Solution 10.3-2 **Fixed-end beam (uniform load)**

Select M_A as the redundant reaction.

REACTIONS (FROM SYMMETRY AND EQUILIBRIUM)

$$R_A = R_B = \frac{qL}{2} \qquad M_B = M_A$$

BENDING MOMENT (FROM EQUILIBRIUM)

$$M = R_A x - M_A - \frac{qx^2}{2} = -M_A + \frac{q}{2}(Lx - x^2) \qquad (1)$$

DIFFERENTIAL EQUATIONS

$$EIv'' = M = -M_A + \frac{q}{2}(Lx - x^2)$$

$$EIv' = -M_A x + \frac{q}{2}\left(\frac{Lx^2}{2} - \frac{x^3}{3}\right) + C_1 \qquad (2)$$

B.C. 1 $v'(0) = 0 \quad \therefore C_1 = 0$

$$EIv = -\frac{M_A x^2}{2} + \frac{q}{2}\left(\frac{Lx^3}{6} - \frac{x^4}{12}\right) + C_2 \qquad (3)$$

B.C. 2 $v(0) = 0 \quad \therefore C_2 = 0$

B.C. 3 $v(L) = 0 \quad \therefore M_A = \frac{qL^2}{12}$

REACTIONS

$$R_A = R_B = \frac{qL}{2} \qquad M_A = M_B = \frac{qL^2}{12} \quad \longleftarrow$$

SHEAR FORCE (FROM EQUILIBRIUM)

$$V = R_A - qx = \frac{q}{2}(L - 2x) \quad \longleftarrow$$

BENDING MOMENT (FROM EQ. 1)

$$M = -\frac{q}{12}(L^2 - 6Lx + 6x^2) \quad \longleftarrow$$

SLOPE (FROM EQ. 2)

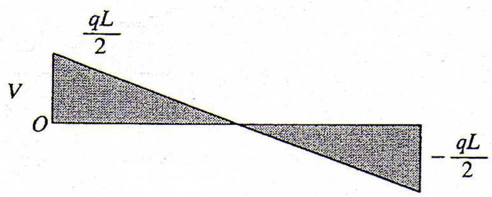
$$v' = -\frac{qx}{12EI}(L^2 - 3Lx + 2x^2) \quad \longleftarrow$$

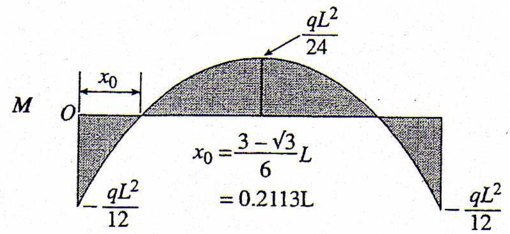
DEFLECTION (FROM EQ. 3)

$$v = -\frac{qx^2}{24EI}(L - x)^2 \quad \longleftarrow$$

$$\delta_{max} = -v\left(\frac{L}{2}\right) = \frac{qL^4}{384EI}$$

SHEAR-FORCE AND BENDING-MOMENT DIAGRAMS





Solution 10.3-5 Propped cantilever beam

Triangular load $q = q_0(L - x)/L$

DIFFERENTIAL EQUATIONS

$$EIv'''' = -q = -\frac{q_0}{L}(L - x) \quad (1)$$

$$EIv''' = V = -q_0 x + \frac{q_0 x^2}{2L} + C_1 \quad (2)$$

$$EIv'' = M = -\frac{q_0 x^2}{2} + \frac{q_0 x^3}{6L} + C_1 x + C_2 \quad (3)$$

$$EIv' = -\frac{q_0 x^3}{6} + \frac{q_0 x^4}{24L} + C_1\frac{x^2}{2} + C_2 x + C_3 \quad (4)$$

$$EIv = -\frac{q_0 x^4}{24} + \frac{q_0 x^5}{120L} + C_1\frac{x^3}{6} + C_2\frac{x^2}{2} + C_3 x + C_4 \quad (5)$$

