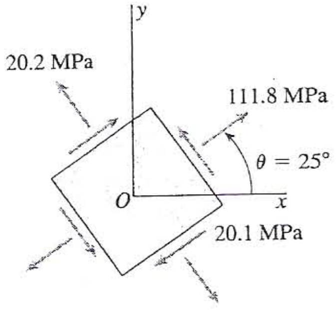
Solution 7.2-2 Plane stress (angle θ)



$\sigma_x = 80$ MPa $\quad \sigma_y = 52$ MPa $\quad \tau_{xy} = 48$ MPa

$\theta = 25°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$= 111.8$ MPa ←

$$\tau_{x_1y_1} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

$= 20.1$ MPa ←

$\sigma_{y_1} = \sigma_x + \sigma_y - \sigma_{x_1} = 20.2$ MPa ←

Solution 7.3-15 Plane stress

$\sigma_x = -3000$ psi $\quad \sigma_y = -12{,}000$ psi

$\tau_{xy} = 6000$ psi

(a) PRINCIPAL STRESSES

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} = 1.3333$$

$2\theta_p = 53.13°$ and $\theta_p = 26.57°$

$2\theta_p = 233.13°$ and $\theta_p = 116.57°$

$$\sigma_{x_1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

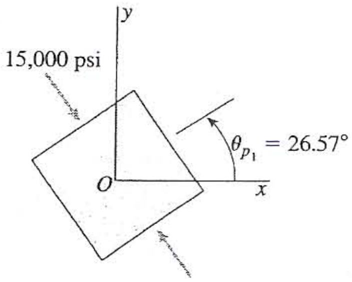
For $2\theta_p = 53.13°$: $\sigma_{x_1} = 0$

For $2\theta_p = 233.13°$: $\sigma_{x_1} = -15{,}000$ psi

Therefore,

$\sigma_1 = 0$ and $\theta_{p_1} = 26.57°$

$\sigma_2 = -15{,}000$ psi and $\theta_{p_2} = 116.57°$ ←



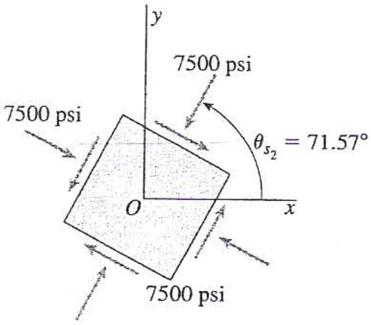
(b) MAXIMUM SHEAR STRESSES

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 7500 \text{ psi}$$

$\theta_{s_1} = \theta_{p_1} - 45° = -18.43°$ and $\tau = 7500$ psi

$\theta_{s_2} = \theta_{p_1} + 45° = 71.57°$ and $\tau = -7500$ psi ←

$$\sigma_{aver} = \frac{\sigma_x + \sigma_y}{2} = -7500 \text{ psi} \leftarrow$$



Solution 7.4-4 Biaxial stress

$\sigma_x = -60$ MPa $\quad \sigma_y = 20$ MPa $\quad \tau_{xy} = 0$

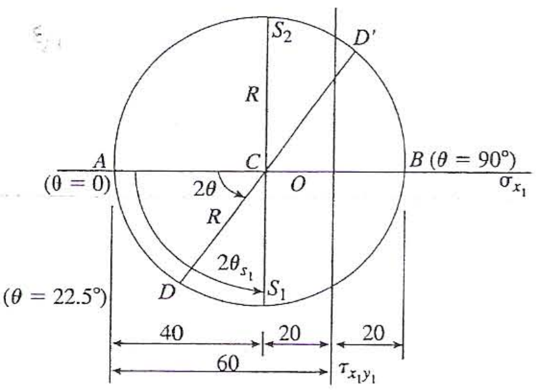
(a) ELEMENT AT $\theta = 22.5°$

(All stresses in MPa)

