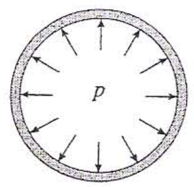
Solution 8.2-1 Spherical tank

CROSS SECTION



Radius: $r = \frac{1}{2}(45 \text{ ft}) = 270 \text{ in.}$

Internal pressure: $p = 400$ psi

Yield stress: $\sigma_Y = 80$ ksi (steel)

Factor of safety: $n = 3.5$

MINIMUM WALL THICKNESS t_{min}

From Eq. (8-1): $\sigma_{max} = \frac{pr}{2t}$ or $\frac{\sigma_Y}{n} = \frac{pr}{2t}$

$$t = \frac{prn}{2\sigma_Y} = 2.36 \text{ in.}$$

Use the next higher 1/4 inch: $t_{min} = 2.50$ in. ←

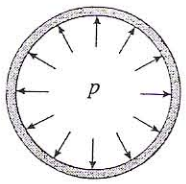
Solution 8.2-11 Pressurized sphere under water

CROSS-SECTION

$r = 4.8$ in. $\quad p_1 = 24$ psi

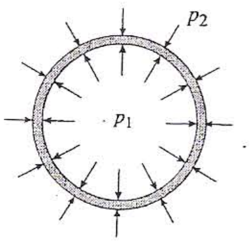
$t = 0.4$ in. $\quad \gamma$ = density of water = 62.4 lb/ft^3

(1) IN AIR: $p_1 = 24$ psi



(1) IN AIR

(2) UNDER WATER: $p_1 = 24$ psi



(2) UNDER WATER

D_0 = depth of water (in.)

$$p_2 = \gamma D_0 = \left(\frac{62.4 \text{ lb/ft}^3}{1728 \text{ in.}^3/\text{ft}^3}\right) D_0 = 0.036111\, D_0 \text{ (psi)}$$

Compressive stress in tank wall equals 90 psi.
(Note: σ is positive in tension.)

$$\sigma = \frac{pr}{2t} = \frac{(p_1 - p_2)r}{2t} \qquad \sigma = -90 \text{ psi}$$

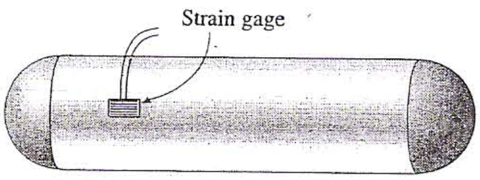
$$-90 \text{ psi} = \frac{(24 \text{ psi} - 0.03611\, D_0)(4.8 \text{ in.})}{2(0.4 \text{ in.})}$$

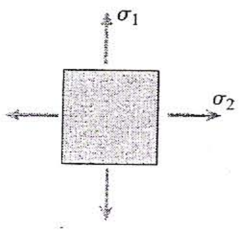
$$= 144 - 0.21667 D_0$$

Solve for D_0: $D_0 = \frac{234}{0.21667}$

$$= 1080 \text{ in.} = 90 \text{ ft} \quad ←$$

Solution 8.3-6 Cylindrical tank





$\tau_{ULT} = 82$ MPa $\quad E = 205$ GPa $\quad \nu = 0.30$

$n = 2$ (factor of safety) $\quad \tau_{max} = \frac{\tau_{ULT}}{n} = 41$ MPa

Find maximum allowable strain reading at the gage.

$$\sigma_1 = \frac{pr}{t} \qquad \sigma_2 = \frac{pr}{2t}$$

From Eq. (8-10):

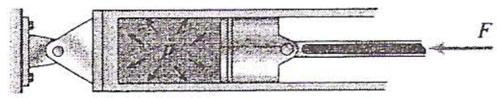
$$\tau_{max} = \frac{\sigma_1}{2} = \frac{pr}{2t} \quad \therefore \quad p_{max} = \frac{2t\tau_{max}}{r}$$

From Eq. (8-11a): $\varepsilon_2 = \frac{pr}{2tE}(1 - 2\nu)$

$$(\varepsilon_2)_{max} = \frac{p_{max} r}{2tE}(1 - 2\nu) = \frac{\tau_{max}}{E}(1 - 2\nu)$$

$$\varepsilon_{max} = \frac{41 \text{ MPa}}{205 \text{ GPa}}(1 - 0.60) = 80 \times 10^{-6} \quad ←$$

Solution 8.3-7 **Cylinder with internal pressure**



$d = 1.80$ in. $\quad r = 0.90$ in.
$F = 3500$ lb $\quad \tau_{allow} = 5500$ psi

Find minimum thickness t_{min}.

Pressure in cylinder: $p = \dfrac{F}{A} = \dfrac{F}{\pi r^2}$

Maximum shear stress (Eq. 8-10):

$$\tau_{max} = \frac{pr}{2t} = \frac{F}{2\pi r t}$$

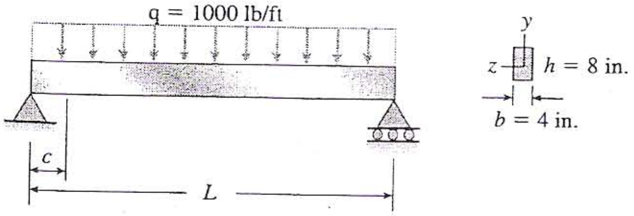
Minimum thickness:

$$t_{min} = \frac{F}{2\pi r \tau_{allow}}$$

Substitute numerical values:

$$t_{min} = \frac{3500 \text{ lb}}{2\pi(0.90 \text{ in.})(5500 \text{ psi})} = 0.113 \text{ in.} \quad \longleftarrow$$

