第二章

布林代數與邏輯閘

2-1 基本定義

- ★封閉性
- ★結合律 $(x*y)*z = x*(y*z)$
- ★交換律 $x*y = y*x$
- ★單位元素 $e*x = x*e = x$,則e爲單位元素
- ★反元素 $x*y = e$
- ★分配律 $x*(y\cdot z) = (x*y)\cdot(x*z)$

2-2 布林代數的公理與定義

★1. (a)運算符號 + 具有封閉性。
(b)運算符號 ·具有封閉性。

★2. (a) + 具有單位元素0：$x+0=0+x=x$
(b) · 具有單位元素1：$x\cdot 1=1\cdot x=x$

★3. (a) + 具有交換律：$x+y=y+x$
(b) · 具有交換律：$x\cdot y=y\cdot x$

布林代數的公理與定義

★4. (a) ·對 + 具有分配性：
$x\cdot(y+z)=(x\cdot y)+(x\cdot z)$
(b) + 對 ·具有分配性：
$x+(y\cdot z)=(x+y)(x+z)$

★5.對任一元素 $x\in B$ ，則存在一元素 $x'\in B$ (稱爲 x 的補數)，使得(a). $x+x'=1$ 且 (b). $x\cdot x'=0$

★6. 至少存在兩個元素 $x,y\in B$ ，使得 $x\neq y$ 。

2-3 布林代數的基本定理與性質

★定理1 (a) $x+x=x$ (b) $x\cdot x=x$

★定理2 (a) $x+1=1$ (b) $x\cdot 0=0$

★定理3 自補定理 $(x')'=x$

★定理4 結合律

(a) $x+(y+z)=(x+y)+z$

(b) $x(yz)=(xy)z$

布林代數的基本定理與性質

★定理5 迪摩根定理

(a) $(x+y)'=x'y'$

(b) $(xy)'=x'+y'$

★定理6 消去定理

(a) $x+xy=x$

(b) $x(x+y)=x$

2-4 布林函數

- 布林函數----

  包含二元變數、常數值1和0以及邏輯運算符號的代數表示式。

- 布林函數之閘電路圖

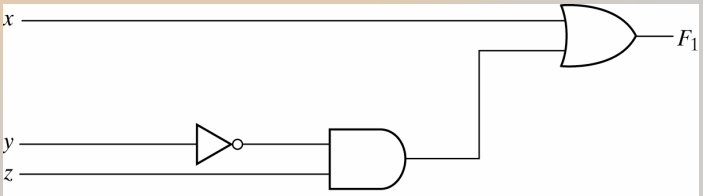
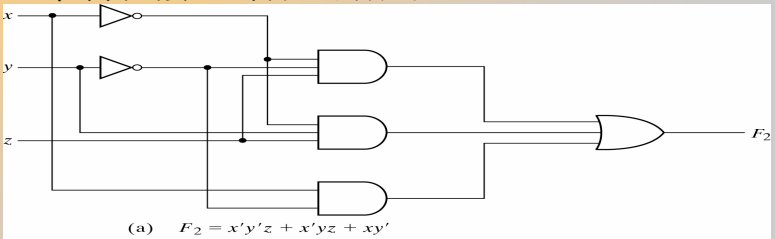


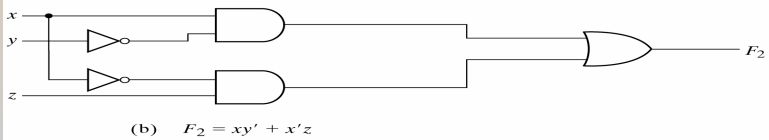
Fig. 2-1 Gate implementation of $F_1 = x + y'z$

布林函數

- 布林函數之閘電路圖



(a) $F_2 = x'y'z + x'yz + xy'$



(b) $F_2 = xy' + x'z$

Fig. 2-2 Implementation of Boolean function F_2 with gates

重合定理與迪摩根定理

- 重合定理
  1. $xy + x'z + yz = xy + x'z$
  2. $(x + y)(x' + z)(y + z) = (x + y)(x' + z)$
- 迪摩根定理
  1. $(A + B + C + ... + F)' = A'B'C'....F'$
  2. $(ABC....F)' = A' + B' + C' + ... + F'$

2-5 正規型式與標準型式

- 正規型式

  全及項(minterm)的和與全或項(maxterm)的積

- 標準型式

  積項和(Sum of Products)

  和項積(Product of Sums)

三個二元變數之全及項與全或項

★ 全及項(minterm)與全或項(maxterm)

			全及項		全或項	
x	y	z	項目	符號	項目	符號
0	0	0	x'y'z'	m_0	x+y+z	M_0
0	0	1	x'y'z	m_1	x+y+z'	M_1
0	1	0	x'yz'	m_2	x+y'+z	M_2
0	1	1	x'yz	m_3	x+y'+z'	M_3
1	0	0	xy'z'	m_4	x'+y+z	M_4
1	0	1	xy'z	m_5	x'+y+z'	M_5
1	1	0	xyz'	m_6	x'+y'+z	M_6
1	1	1	xyz	m_7	x'+y'+z'	M_7

