 = 37.5$ mm

$I_C = 3.3203 \times 10^6 \text{ mm}^4$

REACTIONS OF THE BEAM

$R_A = 2.16$ kN $\quad R_B = 3.24$ kN

MAXIMUM BENDING MOMENT

$M_{max} = R_A(L - d) = R_B(d) = 3888 \text{ N} \cdot \text{m}$

MAXIMUM TENSILE STRESS

$$\sigma_t = \frac{M_{max}c_2}{I_C} = \frac{(3888 \text{ N} \cdot \text{m})(0.0375 \text{ m})}{3.3203 \times 10^6 \text{ mm}^4}$$

$= 43.9$ MPa $\longleftarrow$

MAXIMUM COMPRESSIVE STRESS

$$\sigma_c = \frac{M_{max}c_1}{I_C} = \frac{(3888 \text{ N} \cdot \text{m})(0.0625 \text{ m})}{3.3203 \times 10^6 \text{ mm}^4}$$

$= 73.2$ MPa $\longleftarrow$

Solution 5.6-12 Cantilever beam

DATA $\quad L = 450$ mm $\quad P = 400$ N

$\sigma_{allow} = 60$ MPa

γ = weight density of steel
$= 77.0 \text{ kN/m}^3$

WEIGHT OF BEAM PER UNIT LENGTH

$$q = \gamma\left(\frac{\pi d^2}{4}\right)$$

MAXIMUM BENDING MOMENT

$$M_{max} = PL + \frac{qL^2}{2} = PL + \frac{\pi \gamma d^3 L^2}{8}$$

SECTION MODULUS $S = \dfrac{\pi d^3}{32}$

MINIMUM DIAMETER

$M_{max} = \sigma_{allow} S$

$$PL + \frac{\pi \gamma d^2 L^2}{8} = \sigma_{allow}\left(\frac{\pi d^3}{32}\right)$$

Rearrange the equation:

$$\sigma_{allow} d^3 - 4\gamma L^2 d^2 - \frac{32\, PL}{\pi} = 0$$

(Cubic equation with diameter d as unknown.)

Substitute numerical values (d = meters):

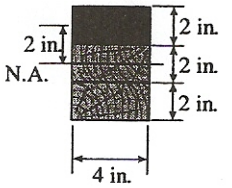
$$(60 \times 10^6 \text{ N/m}^2)d^3 - 4(77{,}000 \text{ N/m}^3)(0.45\text{m})^2 d^2 - \frac{32}{\pi}(400 \text{ N})(0.45 \text{ m}) = 0$$

$60{,}000d^3 - 62.37d^2 - 1.833465 = 0$

Solve the equation numerically:

$d = 0.031614$ m $\quad d_{min} = 31.61$ mm $\longleftarrow$

Solution 5.8-7 **Laminated wood beam on simple supports**



$L = 6 \text{ ft} = 72 \text{ in.}$
$\tau_{allow} = 65 \text{ psi}$
$\sigma_{allow} = .1800 \text{ psi}$

ALLOWABLE LOAD BASED UPON SHEAR STRESS IN THE GLUED JOINTS

$$\tau = \frac{VQ}{Ib} \qquad Q = (4 \text{ in.})(2 \text{ in.})(2 \text{ in.}) = 16 \text{ in.}^3$$

$$V = \frac{P}{2} \qquad I = \frac{bh^3}{12} = \frac{1}{12}(4 \text{ in.})(6 \text{ in.})^3 = 72 \text{ in.}^4$$

$$\tau = \frac{(P/2)(16 \text{ in.}^3)}{(72 \text{ in.}^4)(4 \text{ in.})} = \frac{P}{36} \qquad (P = \text{lb}; \tau = \text{psi})$$

$$P_1 = 36\tau_{allow} = 36\ (65 \text{ psi}) = 2340 \text{ lb}$$

ALLOWABLE LOAD BASED UPON BENDING STRESS

$$\sigma = \frac{M}{S} \qquad M = \frac{PL}{4} = P\left(\frac{72 \text{ in.}}{4}\right) = 18P \text{ (lb-in.)}$$

$$S = \frac{bh^2}{6} = \frac{1}{6}(4 \text{ in.})(6 \text{ in.})^2 = 24 \text{ in.}^3$$

$$\sigma = \frac{(18P \text{ lb-in.})}{24 \text{ in.}^3} = \frac{3P}{4} \qquad (P = \text{lb}; \sigma = \text{psi})$$

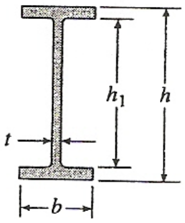
$$P_2 = \frac{4}{3}\sigma_{allow} = \frac{4}{3}(1800 \text{ psi}) = 2400 \text{ lb}$$

ALLOWABLE LOAD

Shear stress in the glued joints governs.