Solution 8.4-3 **Simply supported beam**



$$I = \frac{bh^3}{12} = 170.67 \text{ in.}^4$$

$$A = bh = 32 \text{ in.}^2$$

$$M = \frac{qLc}{2} - \frac{qc^2}{2} = 54{,}000 \text{ lb-in.}$$

$c = 1.0$ ft $\quad L = 10$ ft $\quad V = \dfrac{qL}{2} - qc = 4{,}000$ lb

(a) NEUTRAL AXIS

$$\sigma_x = 0 \quad \sigma_y = 0 \quad \tau_{xy} = -\frac{3V}{2A} = -187.5 \text{ psi}$$

Pure shear: $\sigma_1 = 188$ psi, $\sigma_2 = -188$ psi,
$\tau_{max} = 188$ psi $\longleftarrow$

(Continued)

(b) 2 in. ABOVE THE NEUTRAL AXIS

$$\sigma_x = -\frac{My}{I} = -\frac{(54{,}000 \text{ lb-in.})(2 \text{ in.})}{170.67 \text{ in.}^4} = -632.8 \text{ psi}$$

$$\sigma_y = 0$$

$$\tau_{xy} = -\frac{VQ}{Ib} = -\frac{(4000 \text{ lb})(4 \text{ in.})(2 \text{ in.})(3 \text{ in.})}{(170.67 \text{ in.}^4)(4 \text{ in.})}$$

$$= -140.6 \text{ psi}$$

From Eq. (8-22):

$$\sigma_{1,2} = \frac{\sigma_x}{2} \pm \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = -316.4 \pm 346.2 \text{ psi}$$

$\sigma_1 = 30$ psi $\quad \sigma_2 = -663$ psi $\longleftarrow$

From Eq. (8-24):

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x}{2}\right)^2 + \tau_{xy}^2} = 346 \text{ psi} \quad \longleftarrow$$

(c) TOP OF THE BEAM

$$\sigma_x = -\frac{Mc}{I} = -\frac{(54{,}000 \text{ lb-in.})(4 \text{ in.})}{170.67 \text{ in.}^4} = -1266 \text{ psi}$$

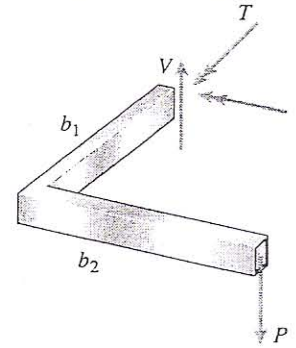
$$\sigma_y = 0 \quad \tau_{xy} = 0$$

Uniaxial stress: $\sigma_1 = 0$, $\sigma_2 = -1266$ psi,
$\tau_{max} = 633$ psi $\longleftarrow$

Solution 8.5-11 **L-shaped bracket**

$P = 150$ lb $\quad b_1 = 20$ in. $\quad b_2 = 30$ in.
$t = 0.125$ in. $\quad h = 3.5$ in. $\quad b = 2.0$ in.

FREE-BODY DIAGRAM OF BRACKET



STRESS RESULTANTS AT THE SUPPORT

Torque: $T = Pb_2 = (150 \text{ lb})(30 \text{ in.}) = 4500$ lb-in.
Moment: $M = Pb_1 = (150 \text{ lb})(20 \text{ in.}) = 3000$ lb-in.
Shear force: $V = P = 150$ lb

PROPERTIES OF THE CROSS SECTION

For torsion:

$$A_m = (b - t)(h - t) = (1.875 \text{ in.})(3.375 \text{ in.}) = 6.3281 \text{ in.}^2$$

For bending: $c = \dfrac{h}{2} = 1.75$ in.

$$I = \frac{1}{12}(bh^3) - \frac{1}{12}(b - 2t)(h - 2t)^3$$

$$= \frac{1}{12}(2.0 \text{ in.})(3.5 \text{ in.})^3 - \frac{1}{12}(1.75 \text{ in.})(3.25 \text{ in.})^3$$

$$= 2.1396 \text{ in.}^4$$

STRESSES AT POINT A ON THE TOP OF THE BRACKET

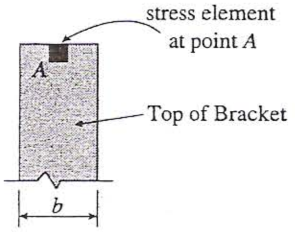
$$\tau = \frac{T}{2tA_m} = \frac{4500 \text{ lb-in.}}{2(0.125 \text{ in.})(6.3281 \text{ in.}^2)} = 2844 \text{ psi}$$

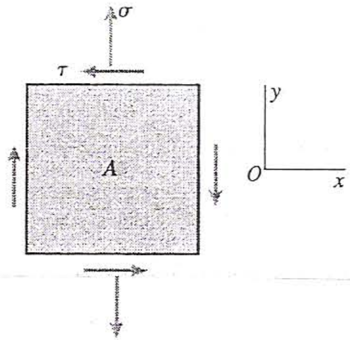
$$\sigma = \frac{Mc}{I} = \frac{(3000 \text{ lb-in.})(1.75 \text{ in.})}{2.1396 \text{ in.}^4} = 2454 \text{ psi}$$

(The shear force V produces no stresses at point A.)

STRESS ELEMENT AT POINT A

(This view is looking downward at the top of the bracket.)





$\sigma_x = 0 \quad \sigma_y = \sigma = 2454$ psi
$\tau_{xy} = -\tau = -2844$ psi

PRINCIPAL STRESSES AND MAXIMUM SHEAR STRESS

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$= 1227 \text{ psi} \pm 3097 \text{ psi}$$

$\sigma_1 = 4324$ psi $\quad \sigma_2 = -1870$ psi

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 3097 \text{ psi}$$

MAXIMUM TENSILE STRESS:

$\sigma_t = 4320$ psi ←