正規型式的轉換

Ex:函數 $F(A,B,C)=\sum(1,4,5,6,7)$

則其補函數

$F'(A,B,C)=\sum(0,2,3)=m_0+m_2+m_3$

利用迪摩根定理求 F' 的補數

則 $F=(m_0+m_2+m_3)'=m_0'\cdot m_2'\cdot m_3'$

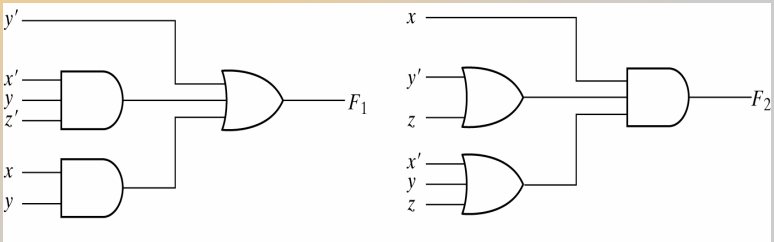
$=M_0\cdot M_2\cdot M_3=\prod(0,2,3)$

故 $m_j'=M_j$

標準型式

★積項和與和項積

$F_1 = y' + xy + x'yz'$ ， $F = x(y' + z)(x' + y + z')$



(a) Sum of Products (b) Product of Sums

Fig. 2-3 Two-level implementation

2-6 其他邏輯運算

★兩個變數所形成之16種布林函數

1.零函數 $F_0 = 0$

2.AND(及) $F_1 = xy$

3.Inhibition(禁止) $F_2 = xy'$

4.Transfer(轉移) $F_3 = x$

5.Inhibition(禁止) $F_4 = x'y$

6.Transfer(轉移) $F_5 = y$

7. Exclusive-OR(互斥-或) $F_6 = xy' + x'y$

8. OR(或) $F_7 = x + y$

其他邏輯運算

9.NOR(反或) $F_8 = (x + y)'$

10.Equivalence(全等) $F_9 = xy + x'y'$

11.Complement(補數) $F_{10} = y'$

12.Implication(意含) $F_{11} = x + y'$

13.Complement(補數) $F_{12} = x'$

14.Implication(意含) $F_{13} = x' + y$

15.NAND(反及) $F_{14} = (xy)'$

16.Identity(單位函數) $F_{15} = 1$

2-7 數位邏輯閘

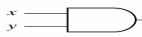
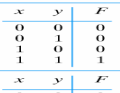
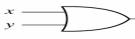
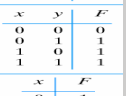
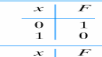
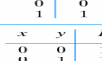
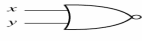
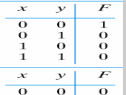
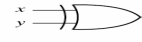
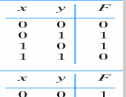
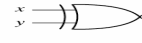
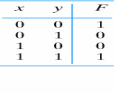
Name	Graphic symbol	Algebraic function	Truth table
AND		$F = xy$	
OR		$F = x + y$	
Inverter		$F = x'$	
Buffer		$F = x$	
NAND		$F = (xy)'$	
NOR		$F = (x + y)'$	
Exclusive-OR (XOR)		$F = xy' + x'y$ $= x \oplus y$	
Exclusive-NOR or equivalence		$F = xy + x'y'$ $= (x \oplus y)'$	

Fig. 2-5 Digital logic gates

正邏輯與負邏輯

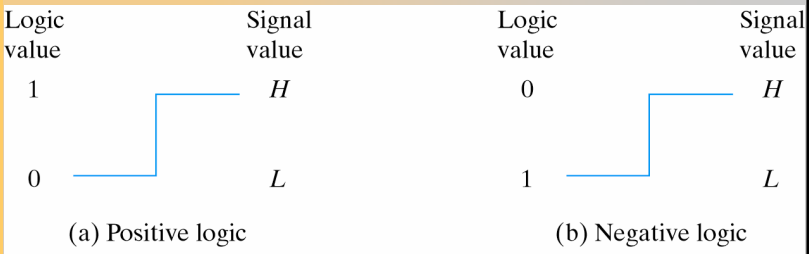


Fig. 2-9 signal assignment and logic polarity

正邏輯與負邏輯

x	y	F
L	L	L
L	H	L
H	L	L
H	H	H

(a) Truth table with H and L

(b) Gate block diagram

x	y	z
0	0	0
0	1	0
1	0	0
1	1	1

(c) Truth table for positive logic

(d) Positive logic AND gate

x	y	z
1	1	1
1	0	1
0	1	1
0	0	0

(e) Truth table for negative logic

(f) Negative logic OR gate

Fig. 2-10 Demonstration of positive and negative logic

2-8 積體電路

- 小型積體元件(SSI)
- 中型積體元件(MSI)
- 大型積體元件(LSI)
- 超大型積體元件(VLSI)

數位邏輯家族

- TTL　電晶體-電晶體邏輯
- ECL　射極-耦合邏輯
- MOS　金屬氧化半導體
- CMOS　互補式金屬氧化半導體

數位邏輯家族

- 扇出數
- 扇入數
- 功率消耗
- 傳播延遲
- 雜訊邊限