$P_{allow} = 2340 \text{ lb}$ ←

Solution 5.10-8 **Bridge girder (simple beam)**



$L = 14 \text{ m}$
$b = 450 \text{ mm} \qquad t = 15 \text{ mm}$
$h = 1860 \text{ mm} \qquad h_1 = 1800 \text{ mm}$
$\sigma_{allow} = 110 \text{ MPa}$
$\tau_{allow} = 50 \text{ MPa}$
$c = h/2 = 930 \text{ mm}$

Eq. (5-47): $I = \frac{1}{12}(bh^3 - bh_1^3 + th_1^3)$

$$= 29.897 \times 10^9 \text{ mm}^4$$

$$S = \frac{I}{c} = \frac{29.897 \times 10^9 \text{ mm}^4}{930 \text{ mm}} = 32.147 \times 10^6 \text{ mm}^3$$

(a) MAXIMUM LOAD BASED UPON BENDING STRESS

$$M_{max} = \frac{qL^2}{8} \qquad \sigma = \frac{M_{max}}{S} \qquad q = \frac{8S\sigma}{L^2}$$

$$q_{max} = \frac{8S\sigma_{allow}}{L^2} = \frac{8(32.147 \times 10^6 \text{ mm}^3)(110 \text{ MPa})}{(14 \text{ m})^2}$$

$$= 144.3 \times 10^3 \text{ N/m} = 144 \text{ kN/m} \quad \leftarrow$$

(b) MAXIMUM LOAD BASED UPON SHEAR STRESS

$$V_{max} = \frac{qL}{2} \qquad \tau_{max} = \frac{V_{max}}{8It}(bh^2 - bh_1^2 + th_1^2) \qquad \text{(Eq. 5-48a)}$$

$$q_{max} = \frac{2V_{max}}{L} = \frac{16It(\tau_{allow})}{L(bh^2 - bh_1^2 + th_1^2)}$$

Substitute numerical values:

$q_{max} = 173.8 \times 10^3 \text{ N/m} = 174 \text{ kN/m}$ ←

NOTE: Bending stress governs. $\quad q_{allow} = 144 \text{ kN/m}$

Solution 5.11-6 **Two wood box beams**

Cross-sectional dimensions are the same.

All dimensions in millimeters.

$b = 200 \quad b_1 = 200 - 2(20) = 160$

$h = 360 \quad h_1 = 360 - 2(20) = 320$

$t = 20$

F = allowable load per nail = 250 N

V = shear force = 3.2 kN

$$I = \frac{1}{12}(bh^3 - b_1h_1^3) = 340.69 \times 10^6 \text{ mm}^4$$

s = longitudinal spacing of the nails

f = shear flow between one flange and both webs

$$f = \frac{2F}{s} = \frac{VQ}{I} \qquad \therefore s_{max} = \frac{2FI}{VQ}$$

(a) BEAM A

$$Q = A_p d_p = (bt)\left(\frac{h-t}{2}\right) = (200)(20)\left(\frac{1}{2}\right)(340)$$

$$= 680 \times 10^3 \text{ mm}^3$$

$$s_A = \frac{2FI}{VQ} = \frac{(2)(250\text{ N})(340.7 \times 10^6\text{ mm}^4)}{(3.2\text{ kN})(680 \times 10^3\text{ mm}^3)}$$

$$= 78.3 \text{ mm} \quad \leftarrow$$

(b) BEAM B

$$Q = A_f d_f = (b - 2t)(t)\left(\frac{h-t}{2}\right) = (160)(20)\frac{1}{2}(340)$$

$$= 544 \times 10^3 \text{ mm}^3$$

$$s_B = \frac{2FI}{VQ} = \frac{(2)(250\text{ N})(340.7 \times 10^6\text{ mm}^4)}{(3.2\text{ kN})(544 \times 10^3\text{ mm}^3)}$$

$$= 97.9 \text{ mm} \quad \leftarrow$$

(c) BEAM B IS MORE EFFICIENT because the shear flow on the contact surfaces is smaller and therefore fewer nails are needed. $\leftarrow$