B.C. 1 $v''(L) = 0$ $\quad \therefore C_1 L + C_2 = \frac{q_0 L^2}{3} \quad (6)$

B.C. 2 $v'(0) = 0$ $\quad \therefore C_3 = 0$

B.C. 3 $v(0) = 0$ $\quad \therefore C_4 = 0$

B.C. 4 $v(L) = 0$ $\quad \therefore C_1 L + 3C_2 = \frac{q_0 L^2}{5} \quad (7)$

Solve Eqs. (6) and (7):

$$C_1 = \frac{2q_0 L}{5} \qquad C_2 = -\frac{q_0 L^2}{15}$$

SHEAR FORCE (EQ. 2)

$$V = \frac{q_0}{10L}(4L^2 - 10Lx + 5x^2)$$

REACTIONS $\quad R_A = V(0) = \frac{2q_0 L}{5} \quad \leftarrow$

$$R_B = -V(L) = \frac{q_0 L}{10} \quad \leftarrow$$

From equilibrium:

$$M_A = \frac{q_0 L^2}{6} - R_B L = \frac{q_0 L^2}{15} \quad \leftarrow$$

DEFLECTION CURVE (FROM EQ. 5)

$$EIv = -\frac{q_0 x^4}{24} + \frac{q_0 x^5}{120L} + \frac{2q_0 L}{5}\left(\frac{x^3}{6}\right) - \frac{q_0 L^2}{15}\left(\frac{x^2}{2}\right)$$

or

$$v = -\frac{q_0 x^2}{120LEI}(4L^3 - 8L^2 x + 5Lx^2 - x^3) \quad \leftarrow$$

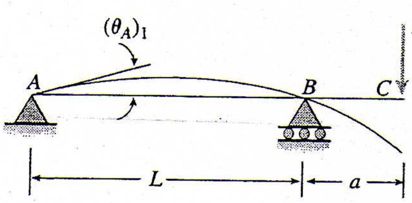
Solution 10.4-3 Beam with an overhang

Select M_A as redundant.

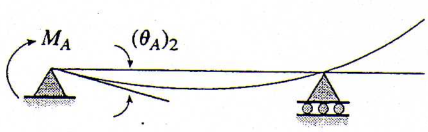
EQUILIBRIUM

$$R_A = \frac{1}{L}(M_A + Pa) \quad R_B = \frac{1}{L}(M_A + PL + Pa)$$

RELEASED STRUCTURE AND FORCE-DISPL. EQS.



$$(\theta_A)_1 = \frac{PaL}{6EI}$$



$$(\theta_A)_2 = \frac{M_A L}{3EI}$$

COMPATIBILITY $\quad \theta_A = (\theta_A)_1 - (\theta_A)_2 = 0$

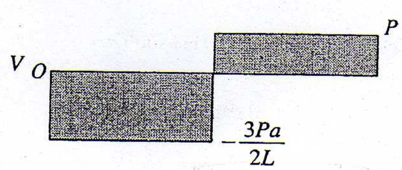
Substitute for $(\theta_A)_1$ and $(\theta_A)_2$ and solve for M_A:

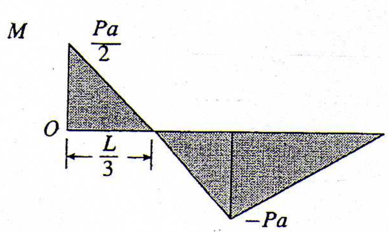
$$M_A = \frac{Pa}{2} \quad \leftarrow$$

OTHER REACTIONS (FROM EQUILIBRIUM)

$$R_A = \frac{3Pa}{2L} \qquad R_B = \frac{P}{2L}(2L + 3a) \quad \leftarrow$$

SHEAR-FORCE AND BENDING-MOMENT DIAGRAMS





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