Solution 9.2-2 Simple beam

$$v = -\frac{q_0 L^4}{\pi^4 EI}\sin\frac{\pi x}{L}$$

$$v' = -\frac{q_0 L^3}{\pi^3 EI}\cos\frac{\pi x}{L}$$

$$v'' = \frac{q_0 L^2}{\pi^2 EI}\sin\frac{\pi x}{L}$$

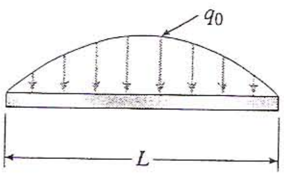
$$v''' = \frac{q_0 L}{\pi EI}\cos\frac{\pi x}{L}$$

$$v'''' = -\frac{q_0}{EI}\sin\frac{\pi x}{L}$$

(a) LOAD (EQ. 9-12c)

$$q = -EIv'''' = q_0 \sin\frac{\pi x}{L} \quad \longleftarrow$$

The load has the shape of a sine curve, acts downward, and has maximum intensity q_0. ←



(b) REACTIONS (EQ. 9-12b)

$$V = EIv''' = \frac{q_0 L}{\pi}\cos\frac{\pi x}{L}$$

At $x = 0$: $\quad V = R_A = \dfrac{q_0 L}{\pi}$ ←

At $x = L$: $\quad V = -R_B = -\dfrac{q_0 L}{\pi}$; $R_B = \dfrac{q_0 L}{\pi}$ ←

(c) MAXIMUM BENDING MOMENT (EQ. 9-12a)

$$M = EIv'' = \frac{q_0 L^2}{\pi^2}\sin\frac{\pi x}{L}$$

For maximum moment, $x = \dfrac{L}{2}$; $M_{max} = \dfrac{q_0 L^2}{\pi^2}$ ←

Solution 9.3-9 Simple beam (couple M_0)

BENDING-MOMENT EQUATION (EQ. 9-12a)

$$EIv'' = M = M_0\left(1 - \frac{x}{L}\right)$$

$$EIv' = M_0\left(x - \frac{x^2}{2L}\right) + C_1$$

$$EIv = M_0\left(\frac{x^2}{2} - \frac{x^3}{6L}\right) + C_1 x + C_2$$

B.C. $v(0) = 0 \quad \therefore C_2 = 0$

B.C. $v(L) = 0 \quad \therefore C_1 = -\dfrac{M_0 L}{3}$

$$v = -\frac{M_0 x}{6\,LEI}(2L^2 - 3Lx + x^2) \quad \longleftarrow$$

MAXIMUM DEFLECTION

$$v' = -\frac{M_0}{6\,LEI}(2L^2 - 6\,Lx + 3x^2)$$

Set $v' = 0$ and solve for x:

$$x_1 = L\left(1 - \frac{\sqrt{3}}{3}\right) \quad \longleftarrow$$

Substitute x_1 into the equation for v:

$$\delta_{max} = -(v)_{x-x_1}$$

$$= \frac{M_0 L^2}{9\sqrt{3}EI} \quad \longleftarrow$$

(These results agree with Case 7, Table G-2.)

Solution 9.3-16 **Simple beam (partial uniform load)**

BENDING-MOMENT EQUATION (EQ. 9-12a)

$$EIv'' = M = \frac{3qLx}{8} - \frac{qx^2}{2} \quad \left(0 \le x \le \frac{L}{2}\right)$$

$$EIv' = \frac{3qLx^2}{16} - \frac{qx^3}{6} + C_1 \quad \left(0 \le x \le \frac{L}{2}\right)$$

$$EIv'' = M = \frac{qL^2}{8} - \frac{qLx}{8} \quad \left(\frac{L}{2} \le x \le L\right)$$

$$EIv' = \frac{qL^2x}{8} - \frac{qLx^2}{16} + C_2 \quad \left(\frac{L}{2} \le x \le L\right)$$

B.C. 1 $(v')_{\text{Left}} = (v')_{\text{Right}}$ at $x = \frac{L}{2}$

$$\therefore C_2 = C_1 - \frac{qL^3}{48}$$

$$EIv = \frac{qLx^3}{16} - \frac{qx^4}{24} + C_1x + C_3 \quad \left(0 \le x \le \frac{L}{2}\right)$$

B.C. 2 $v(0) = 0 \quad \therefore C_3 = 0$

$$EIv = \frac{qL^2x^2}{16} - \frac{qLx^3}{48} + C_1x - \frac{qL^3x}{48} + C_4 \quad \left(\frac{L}{2} \le x \le L\right)$$

B.C. 3 $v(L) = 0 \quad \therefore C_4 = -C_1L - \frac{qL^4}{48}$

B.C. 4 $(v)_{\text{Left}} = (v)_{\text{Right}}$ at $x = \frac{L}{2}$

$$\therefore C_1 = -\frac{3qL^3}{128}$$

$$v = -\frac{qx}{384EI}(9L^3 - 24Lx^2 + 16x^3) \quad \left(0 \le x \le \frac{L}{2}\right) \longleftarrow$$

$$v = -\frac{qL}{384EI}(8x^3 - 24Lx^2 + 17L^2x - L^3) \longleftarrow \quad \left(\frac{L}{2} \le x \le L\right)$$

$$\delta_C = -v\left(\frac{L}{2}\right) = \frac{5qL^4}{768EI} \longleftarrow$$

(These results agree with Case 2, Table G-2.)