$2\theta = 45° \quad \theta = 22.5°$

$2R = 60 + 20 = 80$ MPa $\quad R = 40$ MPa

Point C: $\sigma_{x_1} = -20$ MPa

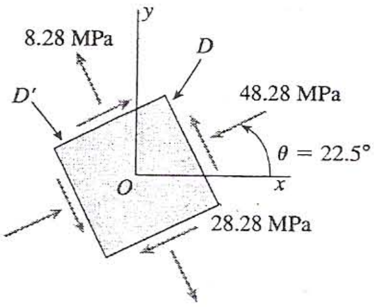


Point D: $\sigma_{x_1} = -20 - R\cos 2\theta = -48.28$ MPa

$\tau_{x_1y_1} = R\sin 2\theta = 28.28$ MPa

Point D': $\sigma_{x_1} = R\cos 2\theta - 20 = 8.28$ MPa

$\tau_{x_1y_1} = -R\sin 2\theta = -28.28$ MPa



(b) MAXIMUM SHEAR STRESSES

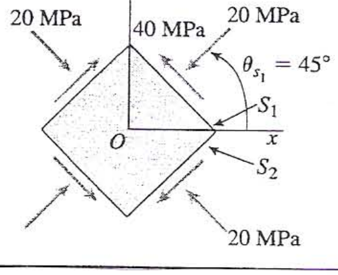
Point S_1: $2\theta_{s_1} = 90° \quad \theta_{s_1} = 45°$

$\tau_{max} = R = 40$ MPa

Point S_2: $2\theta_{s_2} = -90° \quad \theta_{s_2} = -45°$

$\tau_{min} = -R = -40$ MPa

$\sigma_{aver} = -20$ MPa



(a) PRINCIPAL STRESSES

$2\theta_{p_1} = 180° - \alpha = 136.40°$ $\quad \theta_{p_1} = 68.20°$

$2\theta_{p_2} = -\alpha = -43.60°$ $\quad \theta_{p_2} = -21.80°$

Point P_1: $\sigma_1 = R = 43.5$ MPa

Point P_2: $\sigma_2 = -R = -43.5$ MPa

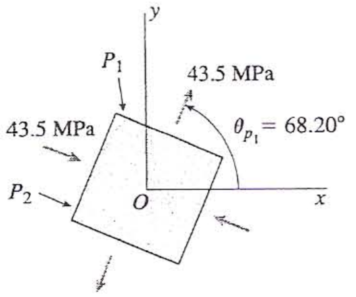
(b) MAXIMUM SHEAR STRESSES

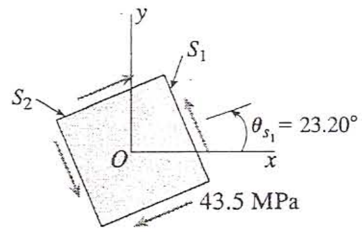
$2\theta_{s_1} = 90° - \alpha = 46.40°$ $\quad \theta_{s_1} = 23.20°$

$2\theta_{s_2} = 2\theta_{s_1} + 180° = 226.40°$ $\quad \theta_{s_2} = 113.20°$

Point S_1: $\sigma_{aver} = 0$ $\quad \tau_{max} = R = 43.5$ MPa

Point S_2: $\sigma_{aver} = 0$ $\quad \tau_{min} = -43.5$ MPa





Solution 7.6-2 Triaxial stress

$\sigma_x = -60$ MPa $\quad \sigma_y = -40$ MPa

$\sigma_z = -40$ MPa

$a = 300$ mm $\quad b = 150$ mm $\quad c = 150$ mm

$E = 200$ GPa $\quad \nu = 0.30$ (steel)

(a) MAXIMUM SHEAR STRESS

$\sigma_1 = -40$ MPa $\quad \sigma_2 = -40$ MPa

$\sigma_3 = -60$ MPa

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = 10.0 \text{ MPa} \quad \longleftarrow$$

(b) CHANGES IN DIMENSIONS

Eq. (7-53a): $\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z) = -180.0 \times 10^{-6}$

Eq. (7-53b): $\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_z + \sigma_x) = -50.0 \times 10^{-6}$

Eq. (7-53c): $\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) = -50.0 \times 10^{-6}$

$\Delta a = a\varepsilon_x = -0.0540$ mm (decrease)

$\Delta b = b\varepsilon_y = -0.0075$ mm (decrease) $\longleftarrow$

$\Delta c = c\varepsilon_z = -0.0075$ mm (decrease)

(c) CHANGE IN VOLUME

Eq. (7-56):

$$e = \frac{1 - 2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = -280.0 \times 10^{-6}$$

$V = abc$

$\Delta V = e\,(abc) = -1890 \text{ mm}^3$ (decrease) $\longleftarrow$

(d) STRAIN ENERGY

Eq. (7-57a): $u = \frac{1}{2}(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z)$

$= 0.00740$ MPa

$U = u\,(abc) = 50.0 \text{ N} \cdot \text{m} = 50.0 \text{ J}$ $\longleftarrow$