Solution 9.4-6 **Cantilever beam (parabolic load)**

LOAD EQUATION (EQ. 9-12 c)

$$EIv'''' = -q = -\frac{q_0}{L^2}(L^2 - x^2)$$

$$EIv''' = -\frac{q_0}{L^2}\left(L^2x - \frac{x^3}{3}\right) + C_1$$

B.C. 1 $EIv''' = V \quad EIv'''(L) = 0 \quad \therefore C_1 = \frac{2q_0L}{3}$

$$EIv'' = -\frac{q_0}{L^2}\left(\frac{L^2x^2}{2} - \frac{x^4}{12}\right) + \frac{2q_0L}{3}x + C_2$$

B.C. 2 $EIv'' = M \quad EIv''(L) = 0 \quad \therefore C_2 = -\frac{q_0L^2}{4}$

$$EIv' = -\frac{q_0}{L^2}\left(\frac{L^2x^3}{6} - \frac{x^5}{60}\right) + \frac{q_0Lx^2}{3} - \frac{q_0L^2x}{4} + C_3$$

B.C. 3 $v'(0) = 0 \quad \therefore C_3 = 0$

$$EIv = -\frac{q_0}{L^2}\left(\frac{L^2x^4}{24} - \frac{x^6}{360}\right) + \frac{q_0Lx^3}{9} - \frac{q_0L^2x^2}{8} + C_4$$

B.C. 4 $v(0) = 0 \quad \therefore C_4 = 0$

$$v = -\frac{q_0x^2}{360L^2EI}(45L^4 - 40L^3x + 15L^2x^2 - x^4) \longleftarrow$$

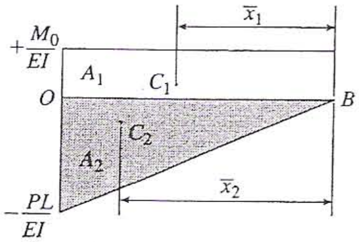
$$\delta_B = -v(L) = \frac{19q_0L^4}{360EI} \longleftarrow$$

$$v' = -\frac{q_0x}{60L^2EI}(15L^4 - 20L^3x + 10L^2x^2 - x^4)$$

$$\theta_B = -v'(L) = \frac{q_0L^3}{15EI} \longleftarrow$$

Solution 9.6-3 Cantilever beam (force P and couple M_0)

M/EI DIAGRAM



NOTE: A_1 is the M/EI diagram for M_0 (rectangle). A_2 is the M/EI diagram for P (triangle).

ANGLE OF ROTATION

Use the sign conventions for the moment-area theorems (page 628 of textbook).

$$A_1 = \frac{M_0 L}{EI} \qquad \bar{x}_1 = \frac{L}{2} \qquad A_2 = -\frac{PL^2}{2EI} \qquad \bar{x}_2 = \frac{2L}{3}$$

$$A_0 = A_1 + A_2 = \frac{M_0 L}{EI} - \frac{PL^2}{2EI}$$

$$\theta_{B/A} = \theta_B - \theta_A = A_0 \qquad \theta_A = 0$$

$$\theta_B = A_0 = \frac{M_0 L}{EI} - \frac{PL^2}{2EI}$$

(θ_B is positive when counterclockwise)

DEFLECTION

Q = first moment of areas A_1 and A_2 with respect to point B

$$Q = A_1\bar{x}_1 + A_2\bar{x}_2 = \frac{M_0 L^2}{2EI} - \frac{PL^3}{3EI}$$

$$t_{B/A} = Q = \delta_B \qquad \delta_B = \frac{M_0 L^2}{2EI} - \frac{PL^3}{3EI}$$

(δ_B is positive when upward)

FINAL RESULTS

To match the sign conventions for θ_B and δ_B used in Appendix G, change the signs as follows.

$$\theta_B = \frac{PL^2}{2EI} - \frac{M_0 L}{EI} \quad \text{(positive clockwise)} \quad \leftarrow$$

$$\delta_B = \frac{PL^3}{3EI} - \frac{M_0 L^2}{2EI} \quad \text{(positive downward)} \quad \leftarrow$$

(These results agree with Cases 4 and 6, Table G-1.)